Computer Analysis and Design of Foundations

Volume III

Reinforced concrete design

Determination of
the contact pressures, settlements, moments
and shear forces of slab foundations by the
method of finite elements

Windows XP/Vista/7/8 - Version 9.3

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Reinforced Concrete Design

1 Introduction

The following chapter gives an overview of the concrete design codes used by the program ELPLA. The program designs reinforced concrete slabs according to one of the following design codes:

EC 2: The European Committee for Standardization, Design of Concrete Structures [3].

DIN 1045: German Institute for Standardization, Design and Construction of Reinforced Concrete [2].

ACI : The American Concrete Institute, Building Code Requirements for Structural Concrete [1].

ECP: The Egyptian Code of Practice, Design and Construction of Reinforced Concrete Structures [4].

2 Concrete properties

Codes usually classify the concrete in different grades according to the value of the maximum compressive strength of standard cylinders or cubes. The following text gives brief information about the concrete properties according to the design codes available in ELPLA.

2.1 Mechanical properties of concrete according to EC 2

EC 2 defines the concrete grade according to both the standard cylinder compressive strength of concrete f_{ck} (characteristic strength) and the standard cube compressive strength of concrete $f_{ck,\, cube}$. The standard cylinder has 15 [cm] diameter and 30 [cm] height while the standard cube has 15 [cm] side length. Thus, grade C 20/25 concrete has f_{ck} = 20 [MN/m^2] and $f_{ck,\, cube}$ = 25 [MN/m^2]. For similar mixes of concrete, the cylinder strength varies from about 70% to 80% of the cube strength.

According to EC 2 the mechanical properties of concrete, characteristic strength f_{ck}, concrete cube strength $f_{ck,\, cube}$, Young's modulus E_{cm}, and main values of shear strength τ_{Rd} can be taken as in Table (1).

The Young's modulus E_{cm} [kN/mm²] for concrete can be calculated from Equation (1).

$$E_{cm} = 9.5\ (f_{ck} + 8)^{1/3} \tag{1}$$

Where f_{ck} in [MN/m²].

Table (1) Mechanical properties of concrete according to EC 2

Concrete grade	C 12/15	C 16/20	C 20/25	C 25/30	C 30/37	C 35/45	C 40/50	C 45/55	C 50/60
f_{ck} [MN/m²]	12	16	20	25	30	35	40	45	50
$f_{ck,\ cube}$ [MN/m²]	15	20	25	30	37	45	50	55	60
E_{cm} [MN/m²]	26000	27500	29000	30500	32000	33500	35000	36000	37000
τ_{Rd} [MN/m²]	0.2	0.22	0.24	0.26	0.28	0.3	0.31	0.32	0.33

2.2 Mechanical properties of concrete according to DIN 1045

DIN 1045 defines the concrete grade according to the standard cube compressive strength of concrete β_{WN} (nominal strength). The standard cube has 20 cm side length. Thus, grade B 25 concrete has β_{WN} = 25 [MN/m²].

According to DIN 1045 the mechanical properties of concrete, nominal strength β_{WN}, compressive strength β_R, main value of shear strength τ_{011}, Young's modulus E and shear modulus G can be taken as in Table (2).

To convert the concrete grade from B-classification of DIN 1045 to C-classification of EC 2, the following relation is used.

$$f_{ck,\ cube} = 0.97\ \beta_{WN} \tag{2}$$

Table (2) Mechanical properties of concrete according to DIN 1045

Concrete grade	B 5	B 10	B 15	B 25	B 35	B 45	B 55
β_{WN} [MN/m^2]	5	10	15	25	35	45	55
β_R [MN/m^2]	3.5	7	10.5	17.5	23	27	30
τ_{011} [MN/m^2]	-	-	0.35	0.5	0.6	0.7	0.8
E [MN/m^2]	-	22000	26000	30000	34000	37000	39000
G [MN/m^2]	-	-	-	13000	14000	15000	16000

2.3 Mechanical properties of concrete according to ACI

ACI defines the concrete grade according to the standard cylinder compressive strength of concrete f'_c (specified strength). The standard cylinder has 15 cm diameter and 30 cm height. Thus, grade C 3500 (or C 25) concrete has f'_c = 3500 psi (f'_c = 25 [MN/m^2]).

According to ACI the mechanical properties of concrete, specified strength f'_c and Young's modulus E_c can be taken as in Table (3).

The Young's modulus E_c [MN/m^2] for normal weight concrete can be calculated from Equation (3):

$$E_c = 4730 \sqrt{f'_c} \quad (3)$$

Where f'_c in [MN/m^2].

Table (3) Mechanical properties of concrete according to ACI.

Concrete grade C	1000 (7)	2000 (14)	3000 (21)	3500 (25)	4000 (28)	5000 (35)	6000 (42)
f'_c [psi]	1000	2000	3000	3500	4000	5000	6000
f'_c [MN/m^2]	7	14	21	25	28	35	42
E_c [MN/m^2]	13000	18000	22000	24000	25000	28000	31000

2.4 Mechanical properties of concrete according to ECP

ECP defines the concrete grade according to the standard cube compressive strength of concrete f_{cu} (characteristic strength). The standard cube has 15 cm side length. Thus, grade C 250 concrete has f_{cu} = 250 [kg/cm^2].

According to ECP the mechanical properties of concrete, characteristic strength f_{cu}, concrete cylinder strength f'_c, compressive stress of concrete for bending or compression with big eccentricity f_c, and main value of punching shear strength q_{cp} can be taken as in Table (4).
The Young's modulus E_c [kg/cm^2] for concrete can be calculated from Equation (4), in which the part of reinforcement is left out of consideration.

$$E_c = 14000 \sqrt{f_{cu}} \qquad (4)$$

Where f_{cu} in [kg/cm^2].

Table (4) Mechanical properties of concrete according to ECP

Concrete grade	C 150	C 175	C 200	C 225	C 250	C 275	C 300
f_{cu} [kg/cm^2]	150	175	200	225	250	275	300
$f'_c = 0.8\ f_{cu}$ [kg/cm^2]	120	140	160	180	200	220	250
f_c [kg/cm^2]	65	70	80	90	95	100	105
q_{cp} [kg/cm^2]	7	7	8	8	9	9	10
E_c [kg/cm^2]	$17*10^4$	$19*10^4$	$20*10^4$	$21*10^4$	$22*10^4$	$23*10^4$	$24*10^4$

To convert to [MN/m^2], divide by 10

2.5 Poisson's ratio

Poisson's ratio in general ranges from 0.15 to 0.30 for concrete, and an average value of ν_c = 0.20 may be taken for elastic analysis.

2.6 Shear modulus

The relation between the shear modulus G_c, Young's modulus E_c and Poisson's ratio ν_c is defined in the following equation:

$$G_c = \frac{E_c}{2\ (1 + \nu_c)} \qquad (5)$$

2.7 Coefficient of thermal expansion of concrete

The coefficient of thermal expansion of concrete α_t is varying within the limits of $5*10^{-6}$ and $13*10^{-6}$ depending on the type of concrete. An average value of $\alpha_t = 10*10^{-6}$ is usually used.

2.8 Unit weight of concrete

The unit weight of plain concrete γ_{pc} is usually taken 22 [kN/m^3], while the unit weight of reinforced concrete γ_{rc} is usually taken 25 [kN/m^3]. The unit weight is used in computing the own weight of concrete elements.

3 Properties of steel reinforcement

3.1 Steel reinforcement according to EC 2

EC 2 classifies steel reinforcement into grades corresponding to its strength. Thus, grade BSt 500 steel refers to a steel having a characteristic tensile yield strength of 500 [MN/m^2].

Table (5) gives the characteristic tensile yield strength f_{yk} and the design tensile yield strength f_{yd} according to EC 2.

Table (5) Mechanical properties of steel reinforcement according to EC 2

Steel grade	BSt 220	BSt 420	BSt 500	BSt 550	BSt 600
f_{yk} [MN/m^2]	220	420	500	550	600
f_{yd} [MN/m^2]	191	365	435	478	522

3.2 Steel reinforcement according to DIN 1045

DIN 1045 classifies steel reinforcement into grades corresponding to its strength. Thus, grade BSt 500 steel refers to a steel having a characteristic tensile yield strength of $\beta_S = 500$ [MN/m^2].

Table (6) gives the yield strength β_S and the factor α_S for obtaining the punching shear strength of reinforced concrete according to DIN 1045.

Table (6) Mechanical properties of steel reinforcement according to DIN 1045

Steel grade	BSt 220	BSt 420	BSt 500
β_S [MN/m^2]	220	420	500
α_S	1	1.3	1.4

3.3 Steel reinforcement according to ACI

ACI classifies steel reinforcement into grades corresponding to its strength. Thus, grade 280 steel refers to a steel having a minimum specified yield stress of 280 [MN/m^2].

Table (7) gives the yield stress f_y and ultimate stress f_u of the most common grades of steel used in reinforced concrete structures.

Table (7) Mechanical properties of steel reinforcement according to ACI

Steel grade	S 240	S 280	S 350	S 420
Yield stress f_y [MN/m^2]	240	280	350	420
Ultimate stress f_u [MN/m^2]	360	500	560	630

3.4 Steel reinforcement according to ECP

ECP classifies steel reinforcement into grades corresponding to its strength. Thus, grade S 36/52 steel refers to a steel having yield stress of f_y = 36 [kg/mm^2] and ultimate stress of f_u = 52 [kg/mm^2]. Table (8) gives the yield stress f_y, ultimate stress f_u and working stress f_s of the grades of steel used in reinforced concrete structures according to ECP.

Table (8) Mechanical properties of steel reinforcement according to ECP

Steel grade	Mild steel		High tensile steel	
	S 24/35	S 28/45	S 36/52	S 40/60
Yield stress f_y [kg/cm^2]	2400	2800	3600	4000
Ultimate stress f_u [kg/cm^2]	3500	4500	5200	6000
Working stress f_s [kg/cm^2]	1400	1600	2000	2200

To convert to [MN/m^2], divide by 10

4 Section properties

Figure (1) shows an example for section reinforcement parallel to x-direction. Refereeing to this figure, the default values of section geometries used in the program ELPLA are:

Overall slab thickness	d = 1.0	[m]
Top concrete cover +1/2 bar diameter in x-direction	d_{1x} = 5	[cm]
Bottom concrete cover +1/2 bar diameter in x-direction	d_{2x} = 5	[cm]
Top concrete cover +1/2 bar diameter in y-direction	d_{1y} = 6	[cm]
Bottom concrete cover +1/2 bar diameter in y-direction	d_{2y} = 6	[cm]

The program calculates area of reinforcement steel per meter required for the section as:

Bottom steel parallel to the x-axis	$A_{s,botx}$ [cm²/m]
Top steel parallel to the x-axis	$A_{s,topx}$ [cm²/m]
Bottom steel parallel to the y-axis	$A_{s,boty}$ [cm²/m]
Top steel parallel to the y-axis	$A_{s,topy}$ [cm²/m]

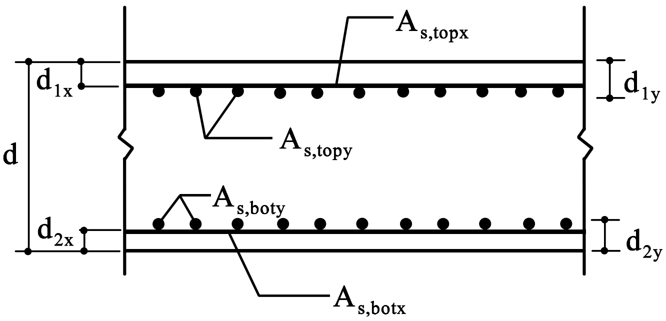

Figure (1) Section geometry and reinforcement parallel to x-direction

4.1 Section thickness

In spit of the slab thickness is defined by element thickness, ELPLA gives all results at nodes. To obtain the reinforcement at nodes, ELPLA determines node thickness corresponding to element thicknesses around it. Figure (2) and Equation (6) show an example to determine the design thickness d for reentered corner node by ELPLA.

$$d = \frac{1}{n} \sum_{i=1}^{n} d_i \tag{6}$$

Where

d	Design thickness at reentered corner node k
d_i	Thickness of the element i around node k
n	Number of elements around node k

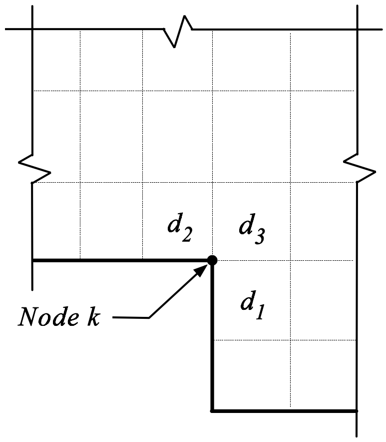

Figure (2) Elements with variable thickness around node k

5 Factored moments

The design load combinations are the various combinations of the prescribed load cases for which the structure needs to be designed. ELPLA considers only one default load factor γ for both dead and live loads. This load factor is used to determine the factored bending moment required to calculate the reinforcement. The factored moment is obtained by factoring the moment by the load factor γ. To consider a set of load combinations for different cases of loadings the user must define the loads multiplied by these load combination factors.
The slab section is then designed for factored moments as a rectangular section. Positive slab moments produce bottom steel while negative slab moments produce top steel.

Table (9) shows the load factors for both dead and live loads according to EC 2, ACI and ECP (limit state method) codes. For both DIN 1045 and ECP (working stress method) codes, the design loads are considered equal to working loads.

Table (9) Load factors according to EC 2, ACI and ECP codes

Design code	load factor for dead load γ_G	Load factor for live load γ_Q
EC 2	1.35	1.5
ACI	1.4	1.7
ECP (limit state method)	1.4	1.6

6 Minimum reinforcement

The minimum areas of steel required for tension and compression reinforcement can be defined in the program ELPLA by the user according to one of the following:

Minimum area of steel in tension per meter, min A_{st} [cm^2], is:

- min $A_{st} = \rho_t * A_c$
- min A_{st} = a certain area of steel
- min A_{st} = maximum value from $\rho_t * A_c$ and the certain area of steel that is defined by the user.

Minimum area of steel in compression per meter, min A_{sc} [cm^2], is:

- min $A_{sc} = \rho_{c1} * A_c$
- min $A_{sc} = \rho_{c2} * A_{st}$
- min A_{sc} = a certain area of steel
- min A_{sc} = maximum value from $\rho_c * A_c$, $\rho_c * A_{st}$ and the certain area of steel that is defined by the user.

where:

ρ_t Steel ratio in tension from area of concrete section [%], 0.15% by default
ρ_{c1} Steel ratio in compression from area of concrete section [%], 0.15% by default
ρ_{c2} Steel ratio in compression from area of steel in tension [%], 20% by default
A_c Area of concrete slab section [cm^2], A_c = d [cm]*100 [cm]
A_{st} Area of tension reinforcement at the section [cm^2]

7 Design scope

As the main stresses in slabs are due to the flexure moments, ELPLA determines the required areas of steel to resist flexure moments only. In such case, reinforcement is calculated using the normal code formulae. Effects due to punching, torsion, shear or any other stress that may exist in the section must be investigated independently by the user.
In the flexural reinforcement design process, the program calculates both the tension and compression reinforcement. Compression reinforcement is added when the applied design moment exceeds the maximum moment capacity of a singly reinforced section. The user has the option of avoiding the compression reinforcement by increasing the effective depth or the grade of concrete.

- The design procedure for different design codes supported by ELPLA is summarized below. The design code symbols are used as far as possible.

8 Design for EC 2

8.1 Design for flexure moment

The design procedure is based on the simplified rectangular stress block shown in Figure (3). The code places a limitation on the neutral axis depth, to safeguard against nonductile failures. When the applied moment exceeds the moment capacity at the is designed balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

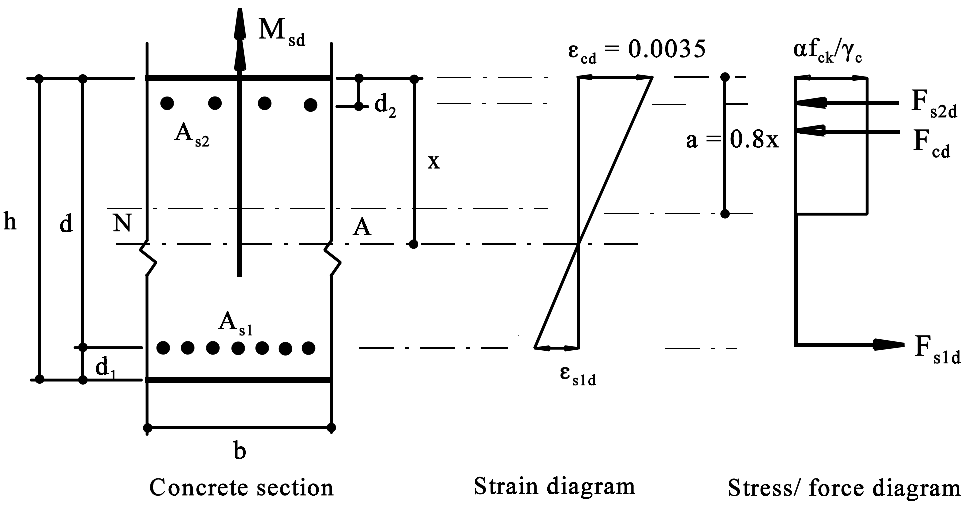

Figure (3) Distribution of stresses and forces according to EC 2

In designing procedure, the normalized moment μ_{sd} and the normalized section capacity $\mu_{sd, lim}$ as a singly reinforced section are obtained first. The reinforcing steel area is determined based on whether μ_{sd} greater than, less than, or equal to $\mu_{sd, lim}$.

The normalized design moment μ_{sd} is given by:

$$\mu_{sd} = \frac{M_{sd}}{bd^2(\alpha f_{cd})} \tag{7}$$

where:

M_{sd} Factored moment [MN.m], $M_{sd} = \gamma M$
M Moment at a section obtained from analysis [MN.m]
γ Total load factor for both dead and live loads, 1.5 by default
b Width of the section to be designed [m], b = 1.0 [m]
d Distance from compression face to tension reinforcement [m]
f_{cd} Design concrete compressive strength [MN/m^2], $f_{cd} = f_{ck}/\gamma_c$
f_{ck} Characteristic compressive cylinder strength of concrete [MN/m^2]

γ_c Partial safety factor for concrete strength, 1.5 by default
α Concrete strength reduction factor for sustained loading, 0.85 by default

The normalized concrete moment capacity $\mu_{sd,\,lim}$ as a singly reinforced section is given by:

$$\mu_{sd,\,\lim} = \alpha_R\, \xi_{\lim}\, (1 - \frac{\alpha_R}{2}\, \xi_{\lim}) \tag{8}$$

where:
ξ_{lim} The limiting value of the ratio x/d
$\xi_{lim} = 0.45$ for concrete grade $\le$ C 40/50, $\xi_{lim} = 0.35$ for concrete grade > C 35/45
x The neutral axis depth [m]
α_R Factor for obtaining depth of compression block, 0.8 by default

- Check if the normalized moment μ_{sd} is not exceeded the normalized section capacity $\mu_{sd,\,lim}$.

Singly reinforced section

If $\mu_{sd} \le \mu_{sd,\,lim}$, then the section is designed as singly reinforced section. The tension reinforcement is calculated as follows:

The normalized steel ratio ω is given by:

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}} \tag{9}$$

The area of tensile steel reinforcement A_{s1} [m^2] is then given by:

$$A_{s1} = \omega \left(\frac{(\alpha f_{cd})\ b\ d}{f_{yd}} \right) \tag{10}$$

where:
f_{yd} Design tensile yield strength of reinforcing steel [MN/m^2], $f_{yd} = f_{yk}/\gamma_s$
f_{yk} Characteristic tensile yield strength of reinforcement [MN/m^2]
γ_s Partial safety factor for steel strength, 1.15 by default

Doubly reinforced section

If $\mu_{sd} > \mu_{sd,\,lim}$, then the section is designed as doubly reinforced section. Both top and bottom reinforcement are required. The compression and tension reinforcement are calculated as follows:

The limiting moment resisted by concrete compression and tensile steel $M_{sd,\,lim}$ [MN.m] as a singly reinforced section is given by:

$$M_{sd,\,\lim} = M_{sd}\,\frac{\mu_{sd,\,\lim}}{\mu_{sd}} \tag{11}$$

Therefore the moment resisted by compression steel and additional tensile steel ΔM [MN.m] is given by:

$$\Delta M = M_{sd} - M_{sd,\,\lim} \tag{12}$$

If the steel stress in compression is assumed to be reached to yield stress, then the required steel $A_{s2} = \Delta A_{s1}$ [m^2] to resist ΔM in tension and compression is given by

$$A_{s2} = \Delta A_{s1} = \frac{\Delta M}{f_{yd}\,(d - d_2)} \tag{13}$$

where

d_2 Concrete cover to center of compression reinforcing [m]

The normalized limiting tensile steel ratio ω required to resist $M_{sd,\,lim}$ is given by:

$$\omega_{\lim} = \alpha_R\,\xi_{\lim} \tag{14}$$

The required tensile reinforcement A_{s1} [m^2] to resist $M_{sd,\,lim} + \Delta M$ is given by:

$$A_{s1} = \omega_{\lim}\left(\frac{(\alpha f_{cd})\,b\,d}{f_{yd}}\right) + \Delta A_{s1} \tag{15}$$

8.2 Check for punching shear

The EC 2 code assumes the critical section for punching shear is at a distance r = 1.5 d around the circumference of the column as shown in Figure (4).

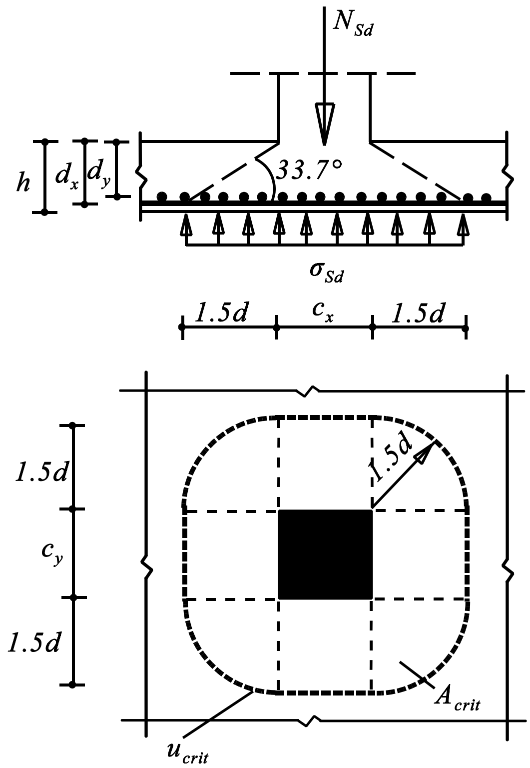

Figure (4) Critical section for punching shear according to EC 2

The punching force at ultimate design load V_{Sd} [MN] is given by:

$$V_{Sd} = N_{Sd} - \sigma_{Sd} A_{crit} \tag{16}$$

where:

N_{Sd} Factored column load [MN]

σ_{Sd} Factored upward soil pressure under the column [MN/m^2]

A_{crit} Area of critical punching shear section [m^2]

$= c_x^2 + 4\, r\, c_x + \pi\, r^2$ for square columns

$= c_x\, c_y + 2\, r\, (c_x + c_y) + \pi\, r^2$ for rectangular columns, $c_x \leq 2\, c_y$ and $2\,(c_x + c_y) \leq 11\, d$

$= \pi\, (D_c + 3\, r)^2 / 4$ for circular columns, $r \leq 3.5\, d$

The design value of the applied shear ν_{Sd} [MN/m] is given by:

$$\nu_{Sd} = \frac{V_{Sd}\ \beta}{u_{crit}} \tag{17}$$

where:

u_{crit} Perimeter of critical punching shear section [m]
$= 4\ c_x + 2\ \pi\ r$ for square columns
$= 2\ (c_x + c_x) + 2\ \pi\ r$ for rectangular columns, $c_x \leq 2\ c_y$ and $2\ (c_x + c_y) \leq 11\ d$
$= \pi\ (D_c + 3\ r)$ for circular columns, $r \leq 3.5\ d$

β Correction factor to consider the irregular shear distribution around the circumference of the column
$\beta = 1.0$ if no eccentricity is expected
For irregular foundation β may be taken as:
$\beta = 1.15$ for interior columns
$\beta = 1.4$ for edge columns
$\beta = 1.5$ for corner columns

Normally, it is impracticable to provide shear reinforcement in slabs and footings. In such cases, concrete alone should resist the punching shear without contribution of the shear reinforcement.

Design shear resistance from concrete alone ν_{Rd1} [MN/m] is given by:

$$\nu_{Rd1} = \tau_{Rd}\ k\ (1.2 + 40\ \rho_1)\ d \tag{18}$$

where:

τ_{Rd} The main value of shear strength [MN/m^2] according to Table (1). The value may be multiplied by 1.2

k Coefficient for consideration of the slab thickness [m], $k = (1.6 - d\ [m]) \geq 1.0$

ρ_1 Steel ratio ranges from $\rho_1 \leq 1.5\%$
$\rho_1 \geq 0.5\%$ (only for foundation with h < 50 cm), $\rho_1 = \sqrt{\rho_{1x}\ \rho_{1y}}$

ρ_{1x} Steel ratio in x-direction [%], $\rho_{1x} = A_{sx} / (b_y\ d_x)$

ρ_{1y} Steel ratio in y-direction [%], $\rho_{1y} = A_{sy} / (b_x\ d_y)$

d Average depth to resist punching shear [m], $d = (d_x + d_x)/2$

d_x Depth to resist punching shear in x-direction [m]

d_y Depth to resist punching shear in x-direction [m]

b_x Width of the section in x-direction [m], $b_x = c_x + 2\ r$

b_y Width of the section in y-direction [m], $b_y = c_y + 2\ r$

A_{sx} Reinforcement in x-direction [m^2]

A_{sy} Reinforcement in y-direction [m^2]

d_x Effective section thickness in x-direction [m]

d_y Effective section thickness in y-direction [m]

- Check if the design value of the applied shear ν_{sd} is not exceeded the concrete shear capacity ν_{Rd1}.

If ν_{sd} # ν_{Rd1}, then the concrete shear capacity is enough to resist the punching stress.

If $\nu_{sd} > \nu_{Rd1}$, then the section is not enough to resist the punching stress. The thickness will have to be increased to resist the punching shear.

9 Design for DIN 1045

9.1 Design for flexure moment

The design procedure is based on the simplified rectangular stress block shown in Figure (5). The code places a limitation on the neutral axis depth, to safeguard against nonductile failures. When the applied moment exceeds the moment capacity at the designed balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

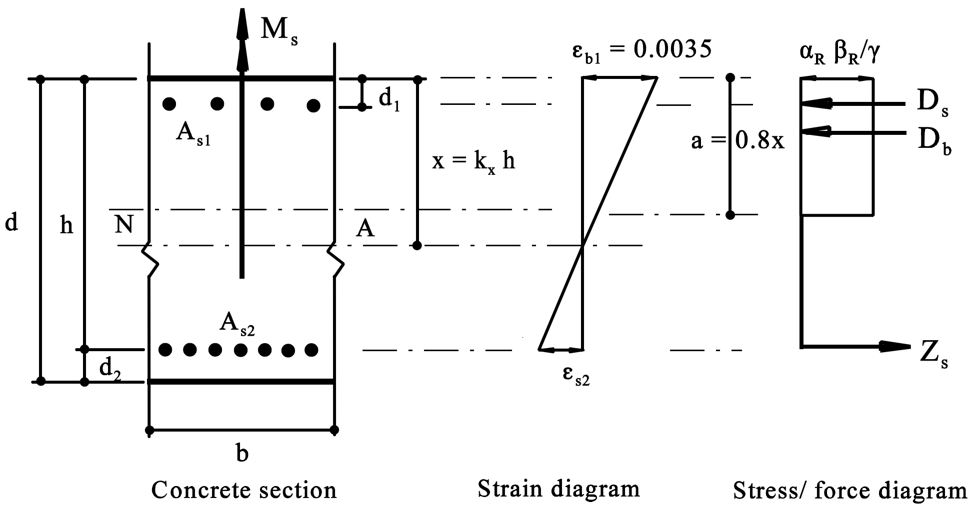

Figure (5) Distribution of stresses and forces according to DIN 1045

In designing procedure, the normalized moment m_s and the normalized section capacity m^*_s as a singly reinforced section are obtained first. The reinforcing steel area is determined based on whether m_s greater than, less than, or equal to m^*_s.

The normalized design moment m_s is given by:

$$m_s = \frac{M_s}{bh^2\left(\frac{\alpha_R \beta_R}{\gamma}\right)} \qquad (19)$$

where:

M_s	Moment at a section obtained from analysis [MN.m]
γ	Saftey factor, 1.75 by default
b	Width of the section to be designed [m], b = 1.0 [m]
h	Distance from compression face to tension reinforcement [m]
β_R	Concrete compressive strength [MN/m^2]
α_R	Concrete strength reduction factor for sustained loading, 0.95 by default

The limiting value of the ratio $k_x = x/h$ of neutral axis to effective depth is given by:

$$k_x = \left(\frac{\varepsilon_{b1}}{\varepsilon_{b1} - \varepsilon_{s2}} \right) \tag{20}$$

where:

ε_{b1} Max. strain in concrete, $\varepsilon_{b1} = 0.0035$
ε_{s2} Max. strain in steel, $\varepsilon_{s2} = -0.003$

The normalized concrete moment capacity m^*_s as a singly reinforced section is given by

$$m^*_s = \chi\, k_x\, (1 - \frac{\chi}{2}\, k_x) \tag{21}$$

where
x The neutral axis depth [m]
χ Factor for obtaining depth of compression block, 0.8 by default

- Check if the normalized moment m_s is not exceeded the normalized section capacity m^*_s.

Singly reinforced section

If $m_s \le m^*_s$, then the section is designed as singly reinforced section. The tension reinforcement is calculated as follows:

The normalized steel ratio ω_M is given by:

$$\omega_M = 1 - \sqrt{1 - 2\, m_s} \tag{22}$$

The area of tensile steel reinforcement A_{s2} [m^2] is then given by:

$$A_{s2} = \omega_M \left(\frac{(\alpha_R \beta_R)\; b\; h}{\beta_S} \right) \tag{23}$$

where:
β_S Tensile yield strength of steel [MN/m^2]

Doubly reinforced section

If $m_s > m^*_s$, then the section is designed as doubly reinforced section. Both top and bottom reinforcement are required. The compression and tension reinforcement are calculated as follows:

The limiting moment resisted by concrete compression and tensile steel M^*_s [MN.m] as a singly reinforced section is given by:

$$M^*_s = M_s \frac{m^*_s}{m_s} \tag{24}$$

Therefore the moment resisted by compression steel and additional tensile steel ΔM_s [MN.m] is given by:

$$\Delta M_s = M_s - M^*_s \tag{25}$$

If the steel stress in compression is assumed to be reached to yield stress, then the required steel $A_{s1} = \Delta A_{s2}$ [m^2] to resist ΔM in tension and compression is given by:

$$A_{s1} = \Delta A_{s2} = \frac{\Delta M_s}{\frac{\beta_S}{\gamma} (h - d_1)} \tag{26}$$

where:

d_1 Concrete cover to center of compression reinforcing [m]

The normalized limiting tensile steel ratio ω^*_M required to resist M^*_s is given by:

$$\omega^*_M = \chi \, k_x \tag{27}$$

The required tensile reinforcement A_{s2} [m^2] to resist $M^*_s + \Delta M_s$ is given by:

$$A_{s2} = \omega^*_M \left(\frac{(\alpha_R \beta_R) \, b \, h}{\beta_S} \right) + \Delta A_{s2} \tag{28}$$

9.2 Check for punching shear

The DIN 1045 code assumes the critical section for punching shear is a circle of diameter d_r around the circumference of the column as shown in Figure (6).

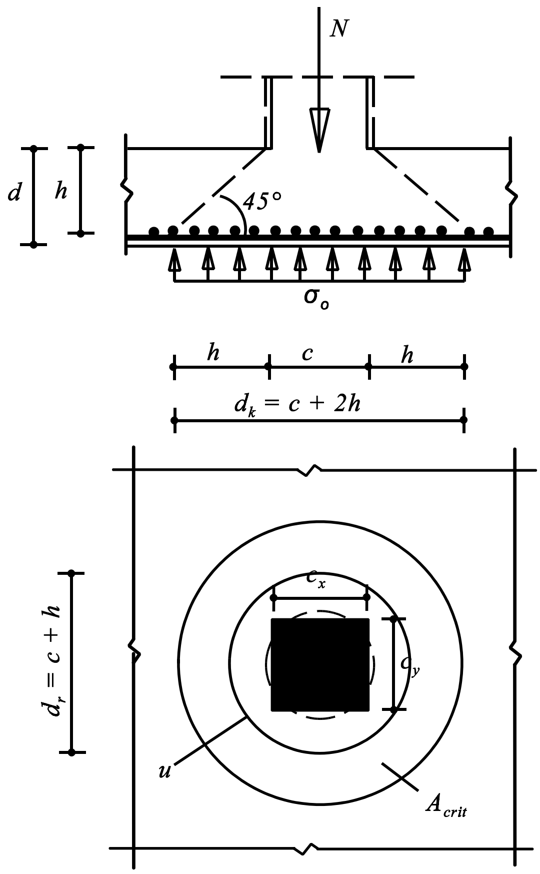

Figure (6) Critical section for punching shear according to DIN 1045

The punching shear force at the section Q_r [MN] is given by:

$$Q_r = N - \sigma_o A_{crit} \qquad (29)$$

where:

c_x	Column side in x-direction [m]
c_y	Column side in y-direction [m]
c	Equivalent diameter to the column size, $c = 1.13 \sqrt{(c_x c_y)}$
h	Depth to resist punching shear [m]
d_k	Diameter of loaded area [m], $d_k = 2h + c$
A_{crit}	Area of critical punching shear section [m^2], $A_{crit} = \pi d_k^2/4$
σ_0	Soil pressure under the column [MN/m^2]
N	Column load [MN]

The punching shear stress τ_r [MN/m^2] is given by:

$$\tau_r = \frac{Q_r}{u\ h} \tag{30}$$

where:

u	Perimeter of critical punching shear section [m], $u = \pi\ d_r$
d_r	Diameter of critical punching shear section [m], $d_r = c+h$

The allowable concrete punching strength τ_{r1} [MN/m^2] is given by:

$$\tau_{r1} = \kappa_1\ \tau_{011} \tag{31}$$

Where:

τ_{011}	The main value of shear strength [MN/m^2] according to Table (2)
α_s	Factor depending on steel grade according to Table (6)
A_{sx}	Reinforcement in x-direction [cm^2/m]
A_{sy}	Reinforcement in y-direction [cm^2/m]
μ_g	Reinforcement grade [%] < 1.5%, $\mu_g = (A_{sx}+A_{sy})/(2h)$
κ_1	Coefficient for consideration of reinforcement, $\kappa_1 = 1.3\ \alpha_s \sqrt{\mu_g}$

Normally, it is impracticable to provide shear reinforcement in slabs and footings. In such cases, concrete alone should resist the punching shear without contribution of the shear reinforcement. The slab thickness is considered to be safe for punching stress, if the punching shear stress is less than the allowable concrete punching strength where:

$$\tau_r \leq \tau_{r1} \tag{32}$$

If the above basic condition is not satisfied, the thickness will have to be increased to resist the punching shear.

10 Design for ACI

10.1 Design for flexure moment

The design procedure is based on the simplified rectangular stress block as shown in Figure (7). The code assumes that the compression carried by concrete is less than 0.75 times that can be carried at the balanced condition.When the applied moment exceeds the moment capacity at the designed balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

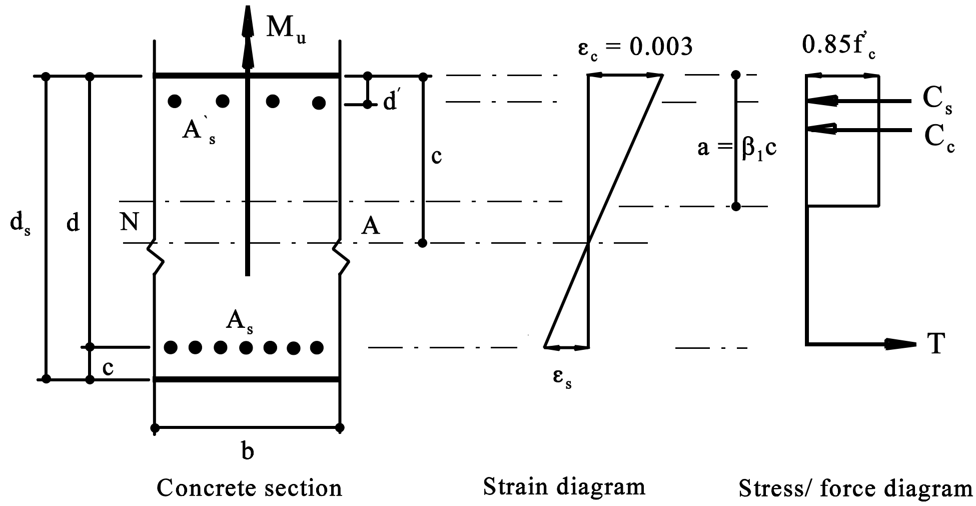

Figure (7) Distribution of stresses and forces according to ACI

In designing procedure, the depth of the compression block a and the maximum allowed depth of compression block a_{max} as a singly reinforced section are obtained first. The reinforcing steel area is determined based on whether a greater than, less than, or equal to a_{max}.

The depth of the compression block a [m] is given by:

$$a = d - \sqrt{d^2 - \frac{2\,|M_u|}{(\alpha f'_c)\;\varphi\;b}} \qquad (33)$$

where:

M_u Factored moment [MN.m], $M_u = \gamma M$
M Moment at a section obtained from analysis [MN.m]
γ Total load factor for both dead and live loads, 1.5 by default
b Width of the section to be designed [m], b = 1.0 [m]

d Distance from compression face to tension reinforcement [m]
f'_c Specified compressive strength of concrete [MN/m^2]
φ Strength reduction factor, 0.9 by default
α Concrete strength reduction factor for sustained loading, 0.85 by default

The factor for obtaining depth of compression block in concrete β_1 is given by:

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 28}{7}\right), \quad 0.65 \le \beta_1 \le 0.85 \tag{34}$$

The depth of neutral axis at balanced condition c_b [m] is given by:

$$c_b = \left(\frac{\varepsilon_{max}}{\varepsilon_{max} + \dfrac{f_y}{E_s}}\right) d \tag{35}$$

where
E_s Modulus of elasticity of reinforcement, assumed as 203900 [MN/m^2], which is equivalent to $29*10^6$ psi
f_y Specified yield strength of flexural reinforcement [MN/m^2]
ε_{max} Max. strain in concrete, $\varepsilon_{max} = 0.003$

The maximum allowed depth of compression block a_{max} [m] is given by:

$$a_{max} = R_{max}\, \beta_1\, c_b \tag{36}$$

where:
R_{max} Factor to obtain maximum allowed depth of compression block, 0.75 by default

- Check if the depth of compression block a is not exceeded the maximum allowed depth of compression block a_{max}.

Singly reinforced section

If $a \le a_{max}$, then the section is designed as singly reinforced section. The area of tensile steel reinforcement A_s [m^2] is then given by:

$$A_s = \frac{M_u}{\varphi\, f_y \left(d - \dfrac{a}{2}\right)} \tag{37}$$

Doubly reinforced section

If $a > a_{max}$, then the section is designed as doubly reinforced section. Both top and bottom reinforcement are required. The compression and tension reinforcement are calculated as follows:

The compressive force C [MN] developed in concrete alone is given by

$$C = (\alpha f'_c)\ b\ a_{\max} \tag{38}$$

The limiting moment resisted by concrete compression and tensile steel M_{lim} [MN.m] as a singly reinforced section is given by:

$$M_{\lim} = C \left(d - \frac{a_{\max}}{2} \right) \varphi \tag{39}$$

Therefore the moment resisted by compression steel and additional tensile steel ΔM [MN.m] is given by:

$$\Delta M = M_u - M_{\lim} \tag{40}$$

If the steel stress in compression is assumed to be reached to yield stress, then the required steel $A'_s = \Delta A_s$ [m^2] to resist ΔM in tension and compression is given by:

$$A'_s = \Delta A_s = \frac{\Delta M}{\varphi\ f_y\ (d - d')} \tag{41}$$

where:

d' Concrete cover to center of compression reinforcing [m]

The required tensile reinforcement A_s [m^2] to resist $M_{lim} + \Delta M$ is given by:

$$A_s = \frac{M_{\lim}}{\varphi\ f_y \left(d - \frac{a_{\max}}{2} \right)} + \Delta A_s \tag{42}$$

10.2 Check for punching shear

The ACI code assumes the critical punching shear section on a perimeter at a distance d/2 from the face of the column as shown in Figure (8).

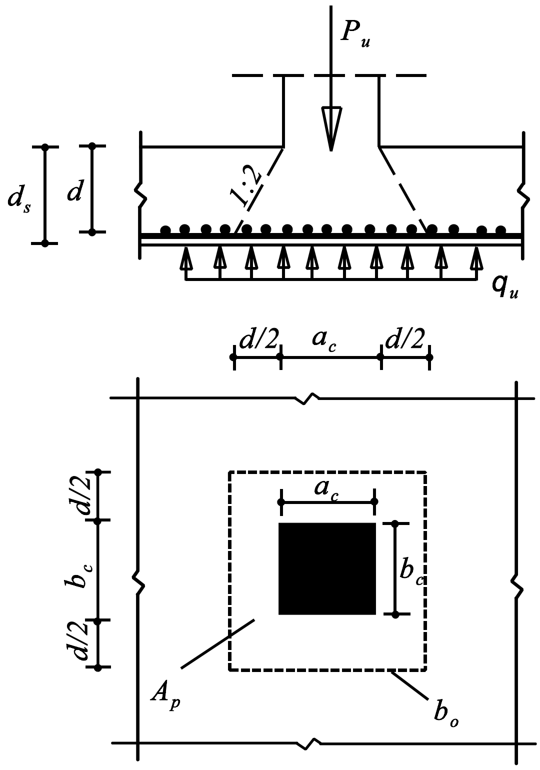

Figure (8) Critical section for Punching shear according to ACI

The nominal concrete punching strength ν_c [MN/m^2] is given by:

$$\nu_c = 0.083 \left(2 + \frac{4}{\beta_c} \right) \sqrt{f'_c} \, , \quad \leq 0.34 \sqrt{f'_c} \tag{43}$$

where:

β_c Ratio of long side to short side of the column

f'_c Specified compressive strength of concrete [MN/m^2]

The allowable concrete punching shear capacity V_c [MN] is given by:

$$V_c = \nu_c \, b_o \, d \tag{44}$$

where:

d	Depth to resist punching shear [m]
b_o	Perimeter of critical punching shear section [m]
	= 4 (a_c+d) for square columns
	= 2 (a_c+b_c+2d) for rectangular columns
	= π (D_c+d) for circular columns
a_c, b_c	Column sides
D_c	Column diameter

The factored punching shear force at a section V_u [MN] is given by:

$$V_u = P_u - q_u A_p \tag{45}$$

where:

P_u	Factored column load [MN]
q_u	Factored upward soil pressure under the column [MN/m^2]
A_p	Area of critical punching shear section [m^2]
	= $(a_c+d)^2$ for square columns
	= $(a_c+d)^2$ $(b_c+d)^2$ for rectangular columns
	= $\pi (D_c+d)^2/4$ for circular columns

Normally, it is impracticable to provide shear reinforcement in slabs and footings. In such cases, concrete alone should resist the punching shear without contribution of the shear reinforcement. The slab thickness is considered to be safe for punching stress, if the factored punching shear force is less than the punching shear capacity of concrete where:

$$V_u \le \varphi V_c \tag{46}$$

where:

φ	Strength reduction factor, is 0.85

If the above basic condition is not satisfied, the thickness will have to be increased to resist the punching shear.

11 Design for ECP (limit state method)

11.1 Design for flexure moment

The design procedure is based on the simplified rectangular stress block shown in Figure (9). The code assumes that the compression carried by concrete is less than 2/3 times that can be carried at the balanced condition. When the applied moment exceeds the moment capacity at the designed balanced condition, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

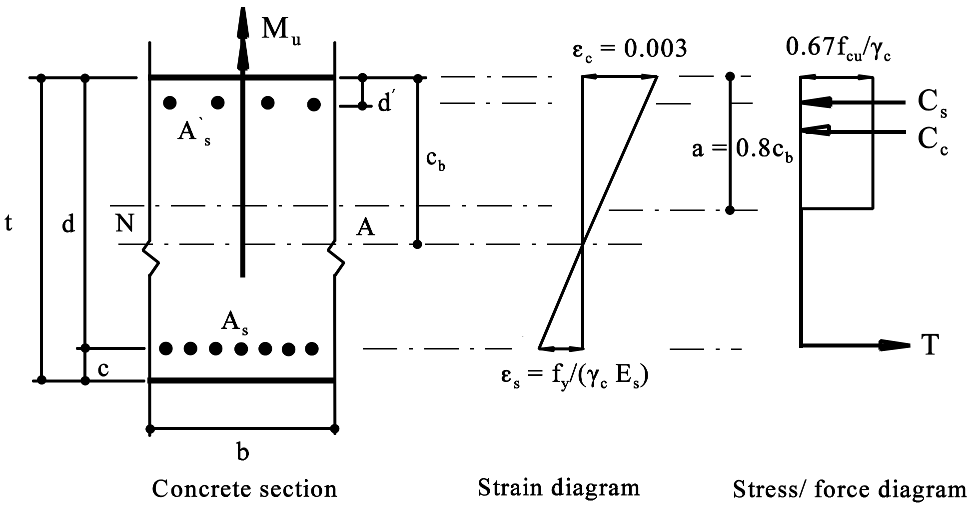

Figure (9) Distribution of stresses and forces according to ECP (limit state method)

In designing procedure, the maximum moment $M_{u,\ max}$ as a singly reinforced section is obtained first. The reinforcing steel area is determined based on whether the factored moment M_u greater than, less than, or equal to $M_{u,\ max}$.

The max value of the ratio $\xi_{max}=c_b/d$ of neutral axis to effective depth is given by:

$$\xi_{max} = \beta \left(\frac{\varepsilon_{max}}{\varepsilon_{max} + \dfrac{f_y}{\gamma_s\ E_s}} \right) \tag{47}$$

where:

c_b The neutral axis depth at balanced condition [m]
E_s Modulus of elasticity of reinforcement, assumed as 200000 [MN/m^2]
f_y Reinforcement yield strength [MN/m^2]
ε_{max} Max. strain in concrete, $\varepsilon_{max}= 0.003$
γ_s Partial safety factor for steel strength, 1.15 by default

β Factor to obtain maximum allowed depth of compression block, 2/3 by default

The max concrete capacity R_{max} as a singly reinforced section is given by:

$$R_{max} \text{ ' } 0.8\ \alpha\ \alpha_R\ \xi_{max}\ (1\ \&\ 0.4\ \xi_{max}) \qquad (48)$$

where:

α Concrete strength reduction factor for sustained loading, 0.85 by default

α_R Factor for obtaining depth of compression block, 0.8 by default

The maximum moment $M_{u,\ max}$ [MN.m] as a singly reinforced section is given by:

$$M_{u,\ max} \text{ ' } R_{max} \frac{f_{cu}}{\gamma_c}\ b\ d^2 \qquad (49)$$

where:

$f!_c$ Specified compressive strength of concrete [MN/m^2]

f_{cu} Concert cube strength [MN/m^2], $f!_c = 0.8\ f_{cu}$

γ_c Partial safety factor for concrete strength, 1.5 by default

b Width of the section to be designed [m], b = 1.0 [m]

d Distance from compression face to tension reinforcement [m]

- Check if the factored moment M_u is not exceeded the maximum allowed moment $M_{u,\ max}$ as a singly reinforced section.

Singly reinforced section

If M_u # $M_{u,\ max}$ then the section is designed as singly reinforced section. The tensile steel reinforcement is calculated as follows:

The concrete capacity R_1 is given by:

$$R_1 \text{ ' } \frac{M_u}{f_{cu}\ b\ d^2} \qquad (50)$$

The normalized steel ratio ω is given by:

$$\omega = 0.8\ \alpha\ \frac{\gamma_s}{\gamma_c} \left(1 - \sqrt{1 - 2.5\ \frac{\gamma_c}{\alpha}\ R_1} \right) \tag{51}$$

The area of tensile steel reinforcement A_s [m^2] is then given by:

$$A_s = \omega\ \frac{f_{cu}}{f_y}\ b\ d \tag{52}$$

Doubly reinforced section

If $M_u > M_{u,\max}$, then the section is designed as doubly reinforced section. Both top and bottom reinforcement are required. The compression and tension reinforcement are calculated as follows:

The moment resisted by compression steel and additional tensile steel ΔM [MN.m] is:

$$\Delta M = M_u - M_{u,\ \max} \tag{53}$$

If the steel stress in compression is assumed to be reached to yield stress, then the required steel, $A'_s = \Delta A_s$ [m^2], to resist ΔM in tension and compression is given by:

$$A'_s = \Delta A_s = \frac{\Delta M}{\frac{f_y}{\gamma_s}\ (d - d')} \tag{54}$$

where:

d' Concrete cover to center of compression reinforcing [m]

The max tensile steel ratio μ_{max} required to resist $M_{u,\max}$ as a singly reinforced section is given by:

$$\mu_{\max} = 0.8\ \alpha\ \alpha_R\ \frac{f_{cu}}{\gamma_c}\ \frac{\gamma_s}{f_y}\ \xi_{\max} \tag{55}$$

where:

γ_s Partial safety factor for steel strength, 1.15 by default

The required tensile reinforcement A_s [m^2] to resist $M_{u,\,max} + \Delta M$ is given by:

$$A_s = \mu_{max}\; b\; d + \Delta A_s \qquad (56)$$

11.2 Check of punching shear

The ECP code for limit state method assumes the critical punching shear section on a perimeter at a distance d/2 from the face of the column as shown in Figure (10).

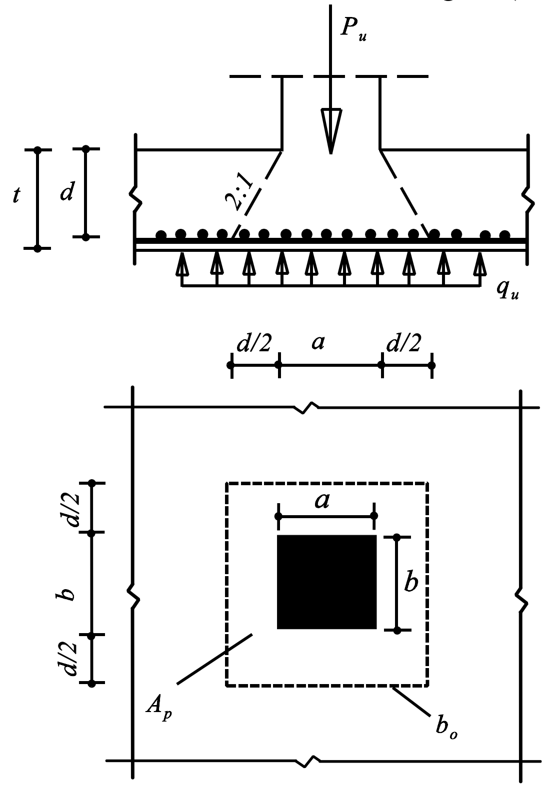

Figure (10) Critical section for Punching shear according to ECP (limit state method)

The factored punching shear force at the section Q_{up} [MN] is given by:

$$Q_{up} = P_u - q_u A_p \qquad (57)$$

where:

P_u Factored column load [MN]

q_u Factored upward soil pressure under the column [MN/m^2]

A_p Area of critical punching shear section [m^2]

= $(a+d)^2$ for square columns

= $(a+d)^2 \ (b+d)^2$ for rectangular columns

= $\pi (D+d)^2/4$ for circular columns

a, b Column sides

D Column diameter

The punching shear stress q_{up} [MN/m^2] is given by:

$$q_{up} = \frac{Q_{up}}{b_o\ d} \tag{58}$$

where:

d Depth to resist punching shear [m]

b_o Perimeter of critical punching shear section [m]
= 4 (a+d) for square columns
= 2 (a+b+2d) for rectangular columns
= π (D+d) for circular columns

The nominal concrete punching strength q_{cup} [MN/m^2] is given by:

$$q_{cup} = 0.316 \left(0.5 + \frac{a}{b}\right) \sqrt{\frac{f_{cu}}{\gamma_c}} \leq 0.316 \sqrt{\frac{f_{cu}}{\gamma_c}} \tag{59}$$

where:

f_{cu} Concert cube strength [MN/m^2], $f'_c = 0.8\ f_{cu}$

f'_c Specified compressive strength of concrete [MN/m^2]

γ_c Partial safety factor for concrete strength, 1.5 by default

a The smallest column side

Normally, it is impracticable to provide shear reinforcement in slabs and footings. In such cases, concrete alone should resist the punching shear without contribution of the shear reinforcement. The slab thickness is considered to be safe for punching stress, if the punching shear stress is less than the nominal concrete punching strength where:

$$q_{cup} \geq q_{up} \tag{60}$$

If the above basic condition is not satisfied, the thickness will have to be increased to resist the punching shear.

12 Design for ECP (working stress method)

12.1 Design for flexure moment

The design procedure is based on the stress diagram shown in Figure (11). In this method, a linearly elastic relationship between stresses and strains is assumed for both the concrete and the reinforcing steel. The codes assumes that the maximum stress produced by the worst combinations of working loads does not exceed a specified allowable working stress value. When the applied moment exceeds the moment capacity at the designed balanced section, the area of compression reinforcement is calculated on the assumption that the additional moment will be carried by compression and additional tension reinforcement.

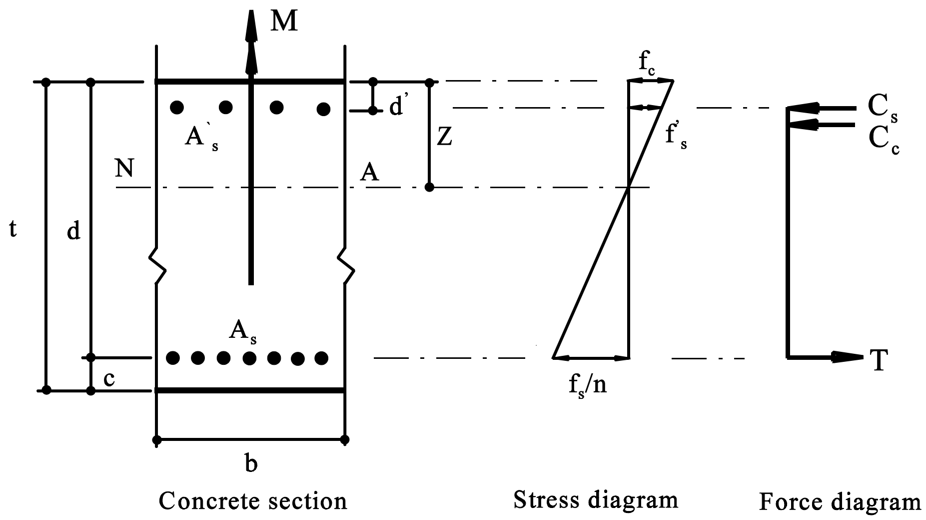

Figure (11) Distribution of stresses and forces according to ECP (working stress method)

In designing procedure, a suitable depth d [m] of the section is assumed first. Then, the maximum depth d_m [m] required to resist the applied moment as a singly reinforced section is obtained. The reinforcing steel area is determined based on whether the assumed depth d greater than, less than, or equal to d_m.

The value of the ratio $\xi = z/d$ of neutral axis to effective depth at balanced condition is given by:

$$\xi = \frac{n}{n + \frac{f_s}{f_c}} \qquad (61)$$

where:

z The neutral axis depth [m]

f_s Tensile stress of steel [MN/m^2]

f_c Compressive stress of concrete [MN/m^2]

n Modular ratio, $n = E_s / E_c$, is the ratio between moduli of elasticity of steel and concrete. The value of the modular ratio is n = 15

The coefficient k_1 to obtain the section depth at balanced condition is given by:

$$k_1 = \sqrt{\frac{2}{f_c\ \xi(1 - \frac{\xi}{3})}} \tag{62}$$

The maximum depth d_m [m] as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M}{b}} \tag{63}$$

where:
M Moment at a section obtained from analysis [MN.m]
b Width of the section to be designed [m], b = 1.0 [m]

- Check if the assumed depth d is not exceeded the maximum depth d_m to resist moment as a singly reinforced section.

Singly reinforced section

If $d \geq d_m$, then the section is designed as singly reinforced section. The tensile steel reinforcement is calculated as follows:

Determine the neutral axis z [m] corresponding to the depth d by iteration from:

$$z = \sqrt{\frac{2\ n\ M\ (d - z)}{b\ f_s\ (d - \frac{z}{3})}} \tag{64}$$

The value of the ratio ξ corresponding to the depth d is given by:

$$\xi = \frac{z}{d} \tag{65}$$

The coefficient k_2 [MN/m^2] to obtain the tensile reinforcement for singly reinforced section, is given by:

$$k_2 = f_s\ (1 - \frac{\xi}{3}) \tag{66}$$

The area of tensile steel reinforcement A_s [m^2] is then given by:

$$A_s = \frac{M}{k_2\ d} \tag{67}$$

The coefficients k_1 and k_2 may be also obtained from the known charts of reinforced concrete. Tables (9) and (10) shows coefficients k_1 and k_2 for singly reinforced rectangular sections. The units used to obtain coefficients k_1 and k_2 in the tables are [MN] and [m].

Doubly reinforced section

If $d_m > d$ then the section is designed as doubly reinforced section. Both top and bottom reinforcement are required. The tension and compression reinforcement is calculated as follows:

The limiting moment resisted by concrete compression and tensile steel M_{lim} [MN.m] as a singly reinforced section is given by:

$$M_{\lim} = \left(\frac{d}{d_m}\right)^2 M \tag{68}$$

Therefore the moment resisted by compression steel and additional tensile steel ΔM [MN.m] is:

$$\Delta M = M - M_{\lim} \tag{69}$$

The coefficient k_2 [MN/m^2] to obtain the tensile steel as a singly reinforced section is given by:

$$k_2 = f_s\ (1 - \frac{\xi}{3}) \tag{70}$$

The area of tensile steel reinforcement A_{s1} [m^2] to resist M_{lim} is then given by:

$$A_{s1} = \frac{M_{\lim}}{k_2\ d} \tag{71}$$

The required additional tensile steel A_{s2} [m^2] to resist ΔM is given by:

$$A_{s2} = \frac{\Delta M}{f_s\,(d - d')} \tag{72}$$

where:
d! Concrete cover to center of compression reinforcing [m]

The total required tensile reinforcement A_s [m^2] to resist $M_{lim} + \Delta M$ is given by:

$$A_s = A_{s1} + A_{s2} \tag{73}$$

The required compression steel A'_s [m^2] to resist ΔM is given by:

$$A'_s = \frac{\Delta M}{f'_s\,(d - d')} \tag{74}$$

where:
f'_s Compressive stress of steel in compression [MN/m^2], which is obtained from:

$$f'_s = n\,f_c\,\frac{z - d'}{z} \tag{75}$$

Table (9) Coefficients k_1 and k_2 for design of singly reinforced rectangular sections according to ECP working stress method (f_s =140-180 [MN/m^2])

f_c [MN/m^2]	f_s =140 [MN/m^2]			f_s = 160 [MN/m^2]			f_s = 180 [MN/m^2]		
	k_1	k_2	ξ	k_1	k_2	ξ	k_1	k_2	ξ
2.0	2.454	132	0.176	2.586	152	0.158	2.711	171	0.143
2.5	2.018	130	0.211	2.121	150	0.190	2.219	170	0.172
3.0	1.727	129	0.243	1.810	148	0.220	1.890	168	0.200
3.5	1.518	127	0.273	1.588	147	0.247	1.654	166	0.226
4.0	1.361	126	0.300	1.420	145	0.273	1.477	165	0.250
4.5	1.238	125	0.325	1.289	144	0.297	1.339	164	0.273
5.0	1.139	124	0.349	1.184	143	0.319	1.228	162	0.294
5.5	1.058	123	0.371	1.098	142	0.340	1.137	161	0.314
6.0	0.990	122	0.391	1.026	141	0.360	1.061	160	0.333
6.5	0.932	121	0.411	0.964	140	0.379	0.996	159	0.351
7.0	0.882	120	0.429	0.911	139	0.396	0.940	158	0.368
7.5	0.838	119	0.446	0.865	138	0.413	0.892	157	0.385
8.0	0.800	118	0.462	0.825	137	0.429	0.849	156	0.400
8.5	0.766	118	0.477	0.789	136	0.443	0.811	155	0.415
9.0	0.736	117	0.491	0.757	136	0.458	0.778	154	0.429
9.5	0.708	116	0.504	0.728	135	0.471	0.747	153	0.442
10.0	0.684	116	0.517	0.702	134	0.484	0.720	153	0.455
10.5	0.661	115	0.529	0.678	134	0.496	0.695	152	0.467
11.0	0.640	115	0.541	0.657	133	0.508	0.673	151	0.478
11.5	0.621	114	0.552	0.637	132	0.519	0.652	151	0.489
12.0	0.604	114	0.563	0.618	132	0.529	0.632	150	0.500

Units in [MN] and [m]. To convert from [MN/m^2] to [kg/cm^2], multiply by 10

$$\textit{The depth of singly reinforced section } d\ [m] = k_1 \sqrt{\frac{M\ [MN.m]}{b\ [m]}}$$

$$\textit{The area of tensile steel reinforcement } A_s\ [m^2] = \frac{M\ [MN.m]}{k_2\ d\ [m]}$$

where b is section width, M is moment about section.

Table (10) Coefficients k_1 and k_2 for design of singly reinforced rectangular sections according to ECP working stress method (f_s =200-240 [MN/m^2])

f_c [MN/m^2]	f_s =200 [MN/m^2]			f_s = 220 [MN/m^2]			f_s = 240 [MN/m^2]		
	k_1	k_2	ξ	k_1	k_2	ξ	k_1	k_2	ξ
2.0	2.831	191	0.130	2.946	211	0.120	3.057	231	0.111
2.5	2.313	189	0.158	2.403	209	0.146	2.490	229	0.135
3.0	1.966	188	0.184	2.040	208	0.170	2.111	227	0.158
3.5	1.718	186	0.208	1.780	206	0.193	1.840	226	0.179
4.0	1.532	185	0.231	1.585	204	0.214	1.637	224	0.200
4.5	1.387	183	0.252	1.433	203	0.235	1.478	222	0.220
5.0	1.270	182	0.273	1.311	201	0.254	1.351	221	0.238
5.5	1.175	181	0.292	1.211	200	0.273	1.247	220	0.256
6.0	1.095	179	0.310	1.127	199	0.290	1.160	218	0.273
6.5	1.027	178	0.328	1.057	197	0.307	1.086	217	0.289
7.0	0.968	177	0.344	0.996	196	0.323	1.022	216	0.304
7.5	0.917	176	0.360	0.943	195	0.338	0.967	214	0.319
8.0	0.873	175	0.375	0.896	194	0.353	0.919	213	0.333
8.5	0.833	174	0.389	0.855	193	0.367	0.876	212	0.347
9.0	0.798	173	0.403	0.818	192	0.380	0.838	211	0.360
9.5	0.766	172	0.416	0.785	191	0.393	0.803	210	0.373
10.0	0.738	171	0.429	0.755	190	0.405	0.772	209	0.385
10.5	0.712	171	0.441	0.728	189	0.417	0.744	208	0.396
11.0	0.688	170	0.452	0.704	189	0.429	0.719	207	0.407
11.5	0.666	169	0.463	0.681	188	0.439	0.695	207	0.418
12.0	0.646	168	0.474	0.660	187	0.450	0.674	206	0.429

Units in [MN] and [m]. To convert from [MN/m^2] to [kg/cm^2], multiply by 10

$$\text{The depth of singly reinforced section } d\ [m] = k_1 \sqrt{\frac{M\ [MN.m]}{b\ [m]}}$$

$$\text{The area of tensile steel reinforcement } A_s\ [m^2] = \frac{M\ [MN.m]}{k_2\ d\ [m]}$$

where b is section width, M is moment about section.

12.2 Check for punching shear

The ECP code for working stress method assumes the critical punching shear section on a perimeter at a distance d/2 from the face of the column as shown in Figure (12).

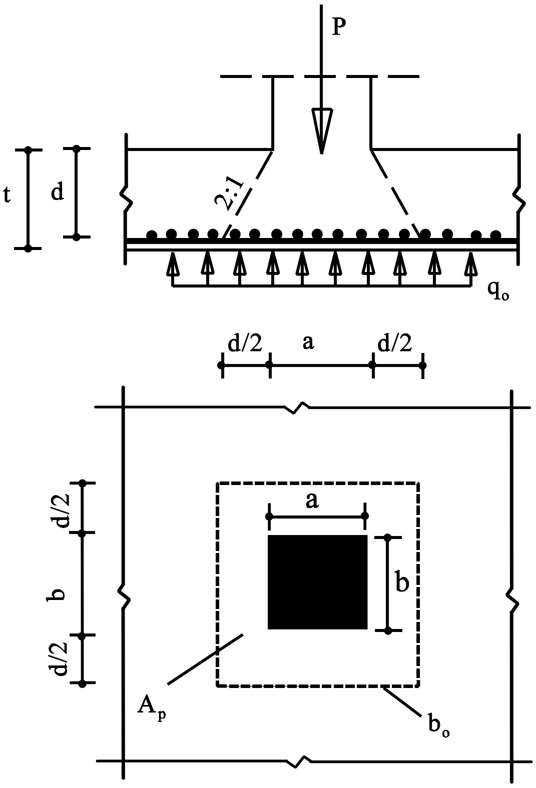

Figure (12) Critical section for punching shear according to ECP (working stress method)

The punching shear force at the section Q_p [MN] is given by:

$$Q_p = P - q_o A_p \qquad (76)$$

where:

P	Column load [MN]
q_o	Upward soil pressure under the column [MN/m^2]
A_p	Area of critical punching shear section [m^2]
	$= (a+d)^2$ for square columns
	$= (a+d)^2 \ (b+d)^2$ for rectangular columns
	$= \pi (D+d)^2/4$ for circular columns
a, b	Column sides
D	Column diameter

The punching shear stress q_p [MN/m^2] is given by:

$$q_p = \frac{Q_p}{b_o\ d} \tag{77}$$

where:
d Depth to resist punching shear [m]
b_o Perimeter of critical punching shear section [m]
= 4 (a+d) for square columns
= 2 (a+b+2d) for rectangular columns
= π (D+d) for circular columns

The allowable concrete punching strength q_{pall} [MN/m^2] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp} \tag{78}$$

where:
q_{cp} The main value of shear strength [MN/m^2] according to Table (4)
a The smallest column side.

Normally, it is impracticable to provide shear reinforcement in slabs and footings. In such cases, concrete alone should resist the punching shear without contribution of the shear reinforcement. The slab thickness is considered to be safe for punching stress, if the punching shear stress is less than the allowable concrete punching strength where:

$$q_p \geq q_p \tag{79}$$

If the above basic condition is not satisfied, the thickness will have to be increased to resist the punching shear.

13 References

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Example 1: Design of a squar footing for different codes

1 Description of the problem

An example is carried out to design a spread footing according to EC 2, DIN 1045, ACI and ECP codes.

A square footing of 0.5 [m] thickness has dimensions of 2.6 [m] * 2.6 [m] is chosen. The footing is support to a column of 0.4 [m] * 0.4 [m], reinforced by 8Φ16 and carries a load of 1276 [kN]. The footing rests on Winkler springs have modulus of subgrade reaction of k_s = 40 000 [kN/m^3]. A thin plain concrete of thickness 0.15 [m] is chosen under the footing and is not considered in any calculation.

2 Footing material and section

The footing material and section are supposed to have the following parameters:

Material properties

Concrete grade according to ECP	C 250			
Steel grade according to ECP	S 36/52			
Concert cube strength	f_{cu} = 250	[kg/cm^2]	= 25	[MN/m^2]
Concert cylinder strength	$f'_c = 0.8\ f_{cu}$		= 20	[MN/m^2]
Compressive stress of concrete	f_c = 95	[kg/cm^2]	= 9.5	[MN/m^2]
Tensile stress of steel	f_s = 2000	[kg/cm^2]	= 200	[MN/m^2]
Reinforcement yield strength	f_y = 3600	[kg/cm^2]	= 360	[MN/m^2]
Young's modulus of concrete	$E_b = 3*10^7$	[kN/m^2]	= 30000	[MN/m^2]
Poisson's ratio of concrete	ν_b = 0.15			
Unit weight of concrete	γ_b = 0.0	[kN/m^3]		

Unit weight of concrete is chosen γ_b = 0.0 to neglect the own weight of the footing.

Section properties

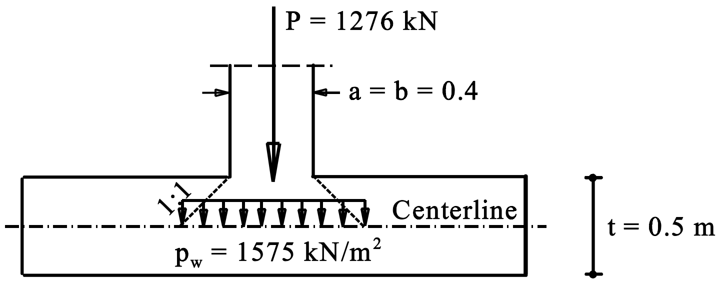
Width of the section to be designed	b = 1.0	[m]
Section thickness	t = 0.50	[m]
Concrete cover + 1/2 bar diameter	c = 5	[cm]
Effective depth of the section	d = t - c = 0.45	[m]
Steel bar diameter	Φ = 18	[mm]

3 Analysis of the footing

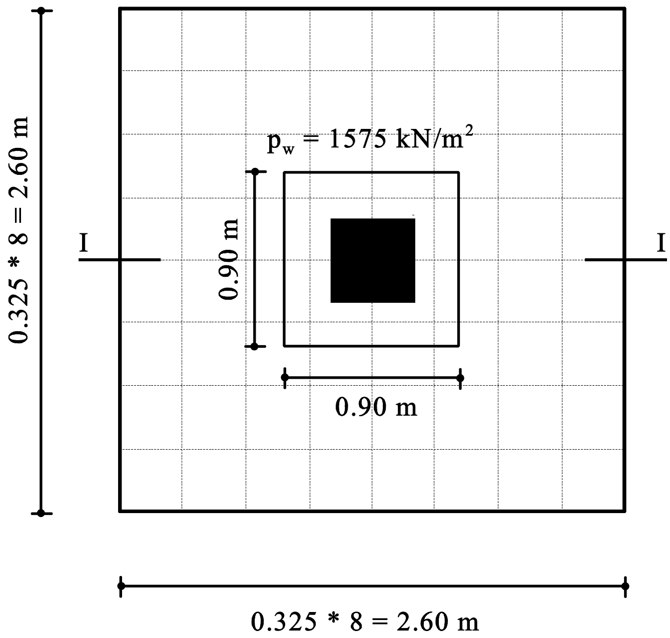
To carry out the analysis, the footing is subdivided into 64 square elements. Each has dimensions of 0.325 m * 0.325 m as shown in Figure (13).

If a point load represents the column load on the mesh of fine finite elements, the moment under the column will be higher than the real moment. In addition, to take effect of the load distribution through the footing thickness, the column load is distributed outward at 45 [°] from the column until reaching the center line of the footing. Therefore, the column load is distributed at center line of the footing on an area of $(a+d)^2$ as shown in Figure (13). Figure (14) shows the calculated contact pressure q [kN/m^2] while Figure (15) shows the bending moment m_x [kN.m/m] at the critical section I-I of the footing.

For the different codes, the footing is designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained. Finally, a comparison among the results of the four codes is presented.

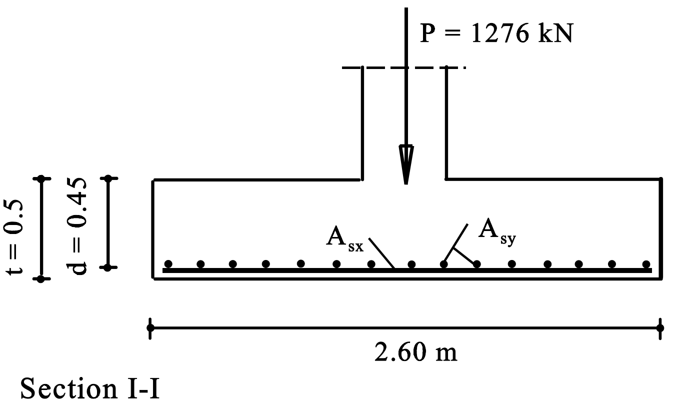


b) Section I-I



a) Plan

Figure (13) Footing dimensions and distribution of column load through the footing



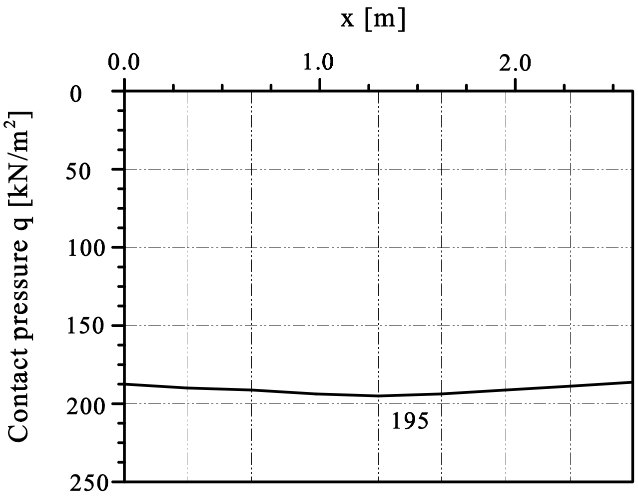


Figure (14) Contact pressure q [kN/m^2] at section I-I

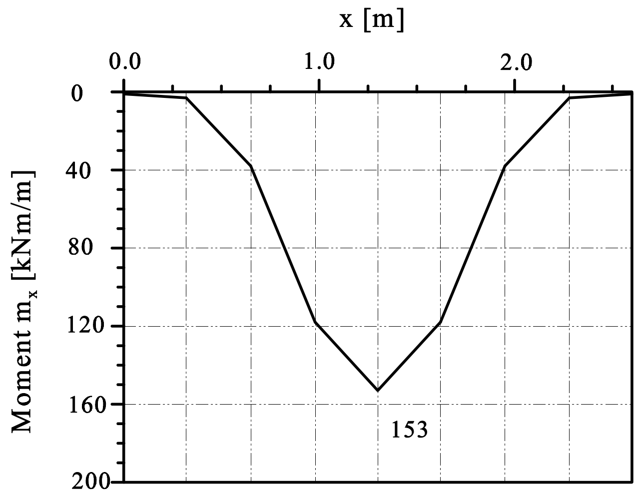


Figure (15) Moment m_x [kN.m/m] at section I-I

4 Design for EC 2

4.1 Design for flexure moment

Material

Concrete grade C 250 (ECP) = C 20/25 (EC 2)
Steel grade S 36/52 (ECP) = BSt 360 (EC 2)
Characteristic compressive cylinder strength of concrete $f_{ck} = 20$ [MN/m^2]
Characteristic tensile yield strength of reinforcement $f_{yk} = f_y = 360$ [MN/m^2]
Partial safety factor for concrete strength $\gamma_c = 1.5$
Design concrete compressive strength $f_{cd} = f_{ck}/\gamma_c = 20/1.5 = 13.33$ [MN/m^2]
Partial safety factor for steel strength $\gamma_s = 1.15$
Design tensile yield strength of reinforcing steel $f_{yd} = f_{yk}/\gamma_s = 360/1.15 = 313$ [MN/m^2]

Factored moment

Moment per meter at critical section obtained from analysis M =153 [kN.m] = 0.153 [MN.m]
Total load factor for both dead and live loads $\gamma = 1.5$
Factored moment $M_{sd} = \gamma M = 1.5*0.153 = 0.2295$ [MN.m]

Geometry

Effective depth of the section d = 0.45 [m]
Width of the section to be designed b = 1.0 [m]

Check for section capacity

The limiting value of the ratio x/d is $\xi_{lim} = 0.45$ for $f_{ck} \leq 35$ [MN/m^2]

The normalized concrete moment capacity $\mu_{sd,\,lim}$ as a singly reinforced section is

$$\mu_{sd,\,\lim} = 0.8\ \xi_{\lim}\ (1 - 0.4\ \xi_{\lim})$$

$$\mu_{sd,\,\lim} = 0.8 \times 0.45\ (1 - 0.4 \times 0.45) = 0.295$$

The normalized design moment μ_{sd} is

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{0.2295}{1.0 \times 0.45^2\ (0.85 \times 13.33)} = 0.1$$

$\mu_{sd} = 0.1 < \mu_{sd,\, lim} = 0.295$, then the section is designed as singly reinforced section.

Determination of tension reinforcement

The normalized steel ratio ω is

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.1} = 0.106$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega \left(\frac{(0.85 f_{cd})\ b\ d}{f_{yd}} \right)$$

$$A_s = 0.106 \left(\frac{(0.85 \times 13.33) \times 1.0 \times 0.45}{313} \right) = 0.001727\ [m^2/m]$$

$$A_s = 17.27\ [cm^2/m]$$

Chosen steel 7Φ18/m = 17.8 [cm^2/m]

4.2 Check for punching shear

The critical section for punching shear is at a distance r = 1.5 d around the circumference of the column as shown in Figure (16).

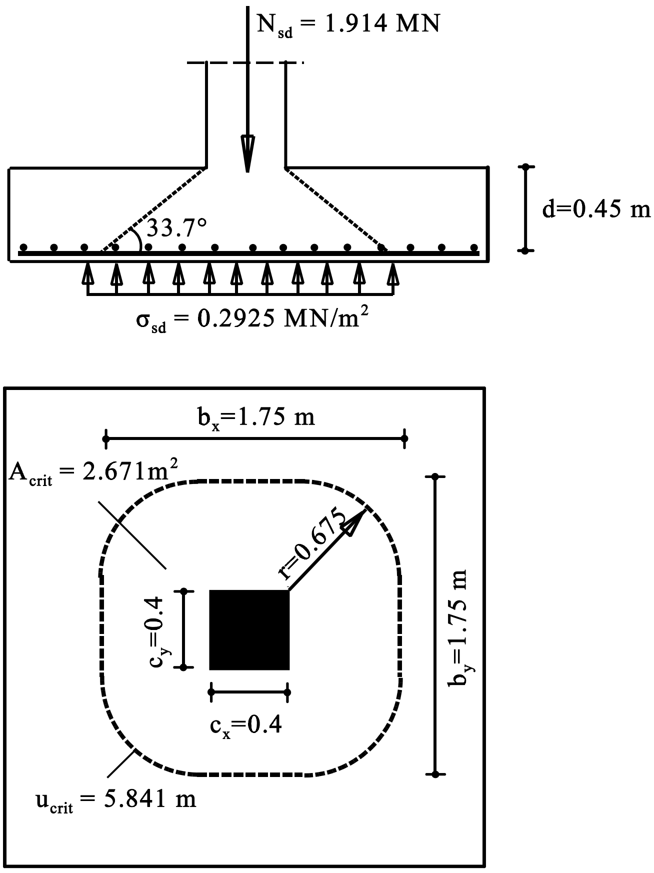


Figure (16) Critical section for Punching shear according to EC 2

Geometry (Figure (16))

Effective depth of the section $d = d_x = d_y = 0.45$ [m]
Column side $c_x = c_y = 0.4$ [m]
Distance of critical punching section from circumference of the column
$r = 1.5\ d = 1.5*0.45 = 0.675$ [m]
Area of critical punching shear section
$A_{crit} = c_x^{\ 2} + 4\ r\ c_x + \pi\ r^2 = (0.4)^2 + 4*0.675*0.4 + \pi\ 0.675^2 = 2.671$ [m²]
Perimeter of critical punching shear section $u_{crit} = 4c_x + 2\ \pi\ r = 4*0.4 + 2\ \pi\ 0.675 = 5.841$ [m]
Width of punching section $b_x = b_y = c_x + 2\ r = 0.4 + 2*0.675 = 1.75$ [m]
Correction factor (where no eccentricity is expected) $\beta = 1.0$
Coefficient for consideration of the slab thickness $k = 1.6 - d = 1.6 - 0.45 = 1.15$ [m] > 1.0 [m]
Reinforcement under the column per meter $A_s = 17.8$ [cm²/m]
Reinforcement at punching section $A_{sx} = A_{sy} = b_x\ A_s = 1.75*17.8 = 31.15$ [cm²]

Steel ratio $\rho_1 = \rho_{1x} = \rho_{1y} = A_{sx}/(b_y\ d_x) = (31.15*10^{-4})/(1.75*0.45) = 0.004 = 0.4$ [%]

Loads and stresses

Column load $N = 1276$ [kN] $= 1.276$ [MN]
Soil pressure under the column $\sigma_o = 195$ [kN/m^2] $= 0.195$ [MN/m^2]
Total load factor for both dead and live loads $\gamma = 1.5$
Factored column load $N_{sd} = \gamma\ N = 1.5*1.276 = 1.914$ [MN]
Factored upward soil pressure under the column $\sigma_{Sd} = \gamma\ \sigma_o = 1.5*0.195 = 0.2925$ [MN/m^2]
Main value of shear strength for concrete C 20/25 according to Table (1)
$\tau_{Rd} = 1.2*0.24 = 0.288$ [MN/m]

Check for section capacity

The punching force at ultimate design load V_{Sd} is

$$V_{Sd} = N_{Sd} - \sigma_{Sd}\ A_{crit}$$

$$V_{Sd} = 1.914 - 0.2925 \times 2.671 = 1.133\ [MN]$$

The design value of the applied shear ν_{Sd} is

$$\nu_{Sd} = \frac{V_{Sd}\ \beta}{u_{crit}}$$

$$\nu_{Sd} = \frac{1.133 \times 1.0}{5.841} = 0.194\ [MN/m]$$

Design shear resistance from concrete alone ν_{Rd1} is

$$\nu_{Rd1} = \tau_{Rd}\ k\ (1.2 + 40\ \rho_1)\ d$$

$$\nu_{Rd1} = 0.288 \times 1.15\ (1.2 + 40 \times 0.004)\ 0.45 = 0.203\ [MN/m]$$

$\nu_{Rd1} = 0.203$ [MN/m] $> \nu_{sd} = 0.194$ [MN/m], the section is safe for punching shear.

5 Design for DIN 1045

5.1 Design for flexure moment

Material

Concrete grade	C 250 (ECP) = B 25 (DIN 1045)	
Steel grade	S 36/52 (ECP) = BSt 360 (DIN 1045)	
Concrete compressive strength		$\beta_R = 17.5$ [MN/m^2]
Tensile yield strength of steel		$\beta_S = 360$ [MN/m^2]
Concrete strength reduction factor for sustained loading		$\alpha_R = 0.95$
Safety factor		$\gamma = 1.75$

Moment

Moment per meter at critical section obtained from analysis M_s =153 [kN.m] = 0.153 [MN.m]

Geometry

Effective depth of the section	h = 0.45 [m]
Width of the section to be designed	b = 1.0 [m]

Check for section capacity

The normalized design moment m_s is

$$m_s = \frac{M_s}{bh^2\left(\frac{\alpha_R \beta_R}{\gamma}\right)}$$

$$m_s = \frac{0.153}{1.0 \times 0.45^2 \left(\frac{0.95 \times 17.5}{1.75}\right)} = 0.07953$$

The limiting value of the ratio k_x of neutral axis to effective depth is

$$k_x = \left(\frac{\varepsilon_{b1}}{\varepsilon_{b1} - \varepsilon_{s2}}\right)$$

$$k_x = \left(\frac{0.0035}{0.0035 + 0.003}\right) = 0.53846$$

The normalized concrete moment capacity m^*_s as a singly reinforced section is

$$m^*_s = \chi \, k_x \, (1 - \frac{\chi}{2} \, k_x)$$

$$m^*_s = 0.8 \times 0.53846 \, (1 - \frac{0.8}{2} \, 0.53846) = 0.337987$$

$m_s = 0.07953 < m^*_s = 0.337987$, then the section is designed as singly reinforced section.

Determination of tension reinforcement

The normalized steel ratio ω_M is

$$\omega_M = 1 - \sqrt{1 - 2 \, m_s}$$

$$\omega_M = 1 - \sqrt{1 - 2 \times 0.07953} = 0.08297$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega_M \left(\frac{(\alpha_R \beta_R) \, b \, h}{\beta_S} \right)$$

$$A_s = 0.08297 \left(\frac{(0.95 \times 17.5) \times 1.0 \times 0.45}{360} \right) = 0.001724 \; [m^2/m]$$

$$A_s = 17.24 \; [cm^2/m]$$

Chosen steel $7\Phi18$/m = 17.8 [cm^2/m]

5.2 Check for punching shear

The critical section for punching shear is a circle of diameter d_r = 0.902 m around the circumference of the column as shown in Figure (16).

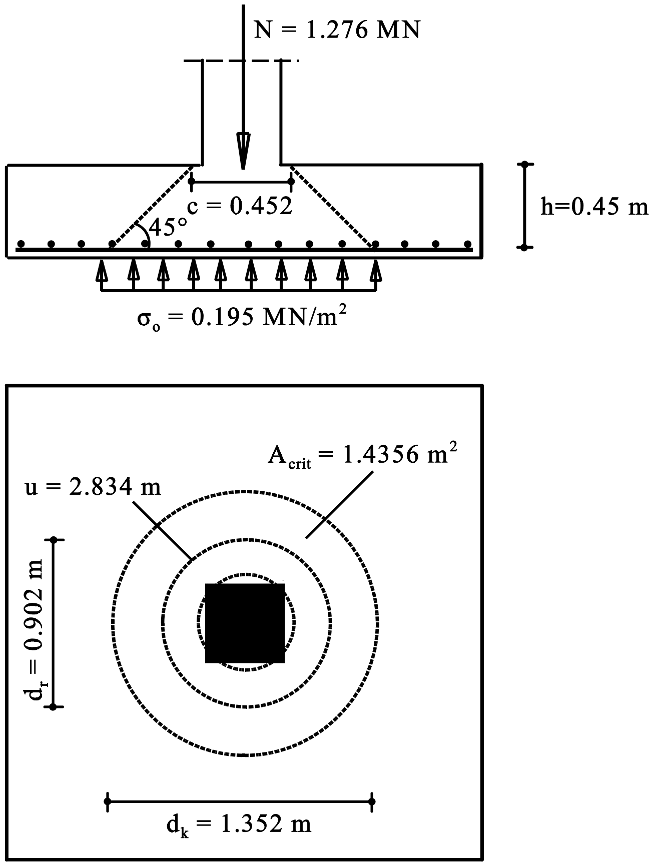


Figure (16) Critical section for Punching shear according to DIN 1045

Geometry (Figure (16))

Effective depth of the section	$h = 0.45$ [m]
Column side	$c_x = c_x = 0.4$ [m]
Average diameter of the column	$c = 1.13*0.4 = 0.452$ [m]
Diameter of loaded area	$d_k = 2\,h + c = 2*0.45+0.452 = 1.352$ [m]
Diameter of critical punching shear section	$d_r = c+h = 0.452+0.45 = 0.902$ [m]
Area of critical punching shear section	$A_{crit} = \pi\, d_k^2/4 = \pi\, 1.352^2/4 = 1.4356$ [m^2]
Perimeter of critical punching shear section	$u = \pi\, d_r = \pi\, 0.902 = 2.834$ [m]
Reinforcement in x-direction	$A_{sx} = A_{sy} = 0.00178$ [m^2/m]

Loads and stresses

Column load	N = 1276 [kN] = 1.276 [MN]
Soil pressure under the column	σ_o = 195 [kN/m^2] = 0.195 [MN/m^2]
Main value of shear strength for concrete B 25 according to Table (2)	τ_{011} = 0.5 [MN/m^2]
Factor depending on steel grade according to Table (6)	α_s = 1.3

Check for section capacity

The punching shear force Q_r is

$$Q_r = N - \sigma_o\, A_{crit}$$

$$Q_r = 1.276 - 0.195 \times 1.4356 = 0.9961 \ [MN]$$

The punching shear stress τ_r is

$$\tau_r = \frac{Q_r}{u\ h}$$

$$\tau_r = \frac{0.9961}{2.834 \times 0.45} = 0.781 \ [MN/m^2]$$

Reinforcement grade μ_g is

$$\mu_g = \frac{A_{sx} + A_{sy}}{2h}$$

$$\mu_g = \frac{0.00178 + 0.00178}{2 \times 0.45} = 0.00396 = 0.396 \ [\%]$$

Coefficient for consideration of reinforcement κ_1 is

$$\kappa_1 = 1.3\ \alpha_s \sqrt{\mu_g}$$

$$\kappa_1 = 1.3 \times 1.3 \sqrt{0.396} = 1.063$$

The allowable concrete punching strength τ_{r1} [MN/m^2] is given by

$$\tau_{r1} = \kappa_1 \, \tau_{011}$$

$$\tau_{r1} = 1.063 \times 0.5 = 0.532 \; [MN/m^2]$$

τ_{r1} = 0.532 [MN/m^2] < τ_r = 0.781 [MN/m^2], the section is unsafe for punching shear. Such situation can be conveniently rectified by increasing the depth of the footing. It will be noticed that the required increase here is 10 [cm].

6 Design for ACI

6.1 Design for flexure moment

Material

Concrete grade C 250 (ECP)
Steel grade S 36/52 (ECP)
Specified compressive strength of concrete $f'_c = 20$ [MN/m²]
Specified yield strength of flexural reinforcement $f_y = 360$ [MN/m²]
Strength reduction factor for flexure $\varphi = 0.9$

Factored moment

Moment per meter at critical section obtained from analysis M =153 [kN.m] = 0.153 [MN.m]
Total load factor for both dead and live loads $\gamma = 1.5$
Factored moment $M_u = \gamma M = 1.5 * 0.153 = 0.2295$ [MN.m]

Geometry

Effective depth of the section d = 0.45 [m]
Width of the section to be designed b = 1.0 [m]

Check for section capacity

The depth of the compression block a is

$$a = d - \sqrt{d^2 - \frac{2\,|M_u|}{(\alpha f'_c)\ \varphi\ b}}$$

$$a = 0.45 - \sqrt{0.45^2 - \frac{2 \times 0.2295}{(0.85 \times 20)\ 0.9 \times 1.0}} = 0.0347\ [m]$$

The factor for obtaining depth of compression block in concrete β_1 is

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 28}{7}\right), \quad 0.65 \le \beta_1 \le 0.85$$

$$\beta_1 = 0.85 - 0.05\left(\frac{20 - 28}{7}\right) = 0.91 > 0.85$$

$$\beta_1 = 0.85$$

The depth of neutral axis at balanced condition c_b is

$$c_b = \left(\frac{\varepsilon_{max}}{\varepsilon_{max} + \frac{f_y}{E_s}} \right) d$$

$$c_b = \left(\frac{0.003}{0.003 + \frac{360}{203900}} \right) 0.45 = 0.283 \ [m]$$

The maximum allowed depth of compression block a_{max} is

$$a_{max} = 0.75 \ \beta_1 \ c_b$$

$$a_{max} = 0.75 \times 0.85 \times 0.283 = 0.18 \ [m]$$

a_{max} = 0.18 [m] > a = 0.0347 [m], then the section is designed as singly reinforced section.

Determination of tension reinforcement

$$A_s = \frac{M_u}{\varphi \ f_y \left(d - \frac{a}{2} \right)}$$

$$A_s = \frac{0.2295}{0.9 \times 360 \left(0.45 - \frac{0.0347}{2} \right)} = 0.001637 \ [m^2/m]$$

$$A_s = 16.37 \ [cm^2/m]$$

Chosen steel 7Φ18/m = 17.8 [cm^2/m]

6.2 Check for punching shear

The critical punching shear section on a perimeter at a distance d/2 = 0.225 [m] from the face of the column as shown in Figure (18).

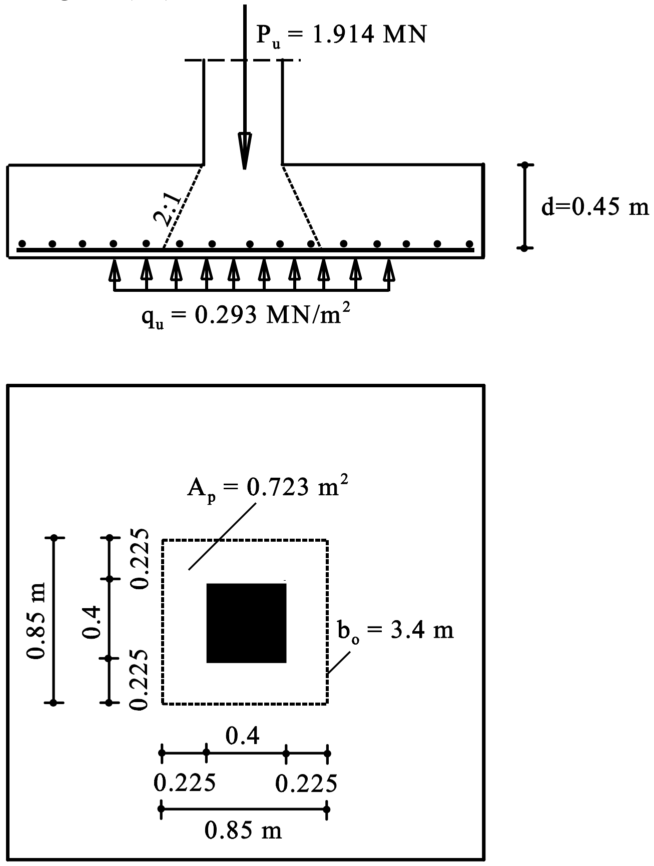


Figure (18) Critical section for Punching shear according to ACI

Geometry (Figure (18))

Effective depth of the section	$d = 0.45$ [m]
Column side	$a_c = b_c = 0.4$ [m]
Area of critical punching shear section	$A_p = (a_c+d)^2 = (0.4+0.45)^2 = 0.723$ [m^2]
Perimeter of critical punching shear section	$b_o = 4\ (a_c+d) = 4\ (0.4+0.45) = 3.4$ [m]
Ratio of long side to short side of the column	$\beta_c = 1.0$

Loads and stresses

Specified compressive strength of concrete	$f'_c = 20$ [MN/m^2]
Strength reduction factor for punching shear	$\varphi = 0.85$
Total load factor for both dead and live loads	$\gamma = 1.5$
Column load	$P_c = 1276$ [kN] $= 1.276$ [MN]

Soil pressure under the column	$q = 195\ [kN/m^2] = 0.195\ [MN/m^2]$
Factored column load	$P_u = \gamma\ P_c = 1.5*1.276 = 1.914\ [MN]$
Factored soil pressure under the column	$q_u = \gamma\ q = 1.5*0.195 = 0.293\ [MN/m^2]$

Check for section capacity

The nominal concrete punching strength ν_c is

$$\nu_c = 0.083 \left(2 + \frac{4}{\beta_c}\right) \sqrt{f'_c}\ , \quad \le 0.34 \sqrt{f'_c}$$

$$\nu_c = 0.083 \left(2 + \frac{4}{1.0}\right) \sqrt{20}\ , \quad \le 0.34 \sqrt{20}$$

$$\nu_c = 1.521\ [MN/m^2]$$

The allowable concrete punching shear capacity V_c is

$$V_c = \nu_c\ b_o\ d$$

$$V_c = 1.521 \times 3.4 \times 0.45 = 2.327\ [MN]$$

The factored punching shear force V_u is

$$V_u = P_u - q_u\ A_p$$

$$V_u = 1.914 - 0.293 \times 0.723 = 1.702\ [MN]$$

The available shear strength is

$$\varphi\ V_c = 0.85 \times 2.327 = 1.978\ [MN]$$

$\varphi\ V_c = 1.978$ [MN] > $V_u = 1.702$ [MN], the section is safe for punching shear.

7 Design for ECP (limit state method)

7.1 Design for flexure moment

Material

Concrete grade	C 250	
Steel grade	S 36/52	
Concert cube strength		$f_{cu} = 25$ [MN/m^2]
Reinforcement yield strength		$f_y = 360$ [MN/m^2]
Partial safety factor for concrete strength		$\gamma_c = 1.5$
Partial safety factor for steel strength		$\gamma_s = 1.15$

Factored moment

Moment per meter at critical section obtained from analysis M =153 [kN.m] = 0.153 [MN.m]
Total load factor for both dead and live loads $\gamma = 1.5$
Factored moment $M_u = \gamma M = 1.5*0.153 = 0.2295$ [MN.m]

Geometry

Effective depth of the section d = 0.45 [m]
Width of the section to be designed b = 1.0 [m]

Check for section capacity

The max value of the ratio ξ_{max} is

$$\xi_{max} = \beta \left(\frac{\varepsilon_{max}}{\varepsilon_{max} + \dfrac{f_y}{\gamma_s\, E_s}} \right)$$

$$\xi_{max} = \frac{2}{3} \left(\frac{0.003}{0.003 + \dfrac{360}{1.15 \times 200000}} \right) = 0.438$$

The max concrete capacity R_{max} as a singly reinforced section is

$$R_{max} = 0.544\ \xi_{max}\ (1 - 0.4\ \xi_{max})$$

$$R_{max} = 0.544 \times 0.438\ (1 - 0.4 \times 0.438) = 0.197$$

The maximum moment $M_{u,\ max}$ as a singly reinforced section is

$$M_{u,\ max} = R_{max} \frac{f_{cu}}{\gamma_c}\ b\ d^2$$

$$M_{u,\ max} = 0.197 \times \frac{25}{1.5} \times 1.0 \times 0.45^2 = 0.665\ [MN.m]$$

$M_{u,\ max} = 0.665 > M_u = 0.2295$, then the section is designed as singly reinforced section.

Determination of tension reinforcement

The concrete capacity R_1 is

$$R_1 = \frac{M_u}{f_{cu}\ b\ d^2}$$

$$R_1 = \frac{0.2295}{25 \times 1.0 \times 0.45^2} = 0.045$$

The normalized steel ratio ω is

$$\omega = 0.521 \left(1 - \sqrt{1 - 4.41\ R_1} \right)$$

$$\omega = 0.521 \left(1 - \sqrt{1 - 4.41 \times 0.045} \right) = 0.055$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega \frac{f_{cu}}{f_y}\ b\ d$$

$$A_s = 0.055 \times \frac{25}{360} \times 1.0 \times 0.45 = 0.001719\ [m^2/m]$$

$$A_s = 17.19\ [cm^2/m]$$

Chosen steel $7\Phi 18$/m = 17.8 [cm^2/m]

7.2 Check for punching shear

The critical punching shear section on a perimeter at a distance d/2 = 0.225 m from the face of the column as shown in Figure (19).

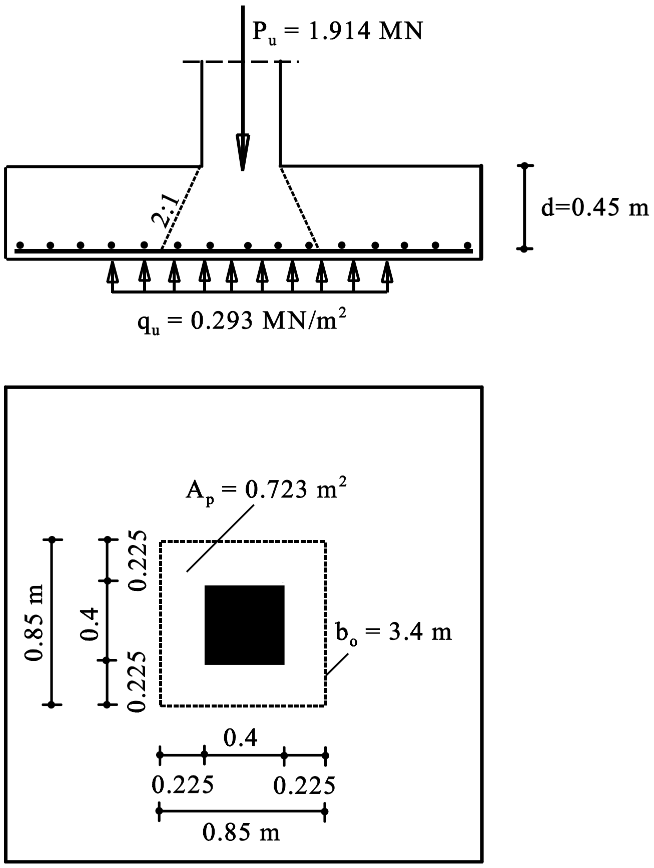


Figure (19) Critical section for Punching shear according to ECP

Geometry (Figure (19))

Effective depth of the section	d = 0.45 [m]
Column side	a = b = 0.4 [m]
Area of critical punching shear section	$A_p = (a+d)^2 = (0.4+0.45)^2 = 0.723$ [m^2]
Perimeter of critical punching shear section	$b_o = 4\ (a+d) = 4\ (0.4+0.45) = 3.4$ [m]

Loads and stresses

Concert cube strength	$f_{cu} = 25$ [MN/m^2]
Total load factor for both dead and live loads	$\gamma = 1.5$
Partial safety factor for concrete strength	$\gamma_c = 1.5$
Column load	P = 1276 [kN] = 1.276 [MN]

Soil pressure under the column — $q_o = 195\ [kN/m^2] = 0.195\ [MN/m^2]$
Factored column load — $P_u = \gamma\ P = 1.5*1.276 = 1.914\ [MN]$
Factored soil pressure under the column — $q_u = \gamma\ q_o = 1.5*0.195 = 0.293\ [MN/m^2]$

Check for section capacity

The factored punching shear force Q_{up} is

$$Q_{up} = P_u - q_u\ A_p$$

$$Q_{up} = 1.914 - 0.293 \times 0.723 = 1.702\ [MN]$$

The punching shear stress q_{up} is

$$q_{up} = \frac{Q_{up}}{b_o\ d}$$

$$q_{up} = \frac{1.702}{3.4 \times 0.45} = 1.112\ [MN/m^2]$$

The nominal concrete punching strength q_{cup} is

$$q_{cup} = 0.316\left(0.5 + \frac{a}{b}\right)\sqrt{\frac{f_{cu}}{\gamma_c}} \le 0.316\sqrt{\frac{f_{cu}}{\gamma_c}}$$

$$q_{cup} = 0.316\left(0.5 + \frac{0.4}{0.4}\right)\sqrt{\frac{25}{1.5}} \le 0.316\sqrt{\frac{25}{1.5}}$$

$$q_{cup} = 1.29\ [MN/m^2]$$

$q_{cup} = 1.29\ [MN/m^2] > q_{up} = 1.112\ [MN/m^2]$, the section is safe for punching shear.

8 Design for ECP (working stress method)

8.1 Design for flexure moment

Material

Concrete grade C 250
Steel grade S 36/52
Compressive stress of concrete $f_c = 95$ [kg/cm^2] = 9.5 [MN/m^2]
Tensile stress of steel $f_s = 2000$ [kg/cm^2] = 200 [MN/m^2]

Moment

Moment per meter at critical section obtained from analysis M =153 [kN.m] = 0.153 [MN.m]

Geometry

Effective depth of the section d = 0.45 [m]
Width of the section to be designed b = 1.0 [m]

Check for section capacity

The value of the ratio ξ is

$$\xi = \frac{n}{n + \frac{f_s}{f_c}}$$

$$\xi = \frac{15}{15 + \frac{200}{9.5}} = 0.416$$

The coefficient k_1 to obtain the section depth at balanced condition is

$$k_1 = \sqrt{\frac{2}{f_c \, \xi \, (1 - \frac{\xi}{3})}}$$

$$k_1 = \sqrt{\frac{2}{9.5\,(0.416\,(1 - \frac{0.416}{3})}} = 0.767$$

The maximum depth d_m as a singly reinforced section is

$$d_m = k_1 \sqrt{\frac{M}{b}}$$

$$d_m = 0.767 \sqrt{\frac{0.153}{1.0}} = 0.3 \ [m]$$

d = 0.45 [m] > d_m = 0.3 [m], then the section is designed as singly reinforced section.

Determination of tension reinforcement

Determine the neutral axis z corresponding to the depth d by iteration from

$$z = \sqrt{\frac{30 \ M \ (d - z)}{b \ f_s \ (d - \frac{z}{3})}}$$

$$z = \sqrt{\frac{30 \times 0.153 \ (0.45 - z)}{1.0 \times 200 \ (0.45 - \frac{z}{3})}} = 0.134 \ [m]$$

The value of the ratio ξ corresponding to the depth d is given by

$$\xi = \frac{0.13}{0.45} = 0.298$$

The coefficient k_2 [MN/m^2] to obtain the tensile reinforcement for singly reinforced section is

$$k_2 = f_s \ (1 - \frac{\xi}{3})$$

$$k_2 = 200 \ (1 - \frac{0.298}{3}) = 180.13$$

The required area of steel reinforcement per meter A_s is

$$A_s = \frac{M}{k_2\ d}$$

$$A_s = \frac{0.153}{180.13 \times 0.45} = 0.001888\ [m^2/m]$$

$$A_s = 18.88\ [cm^2/m]$$

Chosen steel 8Φ18/m = 20.4 [cm^2/m]

8.2 Check for punching shear

The critical punching shear section on a perimeter at a distance d/2 = 0.225 [m] from the face of the column as shown in Figure (20).

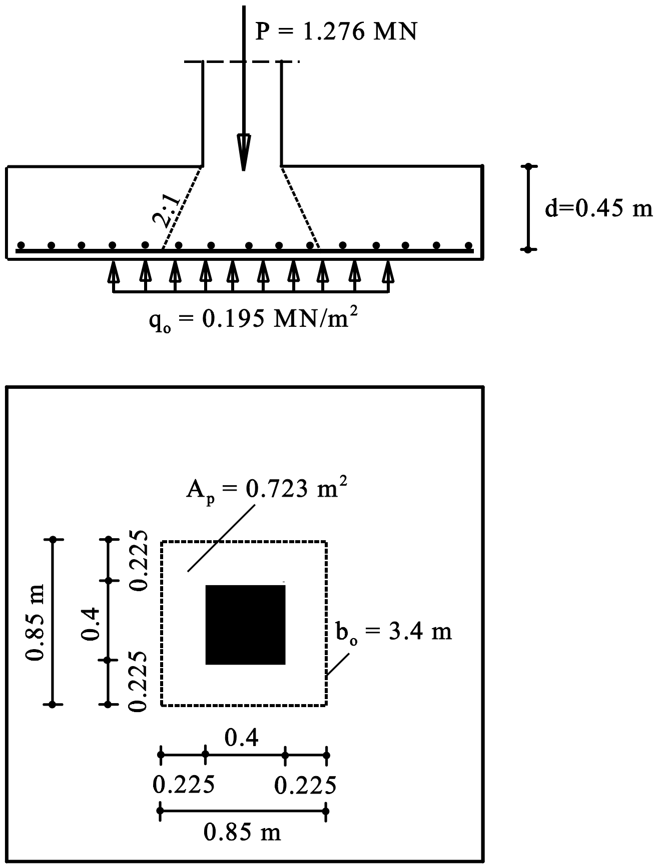


Figure (20) Critical section for Punching shear according to ECP

Geometry (Figure (20))

Effective depth of the section	d = 0.45 [m]
Column side	a = b = 0.4 [m]
Area of critical punching shear section	$A_p = (a+d)^2 = (0.4+0.45)^2 = 0.723$ [m^2]
Perimeter of critical punching shear section	$b_o = 4\,(a+d) = 4\,(0.4+0.45) = 3.4$ [m]

Loads and stresses

Column load	P = 1276 [kN] = 1.276 [MN]
Soil pressure under the column	q_o = 195 [kN/m^2] = 0.195 [MN/m^2]
Main value of shear strength for concrete C 250 according to Table (4)	q_{cp} = 0.9 [MN/m^2]

Check for section capacity

The punching shear force Q_o is

$$Q_p = P - q_o \, A_p$$

$$Q_p = 1.276 - 0.195 \times 0.723 = 1.135 \ [MN]$$

The punching shear stress q_p is given by:

$$q_p = \frac{Q_p}{b_o \, d}$$

$$q_p = \frac{1.135}{3.4 \times 0.45} = 0.742 \ [MN/m^2]$$

The allowable concrete punching strength q_{pall} [MN/m^2] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = \left(0.5 + \frac{0.4}{0.4}\right) 0.9 \leq 0.9$$

$$q_{pall} = 0.9 \ [MN/m^2]$$

$q_{pall} = 0.9$ [MN/m^2] > $q_p = 0.742$ [MN/m^2], the section is safe for punching shear.

9 Comparison of design results

9.1 Comparison of flexure moment results

Table (11) shows a comparison of flexure moment results according to different codes. It can be concluded that design of a footing for the same thickness, load and load factor according to the codes EC 2, DIN 1045, ACI and ECP yields nearly the same reinforcement.

Table (11) Comparison of different design codes for the flexure moment

Design code	Required area of steel [cm^2/m]	Chosen reinforcement A_s
EC 2	17.27	7Φ18/m =17.8 [cm^2/m]
DIN 1045	17.24	7Φ18/m =17.8 [cm^2/m]
ACI	16.37	7Φ18/m =17.8 [cm^2/m]
ECP (limit)	17.19	7Φ18/m =17.8 [cm^2/m]
ECP (working)	18.88	8Φ18/m = 20.4 [cm^2/m]

9.2 Comparison of punching shear results

Table (12) shows a comparison of punching shear results according to different design codes. The calculations using the codes EC 2, DIN 1045, ACI and ECP yield the same results except the code DIN 1045, which increase the depth of the footing 10 [cm].

Table (12) Comparison of different design codes for punching shear

Design code	Action	Resistance	Difference [%]
EC 2	ν_{Sd} = 0.194 [MN/m]	ν_{Rd1} = 0.203 [MN/m]	4.43
DIN 1045	τ_r = 0.781 [MN/m^2]	τ_{r1} = 0.532 [MN/m^2]	-46.81
ACI	V_u = 1.702 [MN]	φ V_c = 1.978 [MN]	13.95
ECP (limit)	q_{up} = 1.112 [MN]	q_{cup} = 1.29 [MN]	13.80
ECP (working)	q_p = 0.742 [MN/m^2]	q_{pall} = 0.9 [MN/m^2]	17.56

Figure (21) shows the footing dimensions and reinforcement according to EC 2, ACI and ECP (limit). Design for ECP (working) increases the reinforcement to 8Φ16/m, while for DIN 1045 increases the thickness of footing to 0.6 [m].

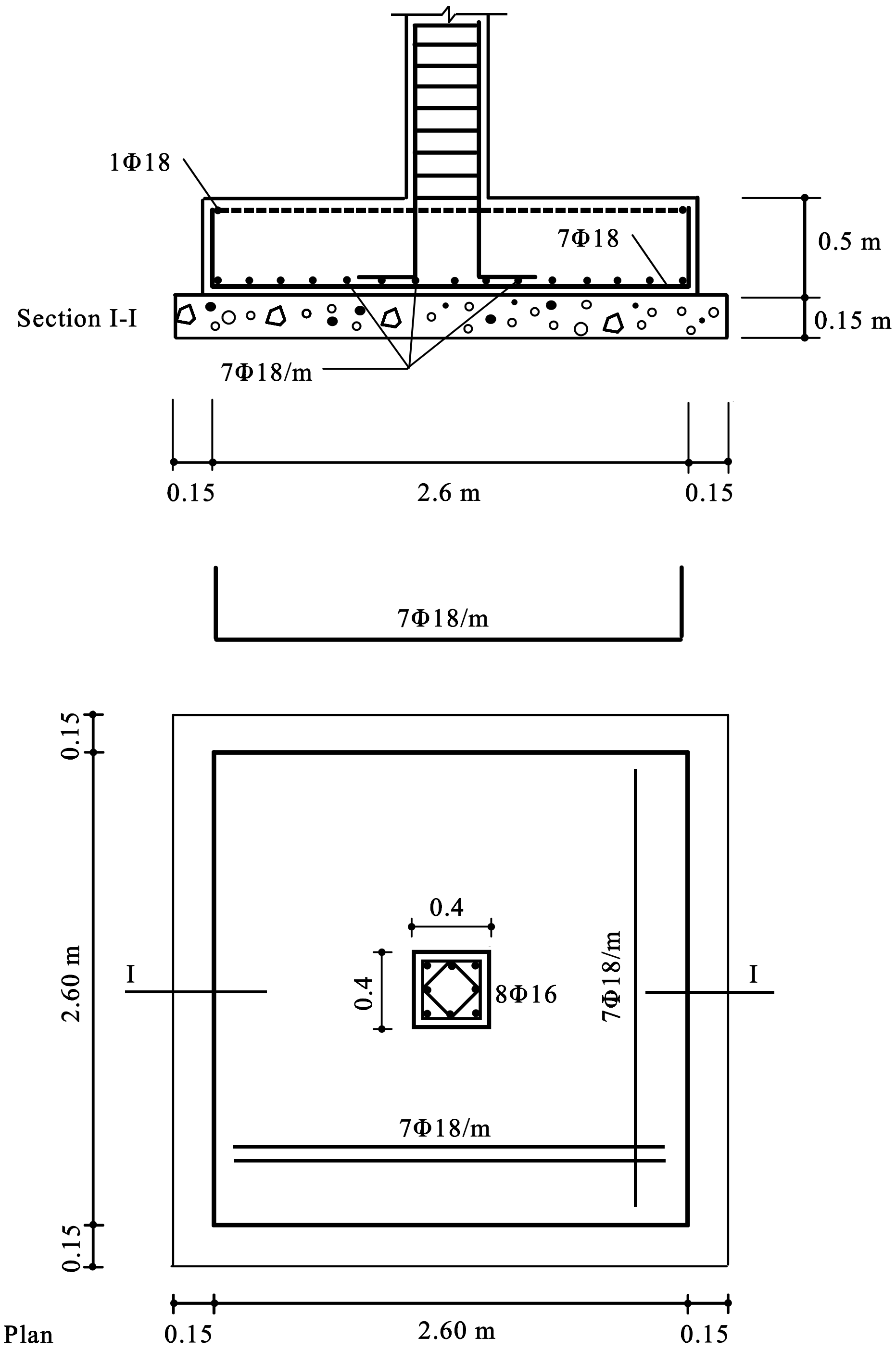


Figure (21) Footing dimensions and reinforcement according to EC 2, ACI and ECP (limit)

Example 2: Design of a square raft for different soil models and codes

1 Description of the problem

Many soil models are used to analysis of raft foundations. Each model gives internal forces for the raft different from that of the others. However, all models are considered save and correct. This example is carried out to show the differences in the design results when the raft is analyzed by different soil models.

A square raft has dimensions of 10 [m] * 10 [m] is chosen. The raft carries four symmetrical loads, each 1200 [kN] as shown in Figure (22). Column sides are 0.50 [m] * 0.50 [m], while column reinforcement is 8Φ19. To carry out the comparison of the different codes and soil models, the raft thickness is chosen d =0.6 [m] for all soil models and design codes.

The raft rests on a homogeneous soil layer of thickness 10 [m] equal to the raft length, overlying a rigid base. The modulus of compressibility of the soil layer is E_s = 10 000 [kN/m^2], while Poisson's ratio of the soil is ν_s = 0.3 [1].

The three subsoil models: simple assumption model, Winkler's model and Continuum model (Isotopic elastic half-space soil medium and Layered soil medium) are represented by four mathematical calculation methods that are available in program ELPLA (Table (13)).

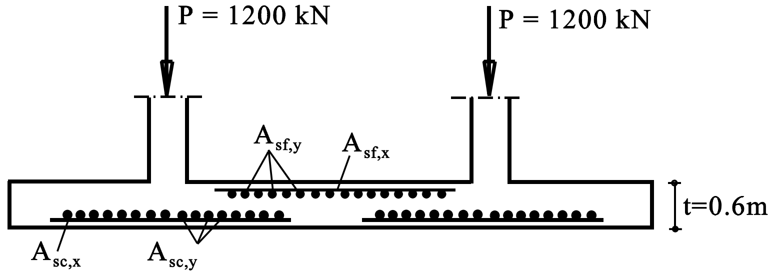
Table (13) Calculation methods

Method No.	Method
1	Linear contact pressure (Simple assumption model)
2	Constant modulus of subgrade reaction (Winkler's model)
5	Modulus of compressibility method for elastic raft on half-space soil medium (Isotopic elastic half-space soil medium - Continuum model)
7	Modulus of compressibility method for elastic raft on layered soil medium (Solving system of linear equations by elimination) (Layered soil medium - Continuum model)

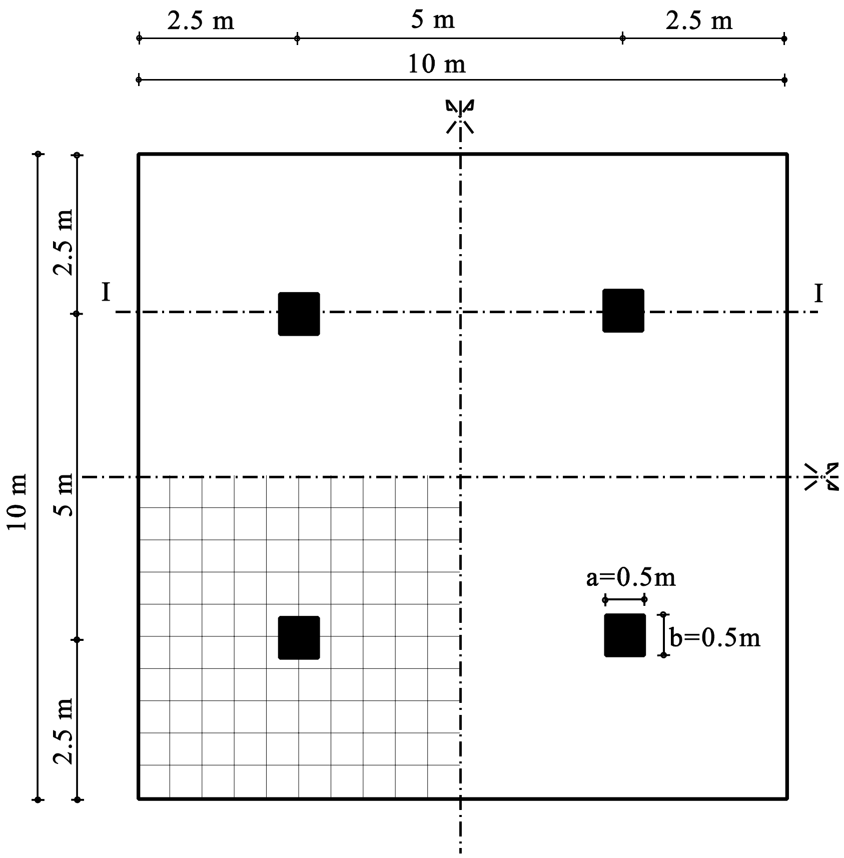
2 Properties of raft material and section

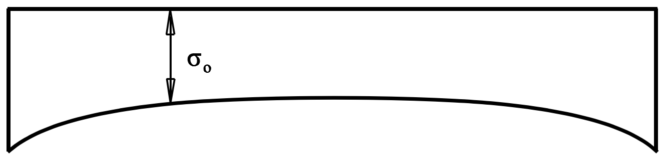
Material properties

Young's modulus of concrete	E_b	= 34 000 000	[kN/m^2]
Poisson's ratio of concrete	ν_b	= 0.20	[1]
Unit weight of concrete	γ_b	= 25	[kN/m^3]

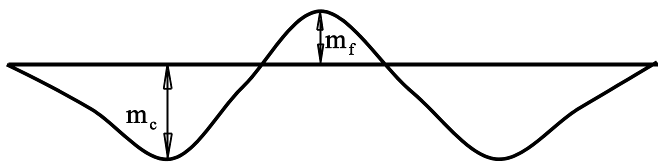


Section I-I





Contact pressure at section I-I



Moment at section I-I

Figure (22) Raft dimensions with mesh and loads

Section properties

Width of the section to be designed	b	= 1.0	[m]
Section thickness	t	= 0.6	[m]
Concrete cover + 1/2 bar diameter	c	= 5	[cm]
Effective depth of the section	d	= t - c = 0.55	[m]
Steel bar diameter	Φ	= 22	[mm]
Minimum area of steel	min A_s	= 5 Φ 19 = 14.2	[cm^2/m]

3 Analysis of the raft

The raft is subdivided into 400 square elements. Each has dimensions of 0.50 [m] * 0.50 [m] yielding to 21 * 21 nodal points for the raft and soil. Taking advantage of the symmetry in shape and load geometry about x-and y-axes, the analysis is carried out considering only a quarter of the raft. Because of the raft symmetry, the design is carried out only for section I-I.

Table (14) shows the contact pressure under the column σ_o, field moment m_f and the column moment m_c at the critical section I-I by application of different soil models. For the different codes, the raft is designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained. Finally, a comparison of the results of the two codes and soil models is presented.

Table (14) Contact pressure σ_o under the column, field moment m_f and column moment m_c at the critical section I-I by application of different soil models

Soil model	σ_o [kN/m^2]	m_c [kN.m/m]	m_f [kN.m/m]
Simple assumption model (1)	63	400	-13
Winkler's model (2)	62	399	-15
Isotopic elastic half-space medium (5)	42	504	136
Layered medium (7)	45	492	111

4 Design for EC 2

4.1 Design for flexure moment

Material

Concrete grade C 30/37
Steel grade BSt 500
Characteristic compressive cylinder strength of concrete $f_{ck} = 30$ [MN/m^2]
Characteristic tensile yield strength of reinforcement $f_{yk} = f_y = 500$ [MN/m^2]
Partial safety factor for concrete strength $\gamma_c = 1.5$
Design concrete compressive strength $f_{cd} = f_{ck}/\gamma_c = 30/1.5 = 20$ [MN/m^2]
Partial safety factor for steel strength $\gamma_s = 1.15$
Design tensile yield strength of reinforcing steel $f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 435$ [MN/m^2]

Factored moment

Total load factor for both dead and live loads $\gamma = 1.5$
Factored column moment $M_{sd} = \gamma m_c$
Factored field moment $M_{sd} = \gamma m_f$

Geometry

Effective depth of the section $d = 0.55$ [m]
Width of the section to be designed $b = 1.0$ [m]

Determination of tension reinforcement

The design of sections is carried out for EC 2 in table forms. Tables (15) and (16) show the design of section I-I.

The normalized design moment μ_{sd} is

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 0.55^2 \times (0.85 \times 20)} = 0.195\ M_{sd}$$

The normalized steel ratio ω is

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.195\ M_{sd}} = 1 - \sqrt{1 - 0.39\ M_{sd}}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega \left(\frac{(0.85 f_{cd})\ b\ d}{f_{yd}} \right)$$

$$A_s = \omega \left(\frac{0.85 \times 20 \times 1.0 \times 0.55}{435} \right) = 0.021493\ \omega\ [m^2/m]$$

$$A_s = 214.943\ \omega\ [cm^2/m]$$

Table (15) Required bottom reinforcement under the column A_{sc} for different soil models

Soil model	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{sc} [cm^2/m]
Simple assumption model (1)	0.600	0.117	0.124	26.76
Winkler's model (2)	0.599	0.116	0.124	26.70
Isotopic elastic half-space medium (5)	0.757	0.147	0.160	34.39
Layered medium (7)	0.737	0.143	0.156	33.43

Table (16) Required top reinforcement in the field A_{sf} for different soil models

Soil model	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{sf} [cm^2/m]
Simple assumption model (1)	0.0197	0.0038	0.0038	0.83
Winkler's model (2)	0.0223	0.0043	0.0044	0.94
Isotopic elastic half-space medium (5)	-	-	-	-
Layered medium (7)	-	-	-	-

Chosen reinforcement

Table (17) shows the number of steel bars under the column and in the field between columns at section I-I considering different soil models. The chosen diameter of steel bars is Φ = 22 [mm].

Table (17) Chosen reinforcement at section I-I for different soil models

Soil model	Chosen reinforcement	
	Bottom Rft under column A_{sc}	Top Rft in the field A_{sf}
Simple assumption model (1)	8 Φ 22 = 30.40 [cm^2/m]	min A_s
Winkler's model (3)	8 Φ 22 = 30.40 [cm^2/m]	min A_s
Isotopic elastic half-space medium (5)	10 Φ 22 = 38.00 [cm^2/m]	min A_s
Layered medium (7)	9 Φ 22 = 34.20 [cm^2/m]	min A_s

4.2 Check for punching shear

The critical section for punching shear is at a distance r = 0.825 [m] around the circumference of the column as shown in Figure (23).

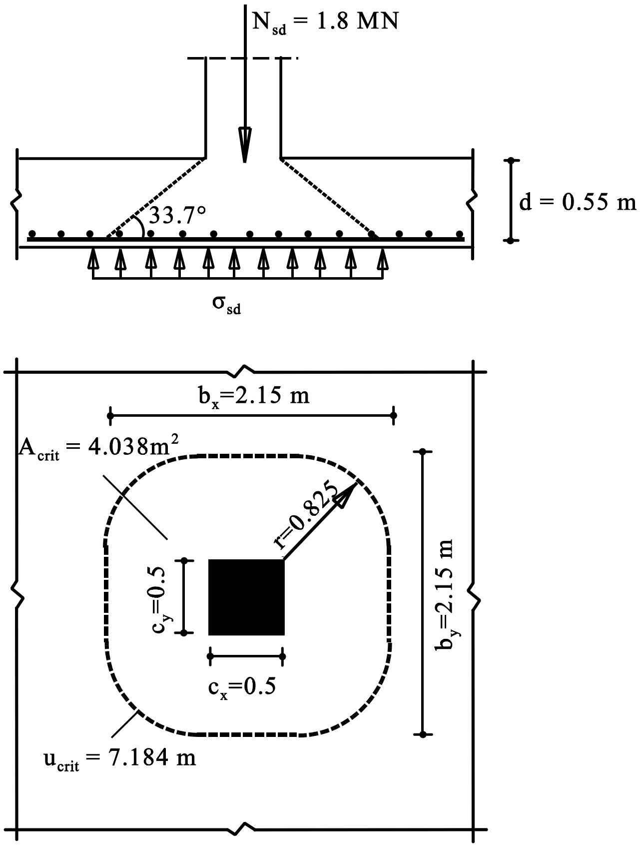


Figure (23) Critical section for Punching shear according to EC 2

Geometry (Figure (23))

Effective depth of the section $d = d_x = d_y = 0.55$ [m]
Column side $c_x = c_y = 0.5$ [m]
Distance of critical punching section from circumference of the column
$r = 1.5\ d = 1.5*0.55 = 0.825$ [m]
Area of critical punching shear section
$A_{crit} = c_x^{\ 2} + 4\ r\ c_x + \pi\ r^2 = (0.5)^2 + 4*0.825*0.5 + \pi\ 0.825^2 = 4.038$ [m²]
Perimeter of critical punching shear section $u_{crit} = 4c_x + 2\ \pi\ r = 4*0.5 + 2\ \pi\ 0.825 = 7.184$ [m]
Width of punching section $b_x = b_y = c_x + 2\ r = 0.5 + 2*0.825 = 2.15$ [m]
Correction factor for interior column $\beta = 1.15$
Coefficient for consideration of the slab thickness $k = 1.6 - d = 1.6 - 0.55 = 1.05$ [m] > 1.0 [m]
Steel ratio $\rho_1 = \rho_{1x} = \rho_{1y} = A_{sx} / (b_y\ d_x) = (A_s*10^{-4}) / (0.55) = 0.00018\ A_s$

Loads and stresses

Total load factor for both dead and live loads	$\gamma = 1.5$
Column load	N = 1200 [kN] =1.2 [MN]
Factored column load	$N_{sd} = \gamma N = 1.5*1.2 = 1.8$ [MN]
Factored upward soil pressure under the column	$\sigma_{Sd} = \gamma\ \sigma_o$

Main value of shear strength for concrete C 30/37 according to Table (1)

$\tau_{Rd} = 1.2*0.28 = 0.336$ [MN/m]

Check for section capacity

The punching force at ultimate design load V_{Sd} is

$$V_{Sd} = N_{Sd} - \sigma_{Sd}\ A_{crit}$$

$$V_{Sd} = 1.8 - 4.038\ \sigma_{Sd}\ [MN]$$

The design value of the applied shear ν_{Sd} is

$$\nu_{Sd} = \frac{V_{Sd}\ \beta}{u_{crit}}$$

$$\nu_{Sd} = \frac{(1.8 - 4.038\ \sigma_{Sd})\ 1.15}{7.184} = 0.288 - 0.646\ \sigma_{Sd}\ [MN/m]$$

Design shear resistance from concrete alone ν_{Rd1} is

$$\nu_{Rd1} = \tau_{Rd}\ k\ (1.2 + 40\ \rho_1)\ d$$

$$\nu_{Rd1} = 0.336(1.05\ (1.2 + 40(0.00018\ A_s)\ 0.55$$

$$\nu_{Rd1} = 0.233 + 0.0014\ A_s\ [MN/m]$$

Table (18) shows the check for punching shear by application of different soil models where for all soil models $\nu_{sd} < \nu_{Rd1}$. Therefore, the section is safe for punching shear.

Table (18) Check for punching shear by application of different soil models

Soil model	σ_{sd} [MN/m^2]	A_s [cm^2/m]	ν_{sd} [MN/m]	ν_{Rd1} [MN/m]
Simple assumption model (1)	0.095	30.40	0.227	$0.276 > \nu_{sd}$
Winkler's model (2)	0.093	30.40	0.228	$0.276 > \nu_{sd}$
Isotopic elastic half-space medium (5)	0.063	38.00	0.247	$0.286 > \nu_{sd}$
Layered medium (7)	0.068	34.20	0.244	$0.281 > \nu_{sd}$

5 Design for DIN 1045

5.1 Design for flexure moment

Material

Concrete grade	B 35	
Steel grade	BSt 500	
Concrete compressive strength		$\beta_R = 23$ [MN/m^2]
Tensile yield strength of steel		$\beta_S = 500$ [MN/m^2]

Geometry

Effective depth of the section	h = 0.55 [m]
Width of the section to be designed	b = 1.0 [m]

Determination of tension reinforcement

The design of sections is carried out for DIN 1045 in table forms. Tables (19) and (20) show the design of section I-I.

The normalized design moment m_s is

$$m_s = \frac{M_s}{bh^2\left(\frac{\alpha_R \beta_R}{\gamma}\right)}$$

$$m_s = \frac{M_s}{1.0 \times 0.55^2 \left(\frac{0.95 \times 23}{1.75}\right)} = 0.264765 \; M_s$$

The normalized steel ratio ω_M is

$$\omega_M = 1 - \sqrt{1 - 2 \; m_s}$$

$$\omega_M = 1 - \sqrt{1 - 2 \times 0.264765 \; M_s} = 1 - \sqrt{1 - 0.5295 \; M_s}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega_M \left(\frac{(\alpha_R \beta_R)\ b\ h}{\beta_S} \right)$$

$$A_s = \omega_M \left(\frac{(0.95 \times 23) \times 1.0 \times 0.55}{500} \right) = 0.024035\ \omega_M\ [m^2/m]$$

$$A_s = 240.35\ \omega_M\ [cm^2/m]$$

Table (19) Required bottom reinforcement under the column A_{sc} for different soil models

Soil model	M_s [MN.m/m]	m_s	ω_M	A_{sc} [cm^2/m]
Simple assumption model (1)	0.400	0.106	0.112	26.97
Winkler's model (2)	0.399	0.106	0.112	26.91
Isotopic elastic half-space medium (5)	0.504	0.134	0.144	34.59
Layered medium (7)	0.492	0.130	0.140	33.64

Table (20) Required top reinforcement in the field A_{sf} for different soil models

Soil model	M_s [MN.m/m]	m_s	ω_M	A_{sf} [cm^2/m]
Simple assumption model (1)	0.013	0.00348	0.00348	0.84
Winkler's model (2)	0.015	0.00394	0.00395	0.95
Isotopic elastic half-space medium (5)	-	-	-	-
Layered medium (7)	-	-	-	-

Chosen reinforcement

Table (21) shows the number of steel bars under the column and in the field between columns at section I-I considering different soil models. The chosen diameter of steel bars is Φ = 22 [mm].

Table (21) Chosen reinforcement at section I-I for different soil models

Soil model	Chosen reinforcement	
	Bottom Rft under column A_{sc}	Top Rft in the field A_{sf}
Simple assumption model (1)	8 Φ 22 = 30.40 [cm^2/m]	min A_s
Winkler's model (3)	8 Φ 22 = 30.40 [cm^2/m]	min A_s
Isotopic elastic half-space medium (5)	10 Φ 22 = 38.00 [cm^2/m]	min A_s
Layered medium (7)	9 Φ 22 = 34.20 [cm^2/m]	min A_s

5.2 Check for punching shear

The critical section for punching shear is a circle of diameter d_r = 1.115 [m] around the circumference of the column as shown in Figure (24).

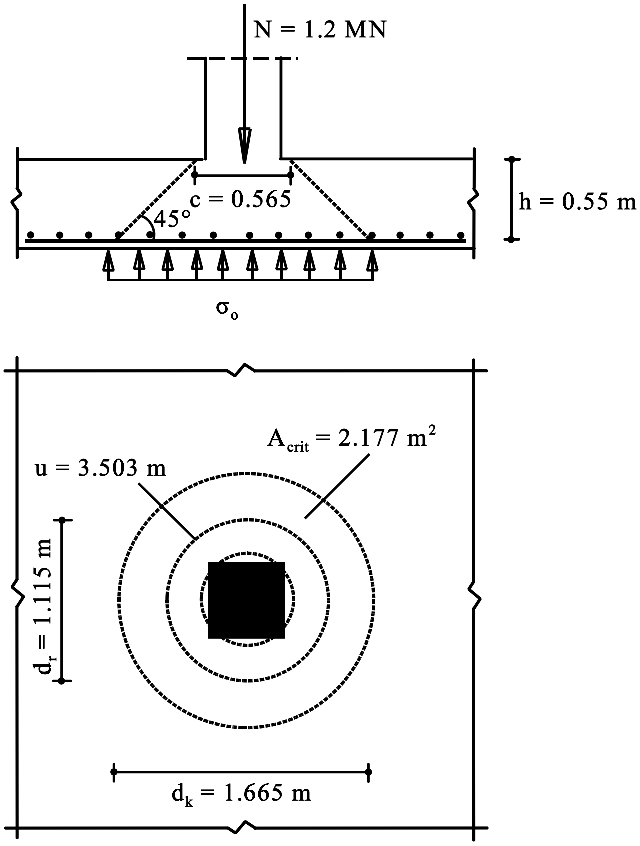


Figure (24) Critical section for Punching shear according to DIN 1045

Geometry (Figure (24))

Effective depth of the section	$h = 0.55$ [m]
Column side	$c_x = c_x = 0.5$ [m]
Average diameter of the column	$c = 1.13*0.5 = 0.565$ [m]
Diameter of loaded area	$d_k = 2\ h + c = 2*0.55+0.565 = 1.665$ [m]
Diameter of critical punching shear section	$d_r = c+h = 0.565+0.55 = 1.115$ [m]
Area of critical punching shear section	$A_{crit} = \pi\ d_k^2/4 = \pi\ 1.665^2/4 = 2.177$ [m^2]
Perimeter of critical punching shear section	$u = \pi\ d_r = \pi\ 1.115 = 3.503$ [m]

Loads and stresses

Column load $N = 1200$ [kN] = 1.2 [MN]

Main value of shear strength for concrete B 35 according to Table (2) $\tau_{011} = 0.6$ [MN/m^2]

Factor depending on steel grade according to Table (6) $\alpha_s = 1.4$

Check for section capacity

The punching shear force Q_r is

$$Q_r = N - \sigma_o A_{crit}$$

$$Q_r = 1.2 - 2.177\ \sigma_o\ [MN]$$

The punching shear stress τ_r is

$$\tau_r = \frac{Q_r}{u\ h}$$

$$\tau_r = \frac{1.2 - 2.177\ \sigma_o}{3.503 \times 0.55} = 0.623 - 1.130\ \sigma_o\ [MN/m^2]$$

Reinforcement grade μ_g is

$$\mu_g = \frac{A_{sx} + A_{sy}}{2h}$$

$$\mu_g = \frac{2\ A_s}{2 \times 0.55 \times 100} = 0.018\ A_s\ [\%]$$

Coefficient for consideration of reinforcement κ_1 is

$$\kappa_1 = 1.3\ \alpha_s \sqrt{\mu_g}$$

$$\kappa_1 = 1.3 \times 1.4 \sqrt{0.018\ A_s} = 0.245\ \sqrt{A_s}$$

The allowable concrete punching strength τ_{r1} [MN/m^2] is

$$\tau_{r1} \;\; ' \;\; \kappa_1 \; \tau_{011}$$

$$\tau_{r1} \;\; ' \;\; 0.2454 \; \sqrt{A_s}\,(0.6 \;\; ' \;\; 0.147 \; \sqrt{A_s} \;\; [MN/m^2]$$

Table (22) shows the check for punching shear by application of different soil models where for all soil models $\tau_r < \tau_{r1}$. Therefore, the section is safe for punching shear.

Table (22) Check for punching shear by application of different soil models

Soil model	σ_o [MN/m^2]	A_s [cm^2/m]	τ_r [MN/m^2]	τ_{r1} [MN/m^2]
Simple assumption model (1)	0.063	30.40	0.552	0.811 > τ_r
Winkler's model (2)	0.062	30.40	0.553	0.811 > τ_r
Isotopic elastic half-space medium (5)	0.042	38.00	0.576	0.906 > τ_r
Layered medium (7)	0.045	34.20	0.572	0.860 > τ_r

6 Comparison between the design according to DIN 1045 and EC 2

Table (23) shows the comparison between the design of the raft according to DIN 1045 and EC 2 by application of different soil model. The comparison is considered only for bottom reinforcement under the column.

It can be concluded from the comparison that if the raft is designed according to EC 2 using a load factor of $\gamma = 1.5$ and DIN 1045, the required reinforcement obtained from EC 2 will be nearly same as that obtained from DIN 1045. Finally, It can be concluded also from Tables (17) and (21) that the chosen reinforcement for both EC 2 and DIN 1045 are identical.

Table (23) Comparison between the design according to DIN 1045 and EC 2

Soil model	A_s [cm²/m] according to		Difference ΔA_s [%]
	DIN 1045	EC 2	
Simple assumption model (1)	26.97	26.76	0.78
Winkler's model (2)	26.91	26.70	0.78
Isotopic elastic half-space medium (5)	34.59	34.39	0.58
Layered medium (7)	33.64	33.43	0.62

Example 3: Design of a raft of high rise building for different soil models and codes

1 Description of the problem

Cruz (1994) under the supervision of the author examined a raft of high rise building by the program ELPLA. He carried out the examination to show the different between the design of rafts according to national code (German code) and Euro code. Here, Kany/ El Gendy (1995) has chosen the same example with some modifications. The accurate method of interpolation is used instead of subareas method to obtain the three-dimensional flexibility coefficient and modulus of subgrade reaction for Continuum and Winkler's models, respectively.

To carry out the comparison between the different design codes and soil models, three different soil models are used to analyze the raft. In this example, three mathematical calculation methods are chosen to represent the three soil models: simple assumption, Winkler's and Continuum models as shown in Table (24).

Table (24) Calculation methods and soil models

Method No.	Calculation method	Soil model
1	Linear contact pressure method	Simple assumption model
3	Variable modulus of subgrade reaction method	Winkler's model
6	Modulus of compressibility method	Continuum model

Figure (25) shows plan of the raft, column loads, dimensions, mesh with section through the raft and subsoil. The following text gives a description of the design properties and parameters.

2 Properties of raft material

Young's modulus of concrete	E_b	= 34 000 000	[kN/m^2]
Poisson's ratio of concrete	ν_b	= 0.20	[1]
Unit weight of concrete	γ_b	= 0	[kN/m^3]

Unit weight of concrete is chosen $\gamma_b = 0.0$ to neglect the self-weight of the raft.

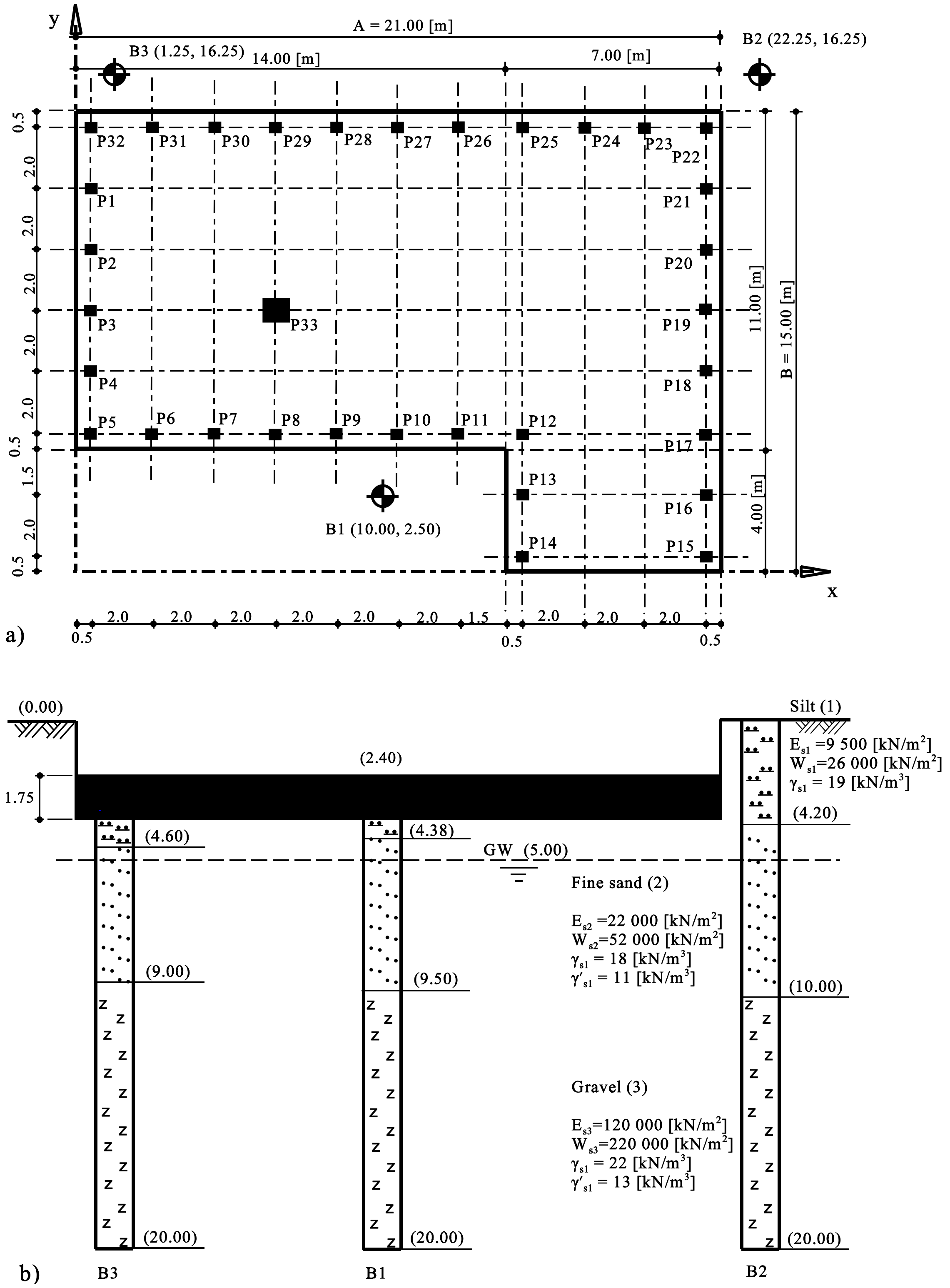


Figure (25) a) Plan of the raft with column loads, dimensions and mesh
b) Section through the raft and subsoil

3 Properties of raft section

To carry out the comparison of the different codes and soil models, the raft thickness is chosen t =1.75 [m] for all soil models and design codes. The raft section has the following parameters:

Width of the section to be designed	b	= 1.0	[m]
Section thickness	t	= 1.75	[m]
Concrete cover + 1/2 bar diameter	c	= 5	[cm]
Effective depth of the section	d	= t - c = 1.70	[m]
Steel bar diameter	Φ	= 25	[mm]
Minimum area of steel	min A_s	= 6 Φ 25 = 29.5	[cm^2/m]

4 Soil properties

Three boring logs characterize the soil under the raft. Each boring has three layers with different materials as shown in Table (25) and Figure (25). Poisson's ratio is constant for all the soil layers. The effect of reloading is taken into account. The general soil parameters are:

Poisson's ratio of the soil layers $\nu_s = 0.25$
Settlement reduction factor for sand according to DIN 4019 $\alpha = 0.66$
Level of foundation depth under ground surface $d_f = 4.15$ [m]

Table (25) Soil properties

Layer No. I	Type of soil	Depth of layer under the ground surface z [m]	Modulus of compressibility of the soil for		Unit weight above ground water γ_s [kN/m^3]	Unit weight under ground water γ'_s [kN/m^3]
			Loading E_s [kN/m^2]	Reloading W_s [kN/m^2]		
1	Silt	4.38/4.2/4.6	9 500	26 000	19	-
2	Fine sand	9.5/10.0/9.0	22 000	52 000	18	11
3	Gravel	20.0	120 000	220 000	22	13

5 Loads on the raft

The raft carries 33 column loads as shown in Figure (25). The ratio of dead to live loads N_{gk} /N_{qk} by the analysis is 70%/ 30%. Table (26) shows the raft loads according to DIN 1045 and EC 2. To obtain the results according to EC 2, the analysis of the raft may be carried out once for both codes due to the given loads. Then the results are multiplied by a global factor of safety $\gamma = 1.395$, which may be obtained through the following relation.

$N_{sd} = N_{gk} + N_{qk} = \gamma_g * G_k + \gamma_q * Q_k$ =1.35 (0.7*P) + 1.5 (0.3*P) = 1.395 P

where:
N_{sd} Design value of action
P Given column load
γ_g Partial factor for dead action, γ_g =1.35
γ_q Partial factor for live action, γ_q = 1.5
G_k Given dead load, N_{gk} = 0.7*P
Q_k Given live load, N_{qk} = 0.3*P
N_{gk} Factored dead load, N_{gk} = 0.7*P
N_{qk} Factored live load, N_{qk} = 0.3*P

Table (26) Loads on the raft

Column No.	Given column load N [kN]	Dead load (70% N) G_k [kN]	Live load (30% N) Q_k [kN]	$N_{gk}=\gamma_g G_k$ (γ_g=1.35) [kN]	$N_{qk}=\gamma_q Q_k$ (γ_q=1.50) [kN]	N_{sd}= $N_{gk} + N_{qk}$ [kN]
P22, P32	980	686	294	926	441	1367
P23 to P31	1350	945	405	1276	608	1883
P16 to P21	1380	966	414	1304	621	1925
P14, P15	1150	805	345	1087	518	1604
P13	1000	700	300	945	450	1395
P1 to P4, P12	1250	875	375	1181	563	1744
P6 to P11	1200	840	360	1134	540	1674
P5	990	693	297	936	446	1381
P33	10490	7343	3147	9913	4720	14634

6 Analysis of the raft

The raft is subdivided into 106 elements. Then, the analysis of the raft according to both the two codes DIN 1045 and EC 2 are carried out by the program ELPLA. The system of linear equations of the Continuum model is solved by iteration (method 6). The maximum difference between the soil settlement s [cm] and the raft deflection w [cm] is considered as an accuracy number. In this example, the accuracy is chosen ε = 0.001 [cm]. Because the raft is subdivided into a mesh of coarse finite elements, modifying the point load that represents the column load by is not necessary.

Determination of main modulus of subgrade reaction k_{sm} for the three boring logs

Main moduli k_{sm} equal to the number of boring logs should be determined. Each modulus corresponding to one of the soil boring logs and is calculated from the elastic material of that boring. The main moduli of subgrade reactions k_{sm} for the three boring logs are:

k_{sm1} = 12936 [kN/m^3]
k_{sm2} = 12799 [kN/m^3]
k_{sm3} = 13109 [kN/m^3]

Determination of variable modulus of subgrade reaction $k_{s.i}$

According to Kany / El Gendy (1995), The raft area is divided into three region types as shown in Figure (26).

Type I: This region is a triangular region. The three boring logs B1 to B3 confine that region. The modulus $k_{s.i}$ for a node inside the triangular region, can be determined by interpolation through the values of k_{sm} for the three boring logs.

Type II: One or more sides of the raft and two boring logs confine this region (regions of B1 and B2, B1 and B3). Assuming a linear interpolation between the values of k_{sm} for the two boring logs, can obtain the modulus $k_{s.i}$ for a node i inside this region.

Type III: One or more sides of the raft and one boring confine this region. The modulus $k_{s.i}$ for a node inside this region is equal to the modulus of that boring. For the considered raft, the regions of type III are outside the raft area.

Figure (27) shows the calculated variable modulus of subgrade reaction $k_{s.i}$ according to the interpolation method. In a similar way to the previous solution for Winkler's model, the three-dimensional coefficient of flexibility can be determined for Continuum model.

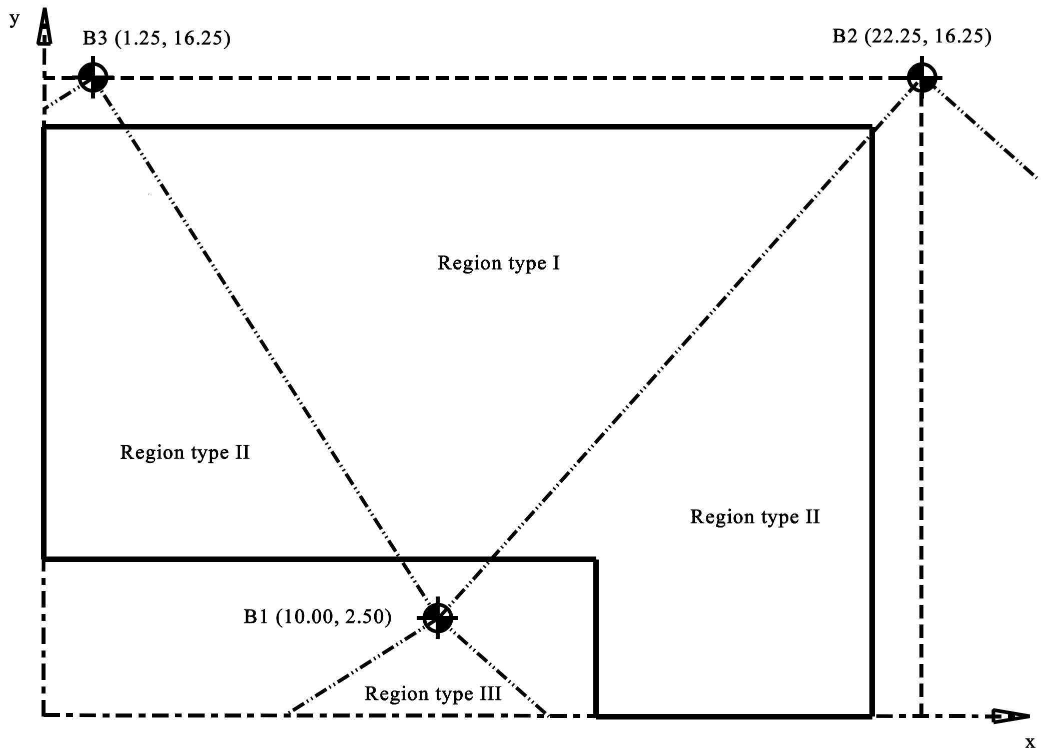


Figure (26) Boring locations and region types I, II and III

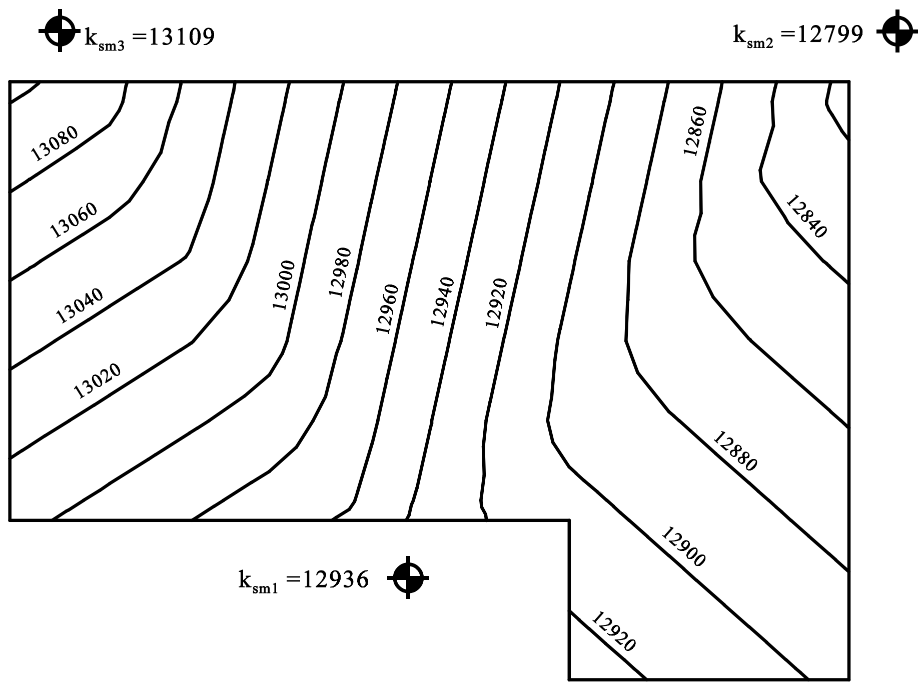


Figure (27) Contour lines for variable modulus of subgrade reaction k_s [kN/m3]

Definition of the critical sections

Two critical sections in x- and y-directions pass through the heavy loaded column P33 are considered as shown in Figure (28). In this example, the design is carried out only for the critical sections x-x and y-y in detail. Figures (29) to (30) and Table (27) show the contact pressure under the column σ_o, field moment m_f and the column moment m_c at the critical sections x-x and y-y by application of different soil models. For the codes DIN 1045 and EC 2, the sections are designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained. Finally, a comparison of the results of the two codes and soil models is presented.

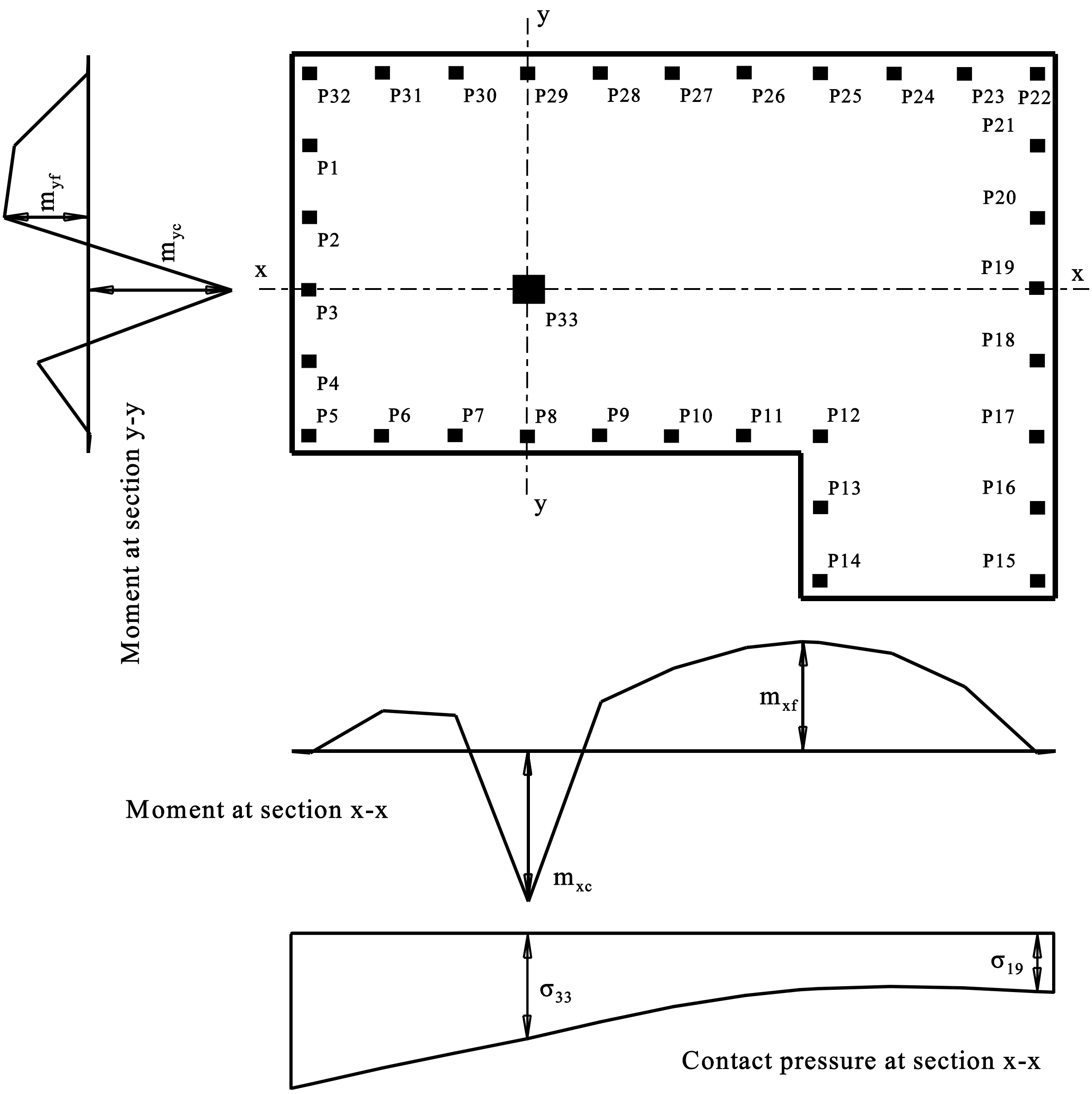


Figure (28) Definition of critical sections in x- and y-directions

Table (27) Contact pressure σ_o under the column, field moment m_f and column moment m_c at the critical sections x-x and y-y by application of different soil models

Soil model	Contact pressure [kN/m²]		Column moment [kN.m/m]		Field moment [kN.m/m]	
	σ_{33}	σ_{19}	m_{xc}	m_{yc}	m_{xf}	m_{yf}
Simple assumption model (1)	221	145	1444	1424	-2182	-1137
Winkler's model (3)	217	164	1827	1527	-1728	-1026
Continuum model (6)	181	159	2320	1866	-1292	-694

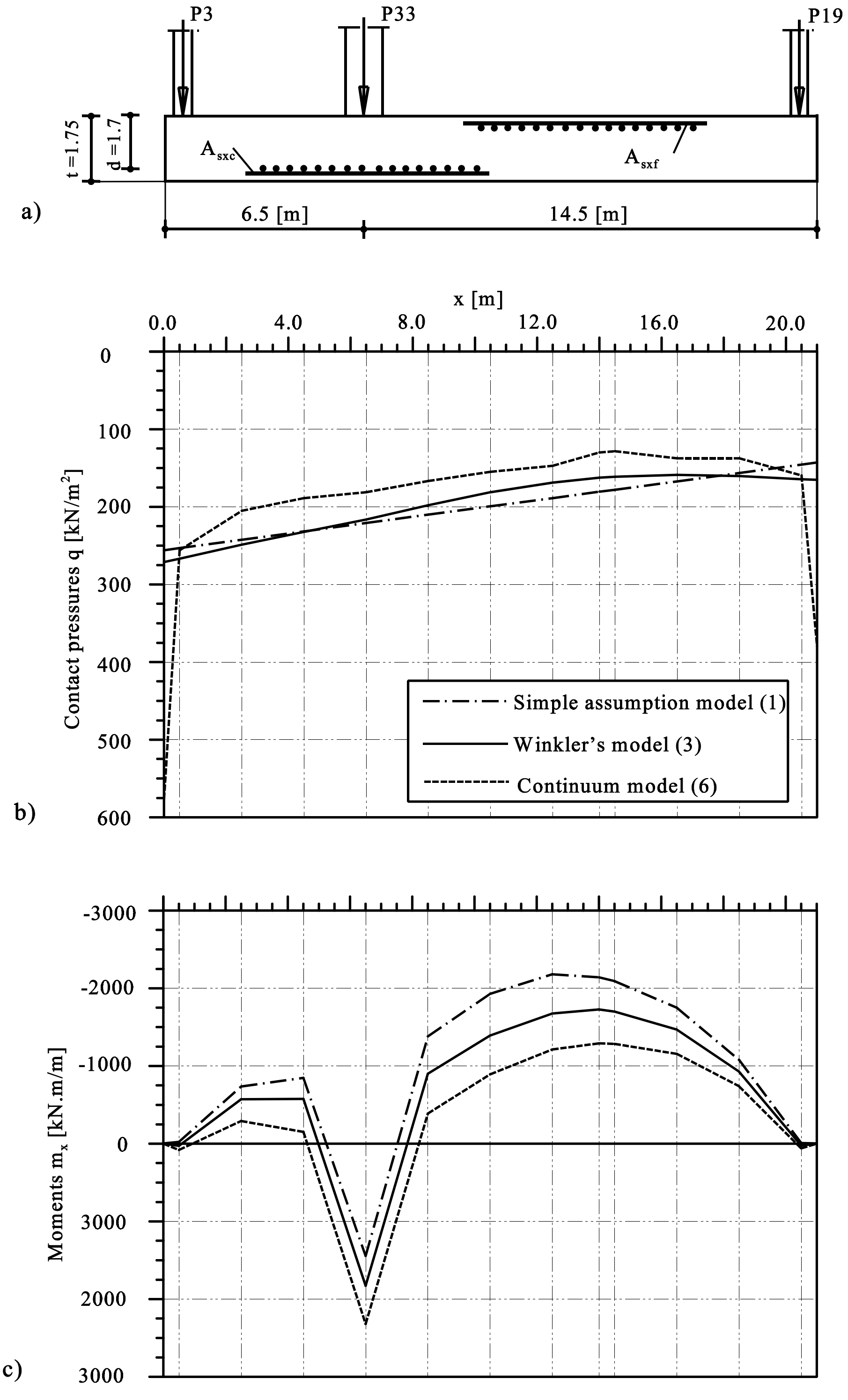


Figure (29) a) Section x-x through the raft
b) Moment m_x [kN.m/m] at section x-x
c) Contact pressure q [kN/m^2] at section x-x

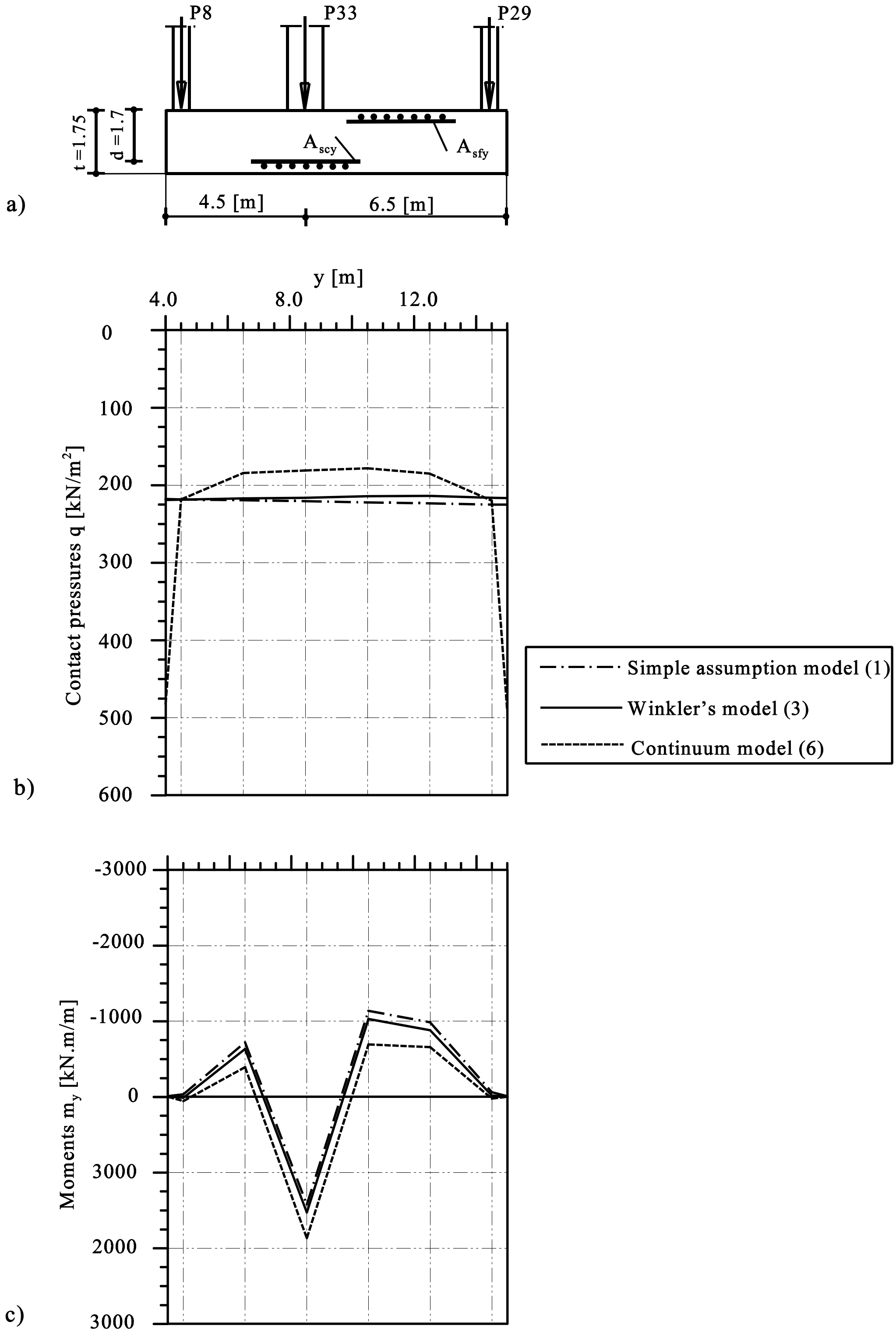


Figure (30) a) Section y-y through the raft
b) Moment m_y [kN.m/m] at section y-y
c) Contact pressure q [kN/m^2] at section y-y

7 Design for EC 2

7.1 Design for flexure moment

Material

Concrete grade	C 30/37	
Steel grade	BSt 500	
Characteristic compressive cylinder strength of concrete		$f_{ck} = 30$ [MN/m^2]
Characteristic tensile yield strength of reinforcement		$f_{yk} = f_y = 500$ [MN/m^2]
Partial safety factor for concrete strength		$\gamma_c = 1.5$
Design concrete compressive strength		$f_{cd} = f_{ck}/\gamma_c = 30/1.5 = 20$ [MN/m^2]
Partial safety factor for steel strength		$\gamma_s = 1.15$
Design tensile yield strength of reinforcing steel		$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 435$ [MN/m^2]

Factored moment

Total load factor for both dead and live loads	$\gamma = 1.395$
Factored column moment	$M_{sd} = \gamma m_c$
Factored field moment	$M_{sd} = \gamma m_f$

Geometry

Effective depth of the section	$d = 1.7$ [m]
Width of the section to be designed	$b = 1.0$ [m]

Determination of tension reinforcement

The design of sections is carried out for EC 2 in table forms. Tables (28) to (31) show the design of sections x-x and y-y.

The normalized design moment μ_{sd} is

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 1.70^2 \times (0.85 \times 20)} = 0.0204\ M_{sd}$$

The normalized steel ratio ω is

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.0204\ M_{sd}} = 1 - \sqrt{1 - 0.0408\ M_{sd}}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega \left(\frac{(0.85 f_{cd})\ b\ d}{f_{yd}} \right)$$

$$A_s = \omega \left(\frac{0.85 \times 20 \times 1.0 \times 1.70}{435} \right) = 0.0664368\ \omega\ [m^2/m]$$

$$A_s = 664.368\ \omega\ [cm^2/m]$$

Table (28) Required bottom reinforcement under the column A_{sxc} for different soil models (section x-x)

Soil model	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{sxc} [cm^2/m]
Simple assumption model (1)	2.014	0.041	0.042	27.89
Winkler's model (3)	2.549	0.052	0.053	35.50
Continuum model (6)	3.236	0.066	0.068	45.40

Table (29) Required top reinforcement in the field A_{sxf} for different soil models (section x-x)

Soil model	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{sxf} [cm^2/m]
Simple assumption model (1)	3.044	0.062	0.064	42.62
Winkler's model (3)	2.411	0.049	0.050	33.52
Continuum model (6)	1.802	0.037	0.038	24.89

Table (30) Required bottom reinforcement under the column A_{syc} for different soil models (section y-y)

Soil model	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{syc} [cm^2/m]
Simple assumption model (1)	1.987	0.041	0.041	27.49
Winkler's model (3)	2.130	0.044	0.044	29.53
Continuum model (6)	2.603	0.053	0.055	36.27

Table (31) Required top reinforcement in the field A_{syf} for different soil models (section y-y)

Soil model	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{syf} [cm^2/m]
Simple assumption model (1)	1.586	0.032	0.033	21.86
Winkler's model (3)	1.431	0.029	0.030	19.69
Continuum model (6)	0.968	0.020	0.020	13.25

Chosen reinforcement

Table (32) and (33) show the number of steel bars under the column and in the field between columns at sections x-x and y-y considering different soil models. The chosen diameter of steel bars is Φ = 25 [mm].

Table (32) Chosen reinforcement at section x-x for different soil models

Soil model	Chosen reinforcement	
	Bottom Rft under the column A_{sxc}	Top Rft in the field A_{sxf}
Simple assumption model (1)	min A_s = 29.50 [cm^2/m]	9 Φ 25 = 44.20 [cm^2/m]
Winkler's model (3)	8 Φ 25 = 39.30 [cm^2/m]	7 Φ 25 = 34.40 [cm^2/m]
Continuum model (6)	10 Φ 25 = 49.10 [cm^2/m]	min A_s = 29.50 [cm^2/m]

Table (33) Chosen reinforcement at section y-y for different soil models

Soil model	Chosen reinforcement	
	Bottom Rft under the column A_{syc}	Top Rft in the field A_{syf}
Simple assumption model (1)	min A_s = 29.50 [cm^2/m]	min A_s = 29.50 [cm^2/m]
Winkler's model (3)	min A_s = 29.50 [cm^2/m]	min A_s = 29.50 [cm^2/m]
Continuum model (6)	8 Φ 25 = 39.30 [cm^2/m]	min A_s = 29.50 [cm^2/m]

7.2 Check for punching shear

7.2.1 Interior column (column P33)

The critical punching shear section for interior column is considered at column P33. The column dimensions are chosen to be 90/90 [cm]. The critical section for punching shear is at a distance r = 2.55 [m] around the circumference of the column as shown in Figure (31).

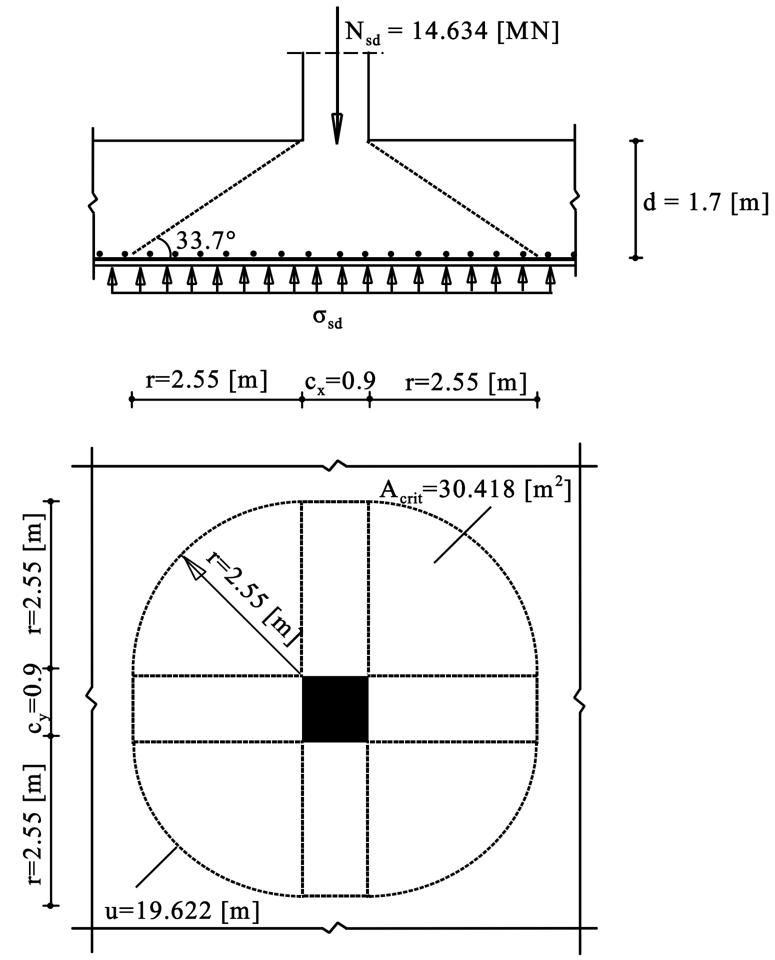


Figure (31) Critical section for Punching shear according to EC 2

Geometry (Figure (31))

Effective depth of the section $d = d_x = d_y = 1.70$ [m]
Column side $c_x = c_y = 0.9$ [m]
Distance of critical punching section from circumference of the column
$r = 1.5\ d = 1.5*1.70 = 2.55$ [m]
Area of critical punching shear section
$A_{crit} = c_x{}^2 + 4\ r\ c_x + \pi\ r^2 = (0.9)^2 + 4*2.55*0.9 + \pi\ 2.55^2 = 30.418$ [m^2]

Perimeter of critical punching shear section $u_{crit} = 4c_x + 2\,\pi\, r = 4*0.9+2\,\pi\, 2.55 = 19.622$ [m]
Width of punching section $b_x = b_y = c_x + 2\,r = 0.9+2*2.55 = 6.0$ [m]
Correction factor for interior column $\beta = 1.15$
Coefficient for consideration of the slab thickness $k = 1.6 - d = 1.6-1.70 = -0.1<1.0$ [m]
k is taken 1.0 [m]
Steel ratio $\rho_{1x} = A_{sx}/(b_y\, d_x) = (A_{sxc}*10^{-4})/(1.70) = 0.0000588\, A_{sxc}$
Steel ratio $\rho_{1y} = A_{sy}/(b_x\, d_y) = (A_{syc}*10^{-4})/(1.70) = 0.0000588\, A_{syc}$
Steel ratio $\rho_1 = \sqrt{\rho_{1x} * \rho_{1y}} = 0.0000588 \sqrt{A_{sxc} * A_{syc}}$

Loads and stresses

Total load factor for both dead and live loads	$\gamma = 1.395$
Column load	$N = 10490$ [kN] $= 10.49$ [MN]
Factored column load	$N_{sd} = \gamma\, N = 1.395*10.49 = 14.634$ [MN]
Factored upward soil pressure under the column	$\sigma_{Sd} = \gamma\, \sigma_o$

Main value of shear strength for concrete C 30/37 according to Table (1)
$\tau_{Rd} = 1.2*0.28 = 0.336$ [MN/m]

Check for section capacity

The punching force at ultimate design load V_{Sd} is

$$V_{Sd} = N_{Sd} - \sigma_{Sd}\, A_{crit}$$

$$V_{Sd} = 14.634 - 30.418\, \sigma_{Sd}\ [MN]$$

The design value of the applied shear ν_{Sd} is

$$\nu_{Sd} = \frac{V_{Sd}\, \beta}{u_{crit}}$$

$$\nu_{Sd} = \frac{(14.634 - 30.418\, \sigma_{Sd})\ 1.15}{19.622} = 0.858 - 1.783\, \sigma_{Sd}\ [MN/m]$$

Design shear resistance from concrete alone ν_{Rd1} is

$$\nu_{Rd1} = \tau_{Rd}\, k\, (1.2 + 40\, \rho_1)\, d$$

$$\nu_{Rd1} = 0.336 \times 1.0 \times (1.2 + 40 \times 0.0000588 \sqrt{A_{sxc} \times A_{sxc}})\ 1.7$$

$$\nu_{Rd1} = 0.68544 + 0.001344 \sqrt{A_{sxc} \times A_{sxc}}\ [MN/m]$$

Table (34) shows the check of punching shear for the interior column P33 by application of different soil models where for all soil models $\nu_{sd} < \nu_{Rd1}$. Therefore, the section is safe for punching shear.

Table (34) Check of punching shear for the interior column P33 by application of different soil models

Soil model	σ_{sd} [MN/m^2]	%(A_{sxc} *A_{sxc}) [cm^2/m]	ν_{sd} [MN/m]	ν_{Rd1} [MN/m]
Simple assumption model (1)	0.308	29.5	0.309	0.725>ν_{sd}
Winkler's model (3)	0.303	34.05	0.318	0.731>ν_{sd}
Continuum model (6)	0.253	43.93	0.407	0.745>ν_{sd}

7.2.2 Exterior column (column P19)

The critical punching shear section for exterior column is considered at column P19 (Figure (32)).

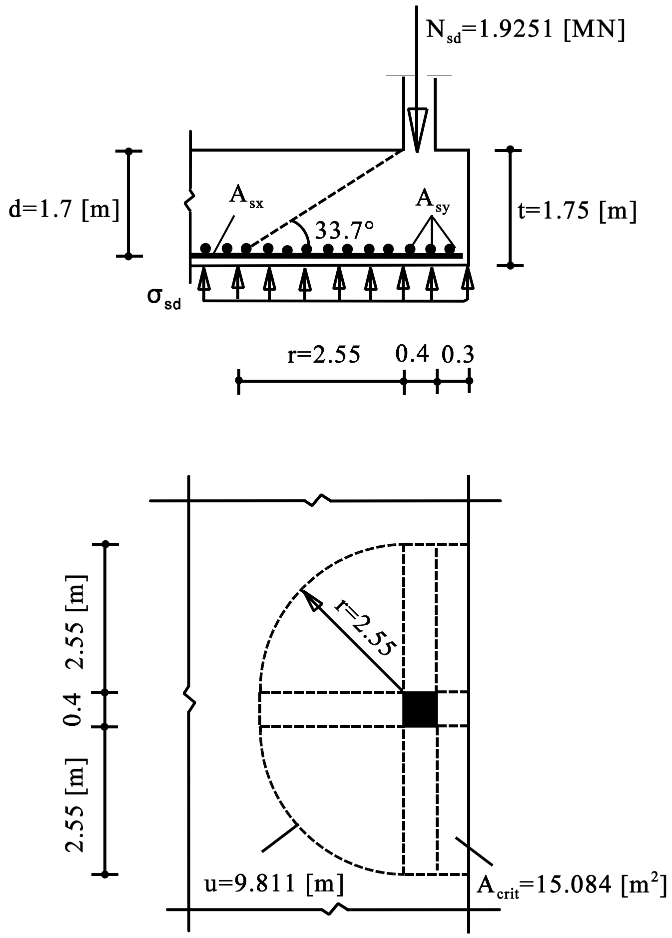


Figure (32) Critical section for Punching shear according to EC 2

Geometry (Figure (32))

Effective depth of the section $d = d_x = d_y = 1.7$ [m]
Column side $c_x = c_y = 0.4$ [m]
Distance of critical punching section from circumference of the column
$r = 1.5\ d = 1.5*1.7 = 2.55$ [m]
Area of critical punching shear section
$A_{crit}=c_x\ c_y +2\ r\ c_x+r\ c_y +0.3(2r+c_y)+0.5\pi\ r^2 = (0.4)^2 +3*2.55*0.4+0.3(2*2.55+0.4)+0.5\pi\ 2.55^2$
$A_{crit}= 15.084$ [m^2]
Perimeter of critical punching shear section
$u_{crit} = 2c_x +c_y +2*0.3+\pi\ r = 3*0.4+2*0.3+\pi\ 2.55=9.811$ [m]

Width of punching section b_x=0.3+c_x +r = 0.3+0.4+2.55 = 3.25 [m]
Width of punching section b_y=c_y +2 r = 0.4+2*2.55 = 5.5 [m]
Correction factor for edge column β = 1.4
Coefficient for consideration of the slab thickness k =1.6 - d = 1.6-1.7 = - 0.1 < 1.0 [m]
k is taken 1.0 [m]
Steel ratio $\rho_1 = \rho_{1x} = \rho_{1y}$ = (min A_s*10^{-4}) /(1.7) = 0.00174

Loads and stresses

Total load factor for both dead and live loads	γ = 1.395
Column load	N = 1380 [kN] =1.38 [MN]
Factored column load	$N_{sd} = \gamma$ N = 1.395*1.38 = 1.9251 [MN]
Factored upward soil pressure under the column	$\sigma_{Sd} = \gamma\ \sigma_o$
Main value of shear strength for concrete C 30/37 according to Table (1)	τ_{Rd} = 1.2*0.28 = 0.336 [MN/m]

Check for section capacity

The punching force at ultimate design load V_{Sd} is

$$V_{Sd} = N_{Sd} - \sigma_{Sd}\ A_{crit}$$

$$V_{Sd} = 1.9251 - 15.084\ \sigma_{Sd}\ [MN]$$

The design value of the applied shear ν_{Sd} is

$$\nu_{Sd} = \frac{V_{Sd}\ \beta}{u_{crit}}$$

$$\nu_{Sd} = \frac{(1.9251 - 15.084\ \sigma_{Sd})\ 1.40}{9.811} = 0.275 - 2.152\ \sigma_{Sd}\ [MN/m]$$

Design shear resistance from concrete alone ν_{Rd1} is

$$\nu_{Rd1} = \tau_{Rd}\ k\ (1.2 + 40\ \rho_1)\ d$$

$$\nu_{Rd1} = 0.336 \times 1.0\ (1.2 + 40 \times 0.00174)\ 1.7$$

$$\nu_{Rd1} = 0.725\ [MN/m]$$

Table (35) shows the check of punching shear for the exterior column P19 by application of different soil models where for all soil models $\nu_{sd} < \nu_{Rd1}$. Therefore, the section is safe for punching shear.

Table (35) Check of punching shear for the exterior column P19 by application of different soil models

Soil model	σ_{sd} [MN/m^2]	%(A_{sxc} *A_{sxc}) [cm^2/m]	ν_{sd} [MN/m]	ν_{Rd1} [MN/m]
Simple assumption model (1)	0.202	29.5	0.160	$0.725>\nu_{sd}$
Winkler's model (3)	0.229	29.5	0.217	$0.725>\nu_{sd}$
Continuum model (6)	0.222	29.5	0.202	$0.725>\nu_{sd}$

8 Design for DIN 1045

8.1 Design for flexure moment

Material

Concrete grade B 35
Steel grade BSt 500
Concrete compressive strength $\beta_R = 23$ [MN/m^2]
Tensile yield strength of steel $\beta_S = 500$ [MN/m^2]

Geometry

Effective depth of the section h = 1.7 [m]
Width of the section to be designed b = 1.0 [m]

Determination of tension reinforcement

The design of sections is carried out for DIN 1045 in table forms. Tables (36) to (39) show the design of sections x-x and y-y.

The normalized design moment m_s is

$$m_s = \frac{M_s}{bh^2\left(\frac{\alpha_R \beta_R}{\gamma}\right)}$$

$$m_s = \frac{M_s}{1.0 \times 1.70^2 \left(\frac{0.95 \times 23}{1.75}\right)} = 0.027713\ M_s$$

The normalized steel ratio ω_M is

$$\omega_M = 1 - \sqrt{1 - 2\ m_s}$$

$$\omega_M = 1 - \sqrt{1 - 2 \times 0.027713\ M_s} = 1 - \sqrt{1 - 0.0554\ M_s}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega_M \left(\frac{(\alpha_R \beta_R)\ b\ h}{\beta_S} \right)$$

$$A_s = \omega_M \left(\frac{0.95 \times 23 \times 1.0 \times 1.70}{500} \right) = 0.07429\ \omega_M\ [m^2/m]$$

$$A_s = 742.9\ \omega_M\ [cm^2/m]$$

Table (36) Required bottom reinforcement under the column A_{sxc} for different soil models (section x-x)

Soil model	M_s [MN.m/m]	m_s	ω_M	A_{sxc} [cm²/m]
Simple assumption model (1)	1.444	0.040	0.041	30.35
Winkler's model (3)	1.827	0.051	0.052	38.62
Continuum model (6)	2.320	0.064	0.067	49.41

Table (37) Required top reinforcement in the field A_{sxf} for different soil models (section x-x)

Soil model	M_s [MN.m/m]	m_s	ω_M	A_{sxf} [cm²/m]
Simple assumption model (1)	2.182	0.061	0.062	46.37
Winkler's model (3)	1.728	0.048	0.049	36.47
Continuum model (6)	1.292	0.036	0.037	27.09

Table (38) Required bottom reinforcement under the column A_{syc} for different soil models (section y-y)

Soil model	M_s [MN.m/m]	m_s	ω_M	A_{syc} [cm^2/m]
Simple assumption model (1)	1.424	0.040	0.040	29.92
Winkler's model (3)	1.527	0.042	0.043	32.13
Continuum model (6)	1.866	0.052	0.053	39.47

Table (39) Required top reinforcement in the field A_{syf} for different soil models (section y-y)

Soil model	M_s [MN.m/m]	m_s	ω_M	A_{syf} [cm^2/m]
Simple assumption model (1)	1.137	0.032	0.032	23.79
Winkler's model (3)	1.026	0.028	0.029	21.43
Continuum model (6)	0.694	0.019	0.019	14.43

Chosen reinforcement

Tables (40) and (41) show the number of steel bars under the column and in the field between columns at sections x-x and y-y considering different soil models. The chosen diameter of steel bars is Φ = 25 [mm].

Table (40) Chosen reinforcement at section x-x for different soil models

Soil model	Chosen reinforcement	
	Bottom Rft under the column A_{sxc}	Top Rft in the field A_{sxf}
Simple assumption model (1)	7 Φ 25 = 34.40 [cm^2/m]	10 Φ 25 = 49.10 [cm^2/m]
Winkler's model (3)	8 Φ 25 = 39.30 [cm^2/m]	8 Φ 25 = 39.30 [cm^2/m]
Continuum model (6)	11 Φ 25 = 54.01 [cm^2/m]	min A_s = 29.50 [cm^2/m]

Table (41) Chosen reinforcement at section y-y for different soil models

Soil model	Chosen reinforcement	
	Bottom Rft under the column A_{syc}	Top Rft in the field A_{syf}
Simple assumption model (1)	min A_s = 29.50 [cm^2/m]	min A_s = 29.50 [cm^2/m]
Winkler's model (3)	7 Φ 25 = 34.40 [cm^2/m]	min A_s = 29.50 [cm^2/m]
Continuum model (6)	9 Φ 25 = 44.20 [cm^2/m]	min A_s = 29.50 [cm^2/m]

8.2 Check for punching shear

8.2.1 Interior column (column P33)

The critical punching shear section for interior column is considered at column P33. The column dimensions are chosen to be 90/90 [cm]. The critical section for punching shear is a circle of diameter d_r = 2.717 [m] around the circumference of the column as shown in Figure (33).

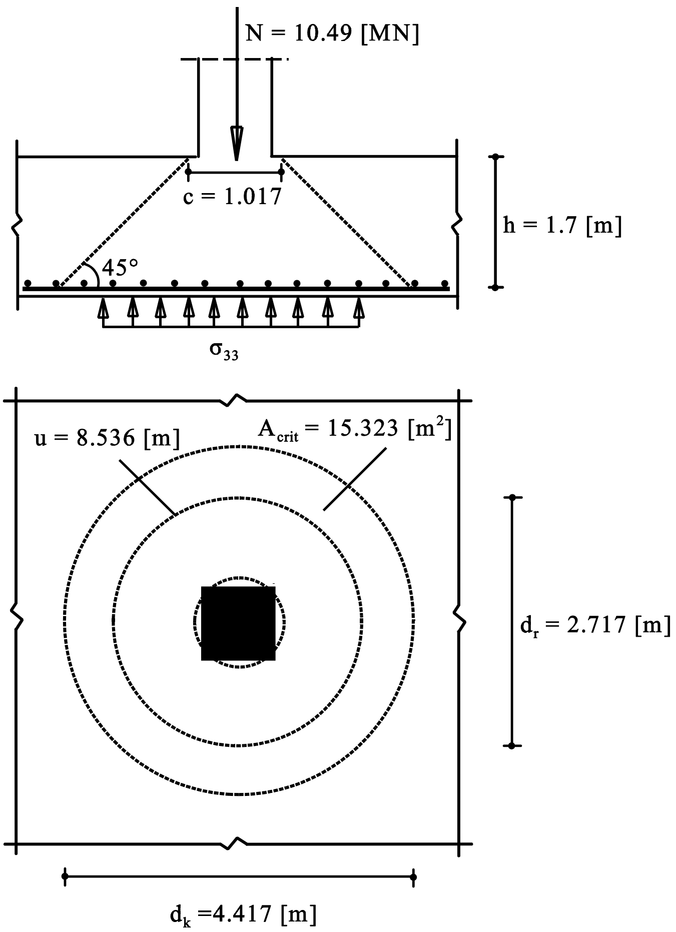


Figure (33) Critical section for Punching shear according to DIN 1045

Geometry (Figure (33))

Effective depth of the section	h = 1.7 [m]
Column side	c_x= c_x = 0.9 [m]
Average diameter of the column	c = 1.13*%(0.9*0.9) =1.017 [m]
Diameter of loaded area	d_k = 2 h+c = 2*1.7+1.017 =4.417 [m]
Diameter of critical punching shear section	d_r = c+h = 1.017+1.7 = 2.717 [m]
Area of critical punching shear section	$A_{crit} = \pi\ d_k^2/4 = \pi\ 4.417^2/4 = 15.323$ [m^2]

Perimeter of critical punching shear section $u = \pi\, d_r = \pi\ 2.717 = 8.536$ [m]

Loads and stresses

Column load $N = 10490$ [kN] $= 10.49$ [MN]
Main value of shear strength for concrete B 35 according to Table (2) $\tau_{011} = 0.6$ [MN/m^2]
Factor depending on steel grade according to Table (6) $\alpha_s = 1.4$

Check for section capacity

The punching shear force Q_r is

$$Q_r = N - \sigma_o\, A_{crit}$$

$$Q_r = 10.49 - 15.323\ \sigma_{33}\ [MN]$$

The punching shear stress τ_r is

$$\tau_r = \frac{Q_r}{u\ h}$$

$$\tau_r = \frac{10.49 - 15.32\ \sigma_{33}}{8.536 \times 1.7} = 0.723 - 1.056\ \sigma_{33}\ [MN/m^2]$$

Reinforcement grade μ_g is

$$\mu_g = \frac{A_{sx} + A_{sy}}{2h}$$

$$\mu_g = \frac{A_{sxc} + A_{syc}}{2 \times 1.7 \times 100} = 0.00294\ \left(A_{sxc} + A_{syc}\right)[\%]$$

Coefficient for consideration of reinforcement κ_1 is

$$\kappa_1 = 1.3\ \alpha_s\ \sqrt{\mu_g}$$

$$\kappa_1 = 1.3 \times 1.4\ \sqrt{0.00294\ \left(A_{sxc} + A_{syc}\right)} = 0.0987\ \sqrt{\left(A_{sxc} + A_{syc}\right)}$$

The allowable concrete punching strength τ_{r1} [MN/m^2] is

$$\tau_{r1} \;\; ' \;\; \kappa_1 \; \tau_{011}$$

$$\tau_{r1} \;\; ' \;\; 0.0987 \sqrt{(A_{sxc} \;\%\; A_{syc})}\,(0.6 \;\; ' \;\; 0.0592 \sqrt{(A_{sxc} \;\%\; A_{syc})}\,[MN/m^2]$$

Table (42) shows the check of punching shear for the interior column P33 by application of different soil models where for all soil models $\tau_r < \tau_{r1}$. Therefore, the section is safe for punching shear.

Table (42) Check for punching shear by application of different soil models

Soil model	σ_{33} [MN/m^2]	$A_{sxc}+A_{scx}$ [cm^2/m]	τ_r [MN/m^2]	τ_{r1} [MN/m^2]
Simple assumption model (1)	0.221	63.90	0.490	0.473 . τ_r
Winkler's model (3)	0.217	73.70	0.494	0.508 > τ_r
Continuum model (6)	0.181	98.21	0.532	0.587 > τ_r

8.2.2 Exterior column (column P19)

The critical punching shear section for exterior column is considered at column P19 (Figure (34)).

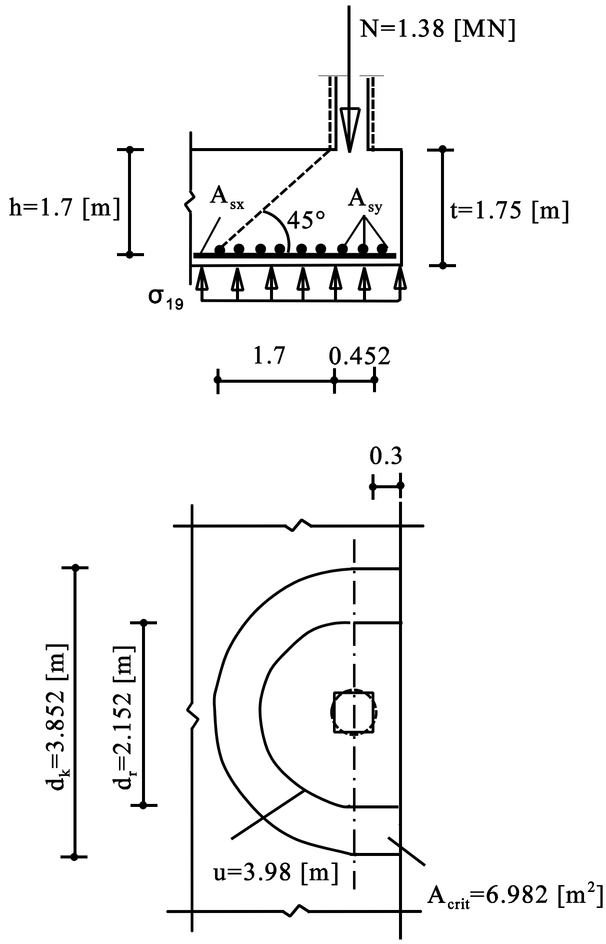


Figure (34) Critical section for Punching shear according to DIN 1045

Geometry (Figure (34))

Effective depth of the section	$h = 1.7$ [m]
Column side	$c_x = c_x = 0.4$ [m]
Average diameter of the column	c = 1.13*%(0.4*0.4) =0.452 [m]
Diameter of loaded area	$d_k = 2\ h + c = 2*1.7+0.452 = 3.852$ [m]
Diameter of critical punching shear section	$d_r = c+h = 0.452+1.7 = 2.152$ [m]
Area of critical punching shear section	$A_{crit} = 0.5\pi d_k^2/4+0.3d_k$
	$A_{crit}=0.5\pi 3.852^2/4+0.3*3.852=6.982$ [m²]
Perimeter of critical punching shear section	$u = 0.5\pi d_r+2*0.3=0.5\pi 2.152+2*0.3$
	u=3.98 [m]

Loads and stresses

Column load $N = 1380$ [kN] $= 1.38$ [MN]

Main value of shear strength for concrete B 35 according to Table (2) $\tau_{011} = 0.6$ [MN/m^2]

Factor depending on steel grade according to Table (6) $\alpha_s = 1.4$

Check for section capacity

The punching shear force Q_r is

$$Q_r = N - \sigma_o A_{crit}$$

$$Q_r = 1.38 - 6.982\ \sigma_{19}\ [MN]$$

The punching shear stress τ_r is

$$\tau_r = \frac{Q_r}{u\ h}$$

$$\tau_r = \frac{1.38 - 6.982\ \sigma_o}{3.98 \times 1.7} = 0.204 - 1.032\ \sigma_{19}\ [MN/m^2]$$

Reinforcement grade μ_g is

$$\mu_g = \frac{A_{sx} + A_{sy}}{2h}$$

$$\mu_g = \frac{2 \times \min A_s}{2 \times 1.7 \times 100} = \frac{2 \times 29.5}{2 \times 1.7 \times 100} = 0.174\ [\%]$$

Coefficient for consideration of reinforcement κ_1 is

$$\kappa_1 = 1.3\ \alpha_s \sqrt{\mu_g}$$

$$\kappa_1 = 1.3 \times 1.4 \sqrt{0.174} = 0.759$$

The allowable concrete punching strength τ_{r1} [MN/m^2] is

$$\tau_{r1} = \kappa_1 \, \tau_{011}$$

$$\tau_{r1} = 0.75 \times 0.6 = 0.456 \ [MN/m^2]$$

Table (43) shows the check of punching shear for the exterior column P19 by application of different soil models where for all soil models $\tau_r < \tau_{r1}$. Therefore, the section is safe for punching shear.

Table (43) Check for punching shear by application of different soil models

Soil model	σ_{19} [MN/m^2]	A_s [cm^2/m]	τ_r [MN/m^2]	τ_{r1} [MN/m^2]
Simple assumption model (1)	0.145	29.5	0.054	0.456 > τ_r
Winkler's model (3)	0.164	29.5	0.035	0.456 > τ_r
Continuum model (6)	0.159	29.5	0.040	0.456 > τ_r

9 Comparison between the design according to DIN 1045 and EC 2

Tables (44) to (47) show the comparison between the design of raft according to DIN 1045 and EC 2 by application of different soil models. The comparison is considered only for required reinforcement due to flexure moment at the critical sections x-x and y-y.

From the comparison, it can be concluded that

- if the raft has the same thickness and is designed according to EC 2 and DIN 1045, the reinforcement obtained from EC 2 will be less than that obtained from DIN 1045 by 9 [%].

- For Continuum model, the contact pressure values at the edges of the raft are higher than those at the middle. Consequently, the positive moment under the column P33 for Continuum model is higher than that for both Simple assumption and Winkler's models, while the negative moment in the filed is less than that of the other models. This relation is valid also for reinforcement.

- The contact pressure for Simple assumption and Winkler's models are quite similar, particularly if the soil is uniform. Therefore, the results of both models are nearly the same.

- If the reinforcement under the column decreases the reinforcement in the field will increase and vice verse. This notice yields for a constant amount of reinforcement in the section. However, the difference in reinforcement calculated by the three models is about 40 [%]. The design of the raft by all methods is considered acceptable in this example.

Table (44) Comparison between the design according to DIN 1045 and EC 2 for required bottom reinforcement A_{sxc} under the column at section x-x

Soil model	A_{sxc} [cm^2/m] according to		Difference ΔA_{sxc} [%]
	DIN 1045	EC 2	
Simple assumption model (1)	30.35	27.89	8.82
Winkler's model (3)	38.62	35.50	8.79
Continuum model (6)	49.41	45.40	8.83

Table (45) Comparison between the design according to DIN 1045 and EC 2 for required top reinforcement A_{sxf} in the field at section x-x

Soil model	A_{sxf} [cm^2/m] according to		Difference ΔA_{sxf} [%]
	DIN 1045	EC 2	
Simple assumption model (1)	46.37	42.62	8.80
Winkler's model (3)	36.47	33.52	8.80
Continuum model (6)	27.09	24.89	8.84

Table (46) Comparison between the design according to DIN 1045 and EC 2 for required bottom reinforcement A_{syc} under the column at section y-y

Soil model	A_{syc} [cm^2/m] according to		Difference ΔA_{syc} [%]
	DIN 1045	EC 2	
Simple assumption model (1)	29.92	27.49	8.84
Winkler's model (3)	32.13	29.53	8.81
Continuum model (6)	39.47	36.27	8.82

Table (47) Comparison between the design according to DIN 1045 and EC 2 for required top reinforcement A_{syf} in the field at section y-y

Soil model	A_{syf} [cm^2/m] according to		Difference ΔA_{syf} [%]
	DIN 1045	EC 2	
Simple assumption model (1)	23.79	21.86	8.83
Winkler's model (3)	21.43	19.69	8.84
Continuum model (6)	14.43	13.25	8.91

10 References

[1] Cruz, L. (1994): “Vergleichsuntersuchungen zur Bauwerk-Boden-Wechselwirkung an eine Hochhaus-Gründungsplatte zwischen den nationalen Normen und den Eurocodes”, Diplomarbeit-Universität-GH Siegen.

[2] Kany, M./El Gendy, M. (1995): “Computing of Beam and Slab Foundations on Three Dimensional Layered Model”, Proceeding of the Sixth International Conference on Computing in Civil and Building Engineering, Berlin.

Example 4: Design of a circular raft for a cylindrical core

1 Description of the problem

Ring or circular rafts can be used for cylindrical structures such as chimneys, silos, storage tanks, TV-towers and other structures. In this case, ring or circular raft is the best suitable foundation to the natural geometry of such structures. The design of circular rafts is quite similar to that of other rafts.

As a design example for circular rafts, consider the cylindrical core wall shown in Figure (35) as a part of five storeys-office building. The diameter of the core wall is 8.0 [m], while the width of the wall is B = 0.3 [m]. The core lies in the center of the building and it does not subject to any significant lateral applied loading. Therefore, the core wall carries only a vertical load of p = 300 [kN/m]. The base of the cylindrical core wall is chosen to be a circular raft of 10.0 [m] diameter with 1.0 [m] ring cantilever. A thin plain concrete of thickness 0.15 [m] is chosen under the raft and is not considered in any calculation.

Two analyses concerning the effect of wall rigidity on the raft are carried out in the actual design. Both by using the Continuum model (method 6) to represent the subsoil. The two cases of analyses are considered as follows:

Case 1: The presence of the core wall is ignored.

Case 2: A height of only one storey is taken into account, where the perimeter wall is modeled by beams having the flexural properties of B = 0.3 [m] width and H = 3.0 [m] height. The choice of this reduced wall height because the wall above the first floor has many openings.

Figure (35) shows plan of the raft, wall load, dimensions and mesh with section through the raft and subsoil. The following text gives a description of the design properties and parameters.

2 Properties of the raft material

The raft is made of reinforcement concrete, which has the following parameters:

Young's modulus of concrete	E_b	$= 3.2 * 10^7$	[kN/m^2]
Poisson's ratio of concrete	ν_b	= 0.20	[1]
Shear modulus of concrete	$G_b = 0.5\ E_b\ (1+\nu_b)$	$=1.3 * 10^7$	[kN/m^2]
Unit weight of concrete	γ_b	= 25	[kN/m^3]

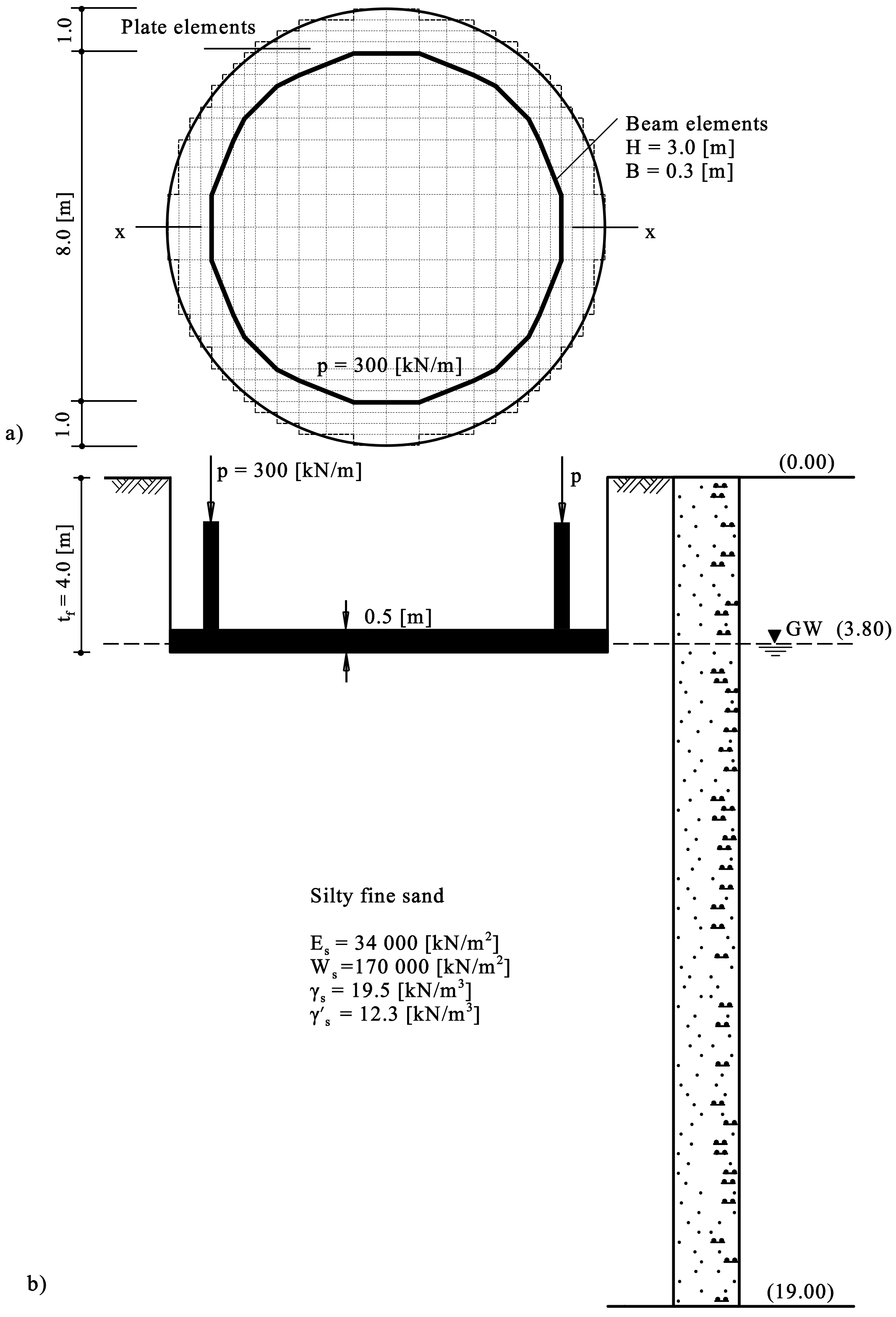


Figure (35) a) Plan of the raft with wall load, dimensions and mesh
b) Section through the raft and subsoil

3 Properties of the raft section

The raft section has the following parameters:

Width of the section to be designed	b	= 1.0	[m]
Section thickness	t	= 0.50	[m]
Concrete cover + 1/2 bar diameter	c	= 5	[cm]
Effective depth of the section	d	= t - c = 0.45	[m]
Steel bar diameter	Φ	= 14	[mm]

Minimum area of steel per meter, min A_s, is

min A_s = 0.15 [%]* concrete section = 0.0015*50*100 = 7.5 [cm^2/m]
take min A_s = 5 Φ 14 [mm/m] = 7.7 [cm^2/m]

4 Soil properties

The core rests on a soil layer of 15.0 [m] of silty fine sand, overlying a rigid base of sandstone as shown in Figure (35). The effect of uplift pressure, reloading of the soil and limit depth of the soil layer are taken into account. The soil layer has the following parameters:

Poisson's ratio	ν_s	= 0.30	[-]
Level of foundation depth under the ground surface	d_f	= 4.0	[m]
Modulus of compressibility for loading	E_s	= 34 000	[kN/m^2]
Modulus of compressibility for reloading	W_s	= 170 000	[kN/m^2]
Unit weight above the ground water	γ_s	= 19.5	[kN/m^3]
Unit weight under the ground water	$\gamma!_s$	= 12.3	[kN/m^3]
Level of water table under ground surface	GW	= 3.8	[m]

5 Analysis of the raft

The raft is subdivided into 404 elements as shown in Figure (3.35). Then, the analysis of the raft is carried out two times for two different structural systems. In the first analysis, the rigidity of the core wall is ignored and only the self-rigidity of 0.5 [m] raft thickness is considered. In the other analysis, the rigidity of the core wall is considered through inserting additional beam elements along the location of the wall on the FE-mesh. The properties of the beam elements (width B = 0.3 [m], height H = 3.0 [m]) are:

Moment of Inertia	$I = B*H^3/12$	= 0.675	[m^4]
Torsional Inertia	$J = H*B^3*(1/3-0.21(B/H)(1-B^4/(12*H^4)))$	= 0.0253	[m^4]

To make better representation for the line loads on the raft, the loads from the wall are modeled as uniform loads acting on the beam elements. In case of the structural system without effect of the wall, beam elements may be remaining in the system while the rigidity of the wall is eliminated by defining all property values of the beam elements by zero except the loads.

The system of linear equations for the Continuum model is solved by iteration (method 6). The maximum difference between the soil settlement s [cm] and the raft deflection w [cm] is considered as an accuracy number. In this example, the accuracy is chosen $\varepsilon = 0.0002$ [cm].

Determination of the limit depth t_s

The level of the soil under the raft in which no settlement occurs or the expected settlement will be very small where can be ignored is determined first as a limit depth of the soil.
The limit depth in this example is chosen to be the level of which the stress due to the raft σ_E reaches the ratio $\xi = 0.2$ of the initial vertical stress σ_V. The stress in the soil σ_E is determined at the characteristic point c of the circular foundation. This stress σ_E is due to the average stress from the raft at the surface $\sigma_O = 108$ [kN/m^2]. At the characteristic point, from the definition of Graßhoff (1955), the settlement if the raft is full rigid will be identical with that if the raft is full flexible. The characteristic point c lies at a distance 0.845 r from the center of the raft as shown in Figure (36). The results of the limit depth calculation are plotted in a diagram as shown in Figure (36). The limit depth is found to be $t_s = 11.23$ [m] under the ground surface.

6 Evaluation and conclusions

To evaluate of analysis results, the results of both analyses are compared together. The following conclusions are drawn:

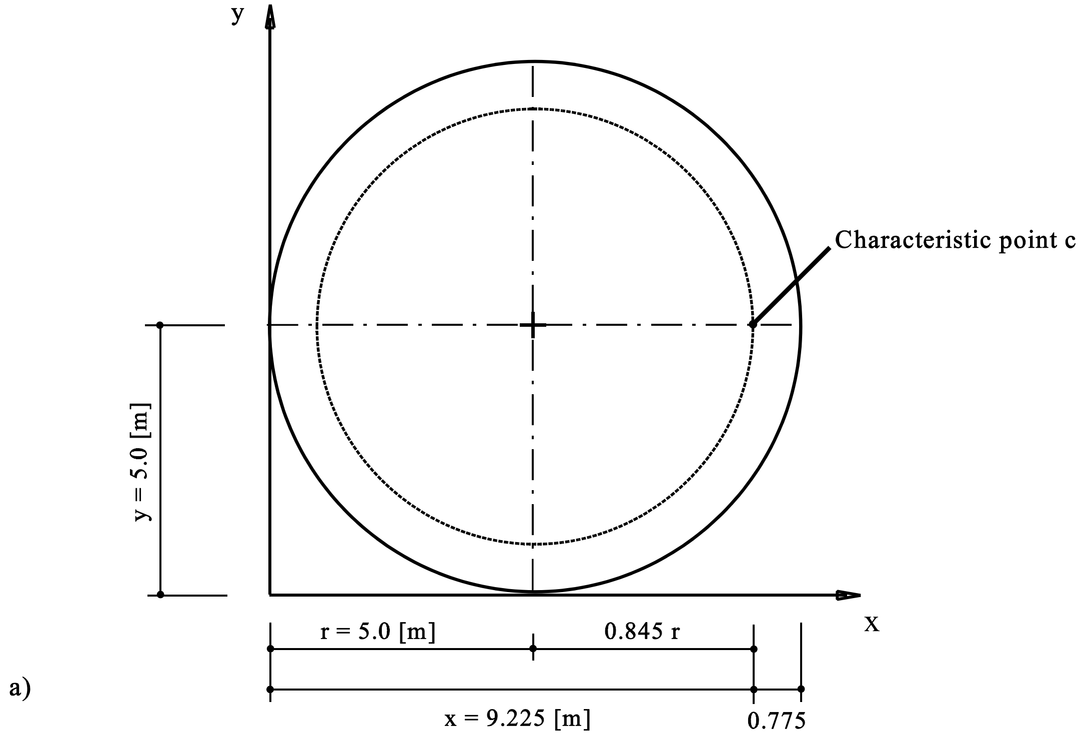
Settlements

Figures (37) and (38) show the extreme values of settlements in x-direction under the raft, while Figure (39) shows the settlements at section x-x under the middle of the raft for both cases of analyses. From the figures, it can be concluded the following:

- The maximum differential settlement across the raft without the effect of the wall (Δs = 0.2 [cm]) is double that with effect of the wall (Δs = 0.1 [cm]).

- The maximum settlements, if the presence of the wall is considered, decrease from 0.49 [cm] to 0.45 [cm] by 9 [%], while the minimum settlements, if the presence of the wall is considered, increase from 0.29 [cm] to 0.35 [cm] by 21 [%].

- The presence of the wall improves the deformation shape where the settlements at the raft edges will decrease, while at the center will increase.



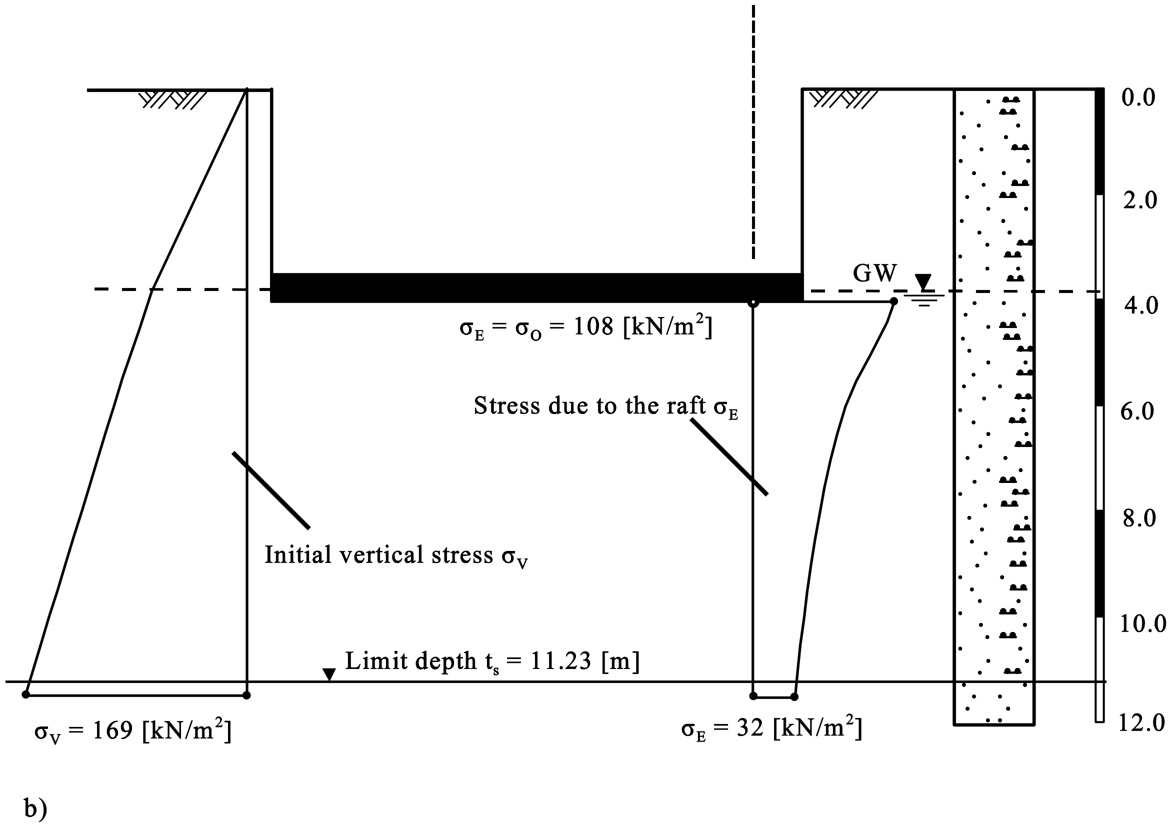


Figure (36) a) Position of characteristic point c
b) Limit depth t_s of the soil under the raft

Contact pressures

Figure (40) shows the contact pressures q at section x-x for both analyses without and with effect of the wall.

- The difference in contact pressures for both analyses is not great along the raft, only a slight difference is found at the center and the edge of the raft.

- If the entire distribution of contact pressure is taken to be uniform (108 [kN/m^2]), in the manner frequently assumed in traditional foundation design, the negative moments will be much higher, while the positive moment will be lower (not shown).

Moments

As the circular raft is a special case of rafts, radial moments m_r are equal to both principal moments h_{m1} and moments m_x in x-direction at the section pass through the center of the raft. In addition, tangential moments m_t are equal to both principal moments h_{m2} and moments m_y in y-direction at the section pass through the center of the raft. Figures (41) to (44) show the contour lines of radial and tangential moments, while Figures (45) and (46) show the vectors of principal moments $h_{m1,2}$ of the raft for both analyses. Figure (47) shows the radial and tangential moments in one figure at section x-x. These results show that:

- The absolute values of negative radial and tangential moments m_r and m_t at the center of the raft in the analysis with effect of the wall ($m_r = m_t = -95$ [kN.m/m]) are lower than that in the analysis without effect of the wall ($m_r = m_t = -124$ [kN.m/m]) by 31 [%]. Therefore, the positive moments m_r and m_t under the wall increase due to taking of wall effect in the analysis.

- The positive radial moments m_r under the wall increases from 98 [kN.m/m] to 130 [kN.m/m] due to taking of wall effect in the analysis by 25 [%].

- Positive tangential moments will occur only, if the analysis considers effect of the wall.

Table (48) shows a comparison of the results at the critical sections for the raft without and wit effect of the wall, which recommends the above conclusions.

Table (48) Settlements, contact pressures, radial and tangential moments at critical sections of the raft for both analyses without and with the effect of wall

Results	Position	Presence of the wall		Difference Δ [%]
		is ignored	is considered	
Settlements s [cm]	Edge	0.49	0.45	9
	Center	0.29	0.35	17
	Under the wall	0.46	0.45	2
Contact pressures q [kN/m^2]	Edge	340	299	14
	Center	71	78	9
	Under the wall	102	102	0
Radial moments m_r [kN.m/m]	Edge	0.0	3	100
	Center	-125	-95	31
	Under the wall	98	130	25
Tangential moments m_t [kN.m/m]	Edge	-17	0.0	-
	Center	-125	-96	30
	Under the wall	-15	22	168

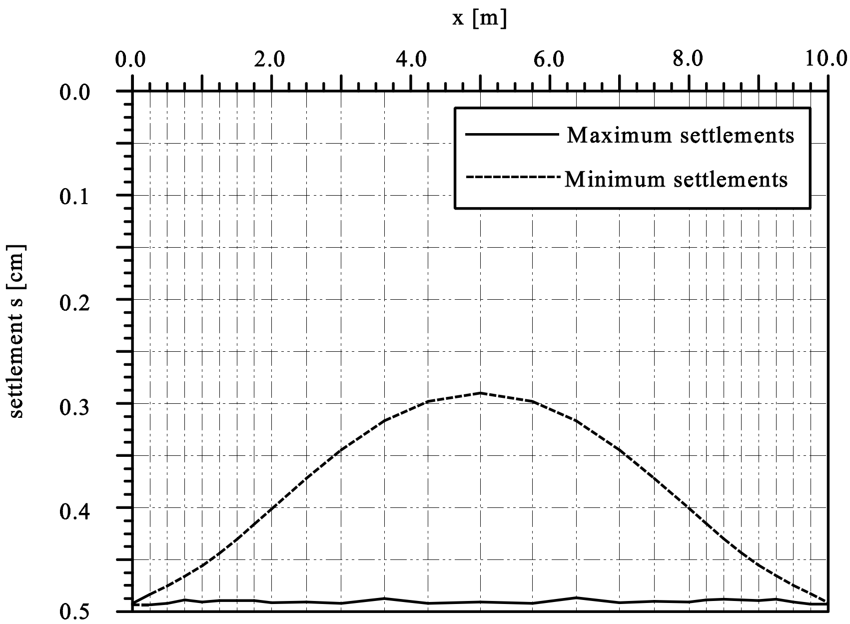


Figure (37) Extreme values of settlements s [cm] in x-direction under the raft without effect of the wall

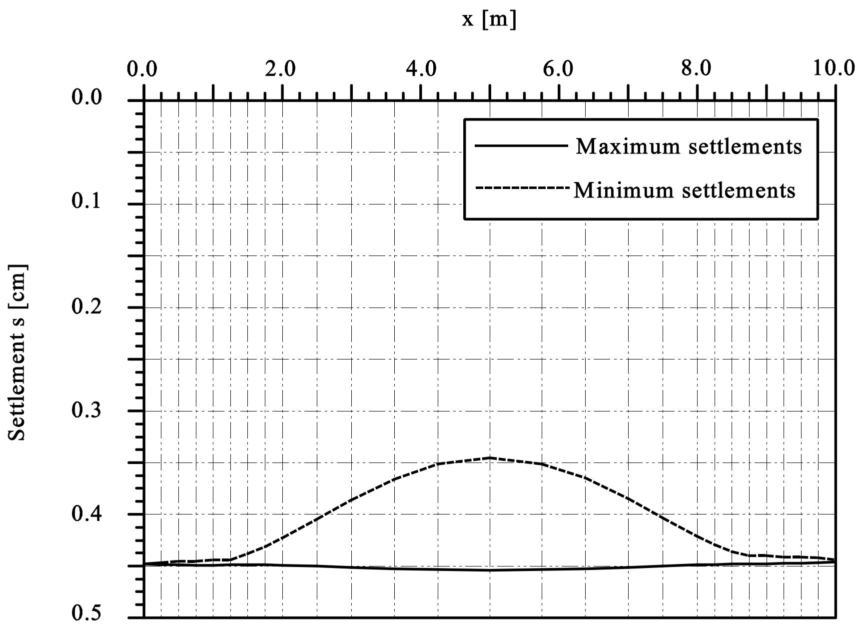


Figure (38) Extreme values of settlements s [cm] in x-direction under the raft with effect of the wall

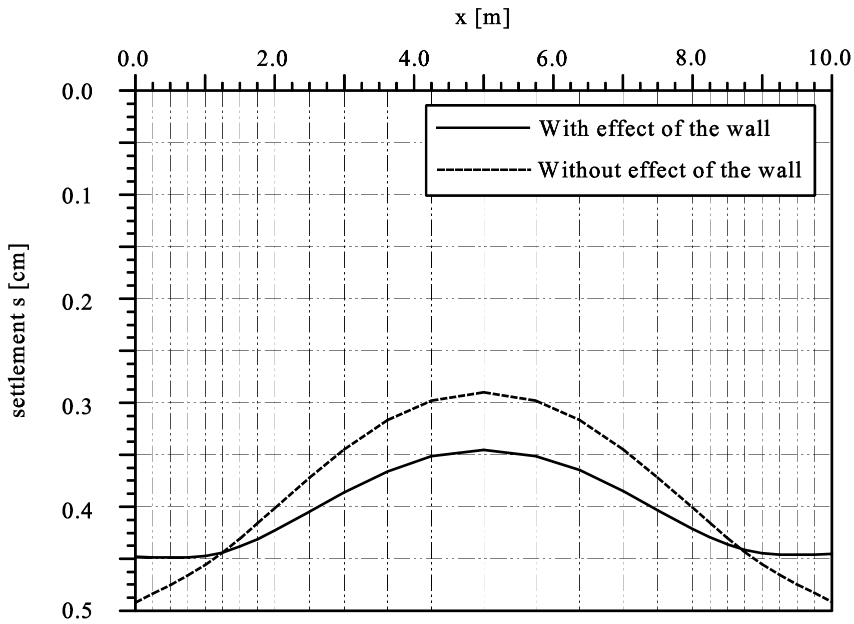


Figure (39) Settlements s [cm] at section x-x

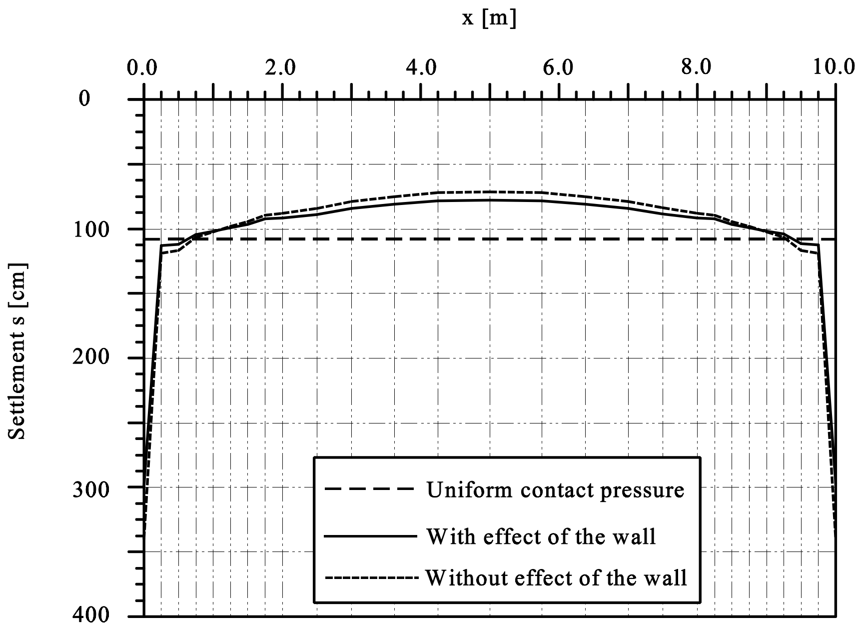


Figure (40) Contact pressures q [kN/m^2] at section x-x

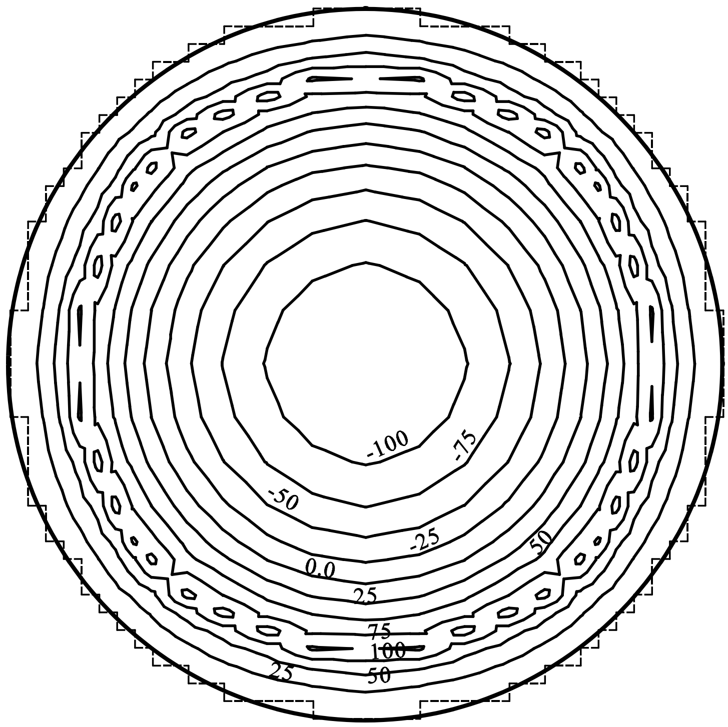


Figure (41) Contour lines of radial moments $m_r = h_{m1}$ [kN.m/m] of the raft without effect of the wall

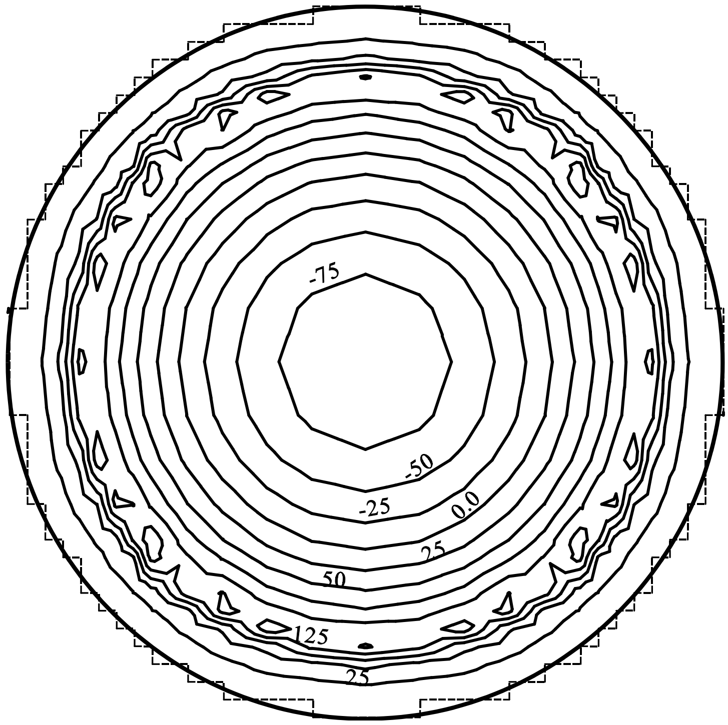


Figure (42) Contour lines of radial moments $m_r = h_{m1}$ [kN.m/m] of the raft with effect of the wall

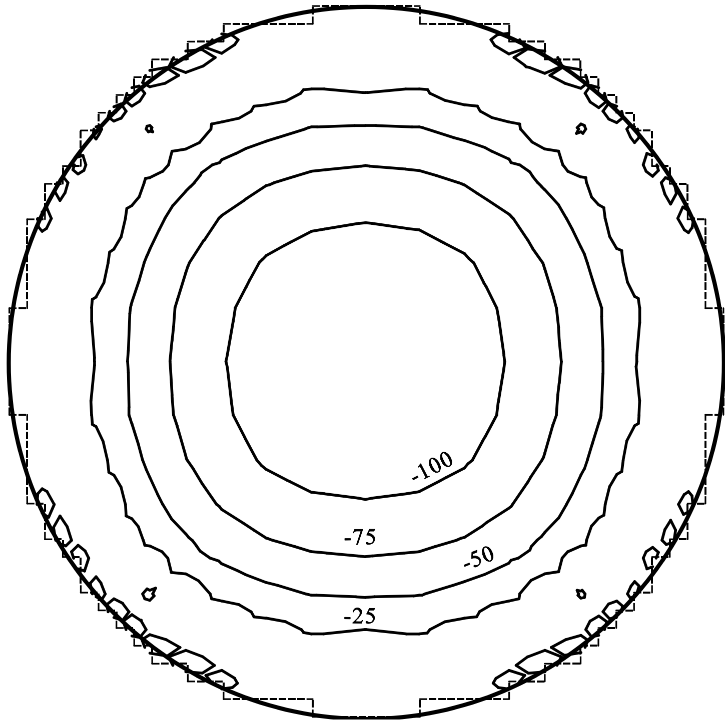


Figure (43) Contour lines of tangential moments $m_t = h_{m2}$ [kN.m/m] of the raft without effect of the wall

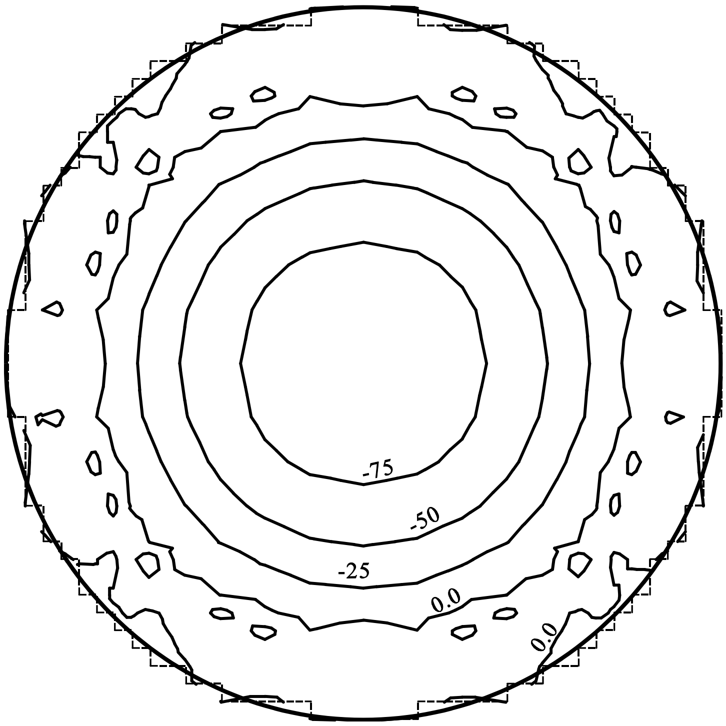


Figure (44) Contour lines of tangential moments $m_t = h_{m2}$ [kN.m/m] of the raft with effect of the wall

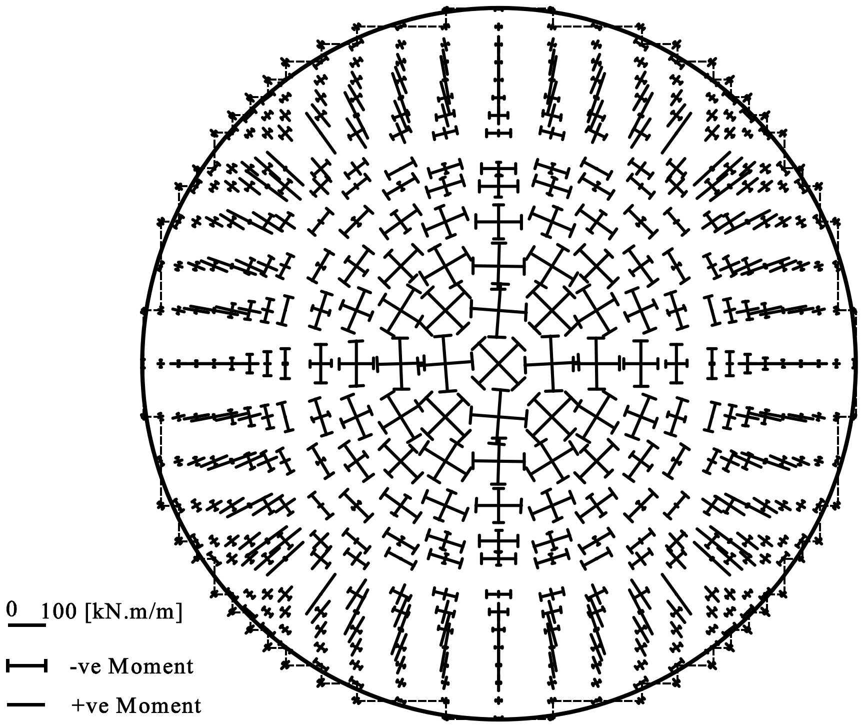


Figure (45) Vectors of principal moments h_{m1} and h_{m2} [kN.m/m] of the raft without effect of the wall

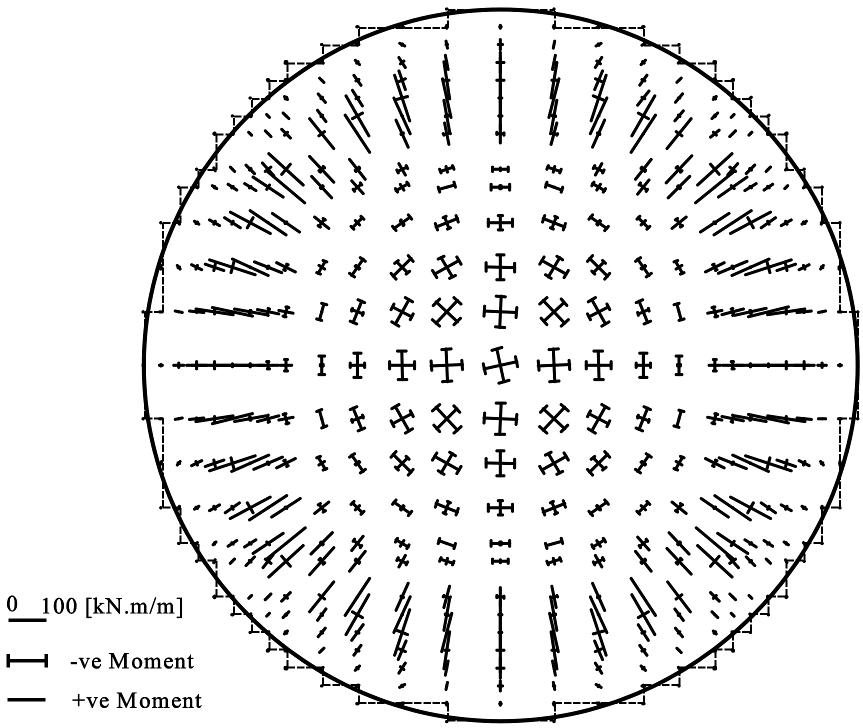


Figure (46) Vectors of principal moments h_{m1} and h_{m2} [kN.m/m] of the raft with effect of the wall

7 Design of the raft for flexure moment according to EC 2

Material

Concrete grade C 30/37
Steel grade BSt 500
Characteristic compressive cylinder strength of concrete $f_{ck} = 30$ [MN/m^2]
Characteristic tensile yield strength of reinforcement $f_{yk} = f_y = 500$ [MN/m^2]
Partial safety factor for concrete strength $\gamma_c = 1.5$
Design concrete compressive strength $f_{cd} = f_{ck}/\gamma_c = 30/1.5 = 20$ [MN/m^2]
Partial safety factor for steel strength $\gamma_s = 1.15$
Design tensile yield strength of reinforcing steel $f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 435$ [MN/m^2]

Factored moment

Total load factor for both dead and live loads $\gamma = 1.395$
Factored radial moment $M_{sd} = \gamma m_r$
Factored tangential moment $M_{sd} = \gamma m_t$

Geometry

Effective depth of the section $d = 0.45$ [m]
Width of the section to be designed $b = 1.0$ [m]

Determination of tension reinforcement

The design of sections is carried out for EC 2 in table forms. Tables (49) to (51) and Figure (47) show the design of critical sections.

The normalized design moment μ_{sd} is

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 0.45^2 \times (0.85 \times 20)} = 0.2905\ M_{sd}$$

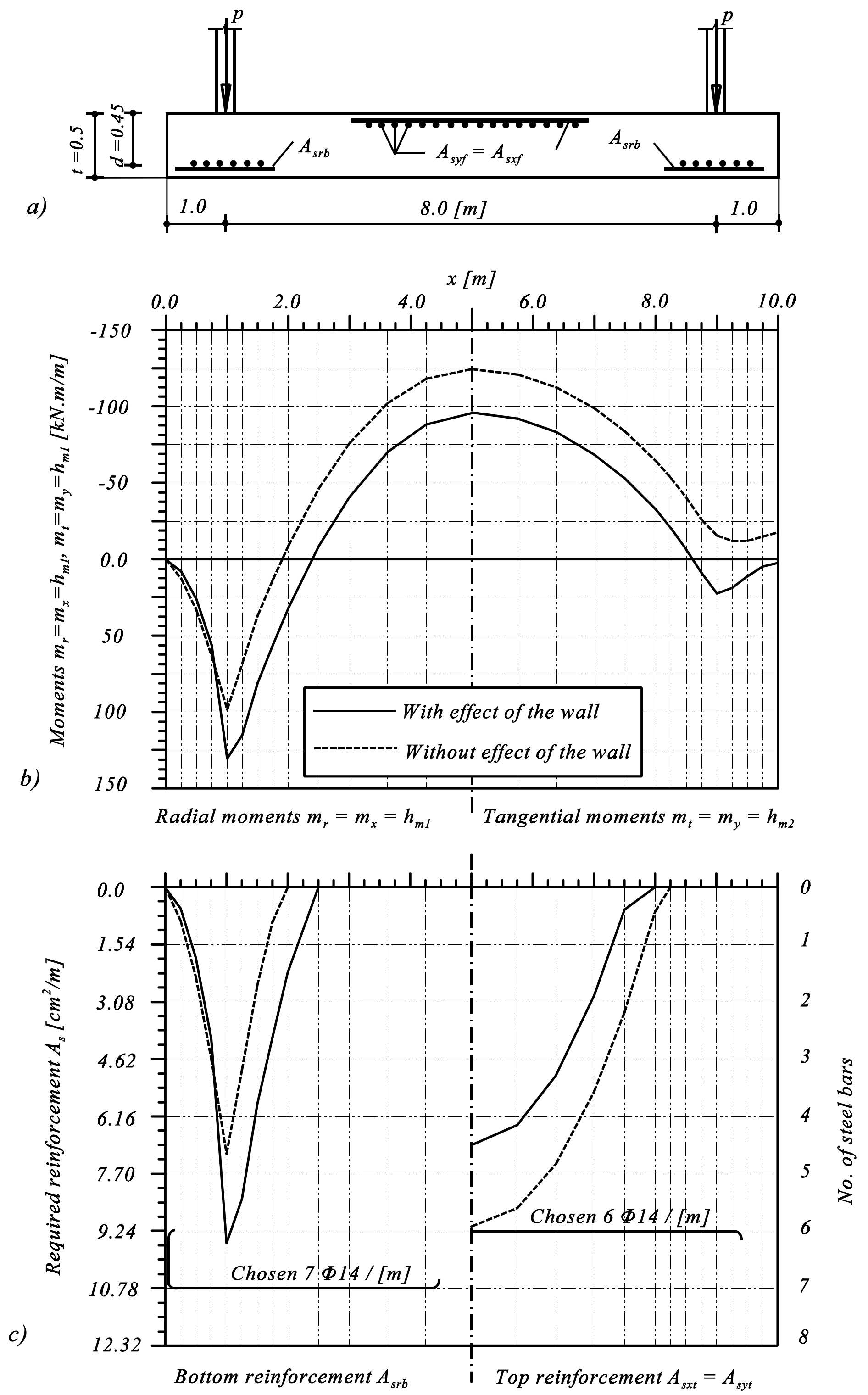


Figure (47) a) Section x-x through the raft
b) Moments $m_r = m_x = h_{m1}$, $m_t = m_y = h_{m2}$ [kN.m/m] at section x-x
c) Main reinforcement A_s at critical sections

The normalized steel ratio ω is

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.2905\ M_{sd}} = 1 - \sqrt{1 - 0.581\ M_{sd}}$$

The required area of steel reinforcement per meter A_s is

$$A_s = \omega \left(\frac{(0.85 f_{cd})\ b\ d}{f_{yd}} \right)$$

$$A_s = \omega \left(\frac{0.85 \times 20 \times 1.0 \times 0.45}{435} \right) = 0.017586\ \omega\ [m^2/m]$$

$$A_s = 175.86\ \omega\ [cm^2/m]$$

Table (49) Required bottom reinforcement in radial direction A_{srb} for the raft without and with effect of the wall

Structural system	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{srb} [cm²/m]
Raft without effect of the wall	0.137	0.0397	0.0405	7.13
Raft with effect of the wall	0.181	0.0527	0.0542	9.52

Table (50) Required bottom reinforcement in tangential direction A_{stb} for the raft without and with effect of the wall

Structural system	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{stb} [cm²/m]
Raft without effect of the wall	-	-	-	-
Raft with effect of the wall	0.0307	0.009	0.009	1.58

Table (51) Required top reinforcement in the field $A_{sxf} = A_{syf}$ for the raft without and with effect of the wall (both x- and y-directions)

Structural system	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{stb} [cm²/m]
Raft without effect of the wall	0.174	0.0507	0.0521	9.15
Raft with effect of the wall	0.133	0.0385	0.0393	6.91

Chosen reinforcement

Table (52) shows the chosen reinforcement for the raft. The bottom reinforcement is chosen to be in radial and tangential directions while the top reinforcement is chosen to be in x- and y-directions. The design considers the maximum reinforcement obtained form both the analyses of the two structural systems. The chosen diameter of steel bars is Φ = 14 [mm].

Table (52) Chosen reinforcement

Bottom reinforcement		Top reinforcement in x- and y-directions $A_{sxt} = A_{syt}$
Radial direction A_{srb}	tangential direction A_{stb}	
7 Φ 14 = 10.78 [cm²/m]	min A_s = 7.7 [cm²/m]	6 Φ 14 = 9.24 [cm²/m]

According to the design of the raft for two structural systems, the raft is reinforced by a square mesh 6 Φ 14 [mm/m] in the upper surface, while the lower surface is reinforced by 7 Φ 14 [mm/m] in radial direction and 5 Φ 14 [mm/m] in tangential direction. In addition, an upper radial and tangential reinforcement 5 Φ 14 [mm/m] are used at the cantilever ring. A small square mesh 5 Φ 14 [mm/m], each side is 1.0 [m] is used at the center of the raft to connect the bottom radial reinforcement. The details of reinforcement of the raft are shown in Figure (48).

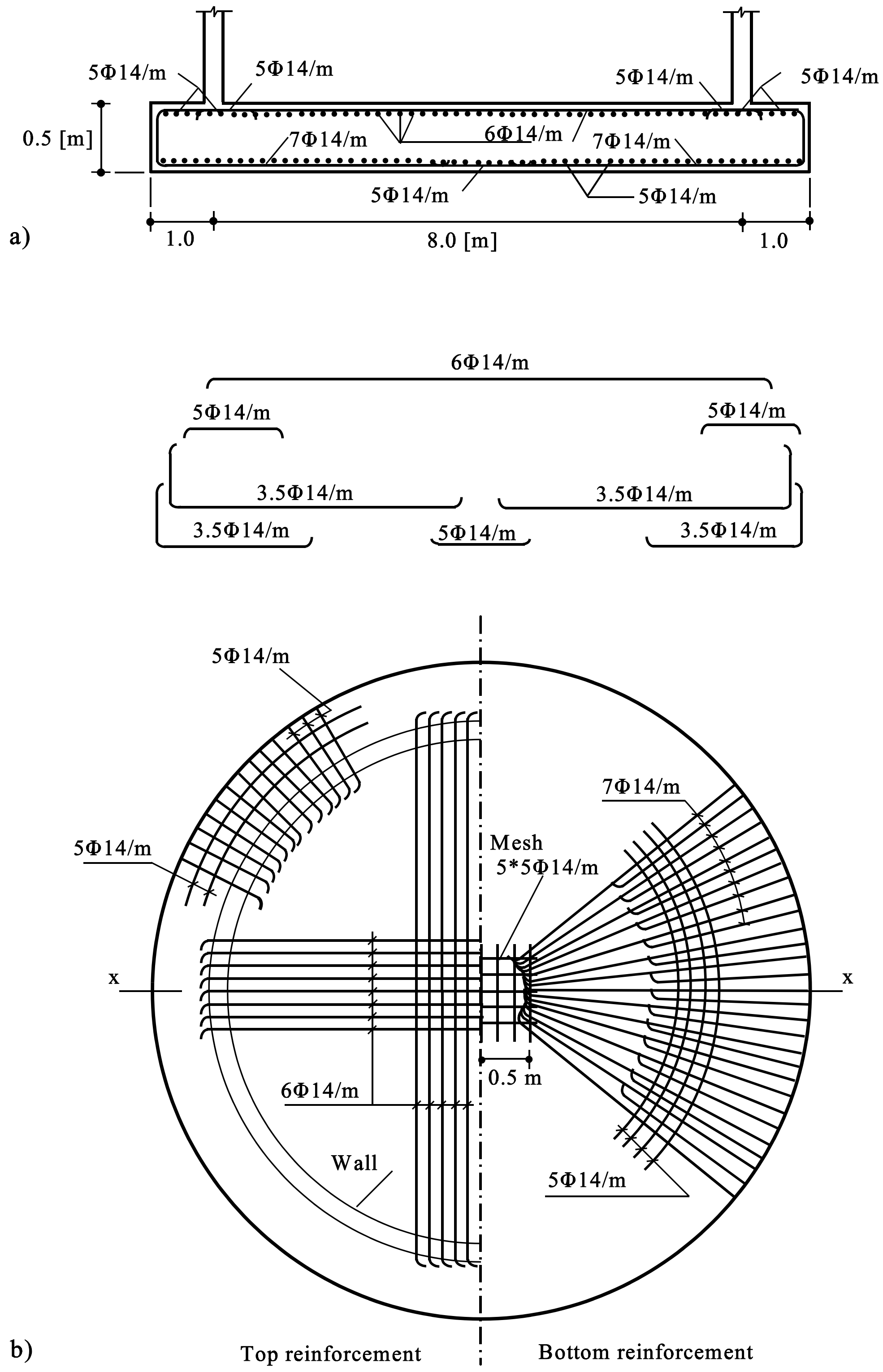


Figure (48) a) Section x-x through the raft with reinforcement
b) Reinforcement of the raft in plan

Example 5: Comparison between flat and ribbed rafts

1 Description of the problem

A ribbed raft may be used where the distance between columns is so great that a flat raft requires excessive depth, with resulting high bending moments. Consequently, the volume of concrete is reduced. A ribbed raft consists of a stiffened slab by girders in x- and y-directions. The girders on the raft may be either down or up the slab. Ribbed rafts can be used for many structures when a flat level for the first floor is not required. Such structures are silos, elevated tanks and various other possible structures. Although this type of foundation has many disadvantages if used in normally buildings, still uses by many designers. Such disadvantages are: the raft needs deep foundation level under the ground surface, fill material on the raft to make a flat level. In addition, a slab on the fill material is required to be constructed for the first floor. The use of the ribbed raft relates to its simplicity in analysis by traditional manners or hand calculations. Particularly, if the columns are arranged in lines. The ribbed raft generally leads to less concrete quantity than the flat raft, especially if the columns have heavy loads and large spans.

In this example two types of rafts, flat and ribbed rafts, are considered as shown in Figure (49). The length of each raft is L = 14.3 [m] while the width is B = 28.3 [m]. Each raft carries 15 column loads and a brick wall load of p = 30 [kN/m] at its edges. Width of ribs is chosen to be b_w = 0.30 [m] equal to the minimum side of columns, while the height of ribs including the slab thickness is chosen to be h_w+h_f = 1.0 [m]. Column dimensions, reinforcement and loads are shown in Table (53). A thin plain concrete of thickness 0.20 [m] is chosen under the raft and is not considered in any calculation.

Table (53) Column models, loads, dimensions and reinforcement

Column	Load [kN]	Dimensions [m*m]	Reinforcement
Model *C1*	781	0.30*0.30	6Φ16
Model *C2*	1562	0.30*0.70	4Φ16 +4Φ19
Model *C3*	3124	0.30*1.40	6Φ22 +6Φ19

Two analyses are carried out to compare between the two structural systems of rafts. In the analyses, the Continuum model is used to represent the subsoil. The two cases of analyses are considered as follows:

- Flat raft for optimal raft thickness.
- Ribbed raft for optimal slab thickness.

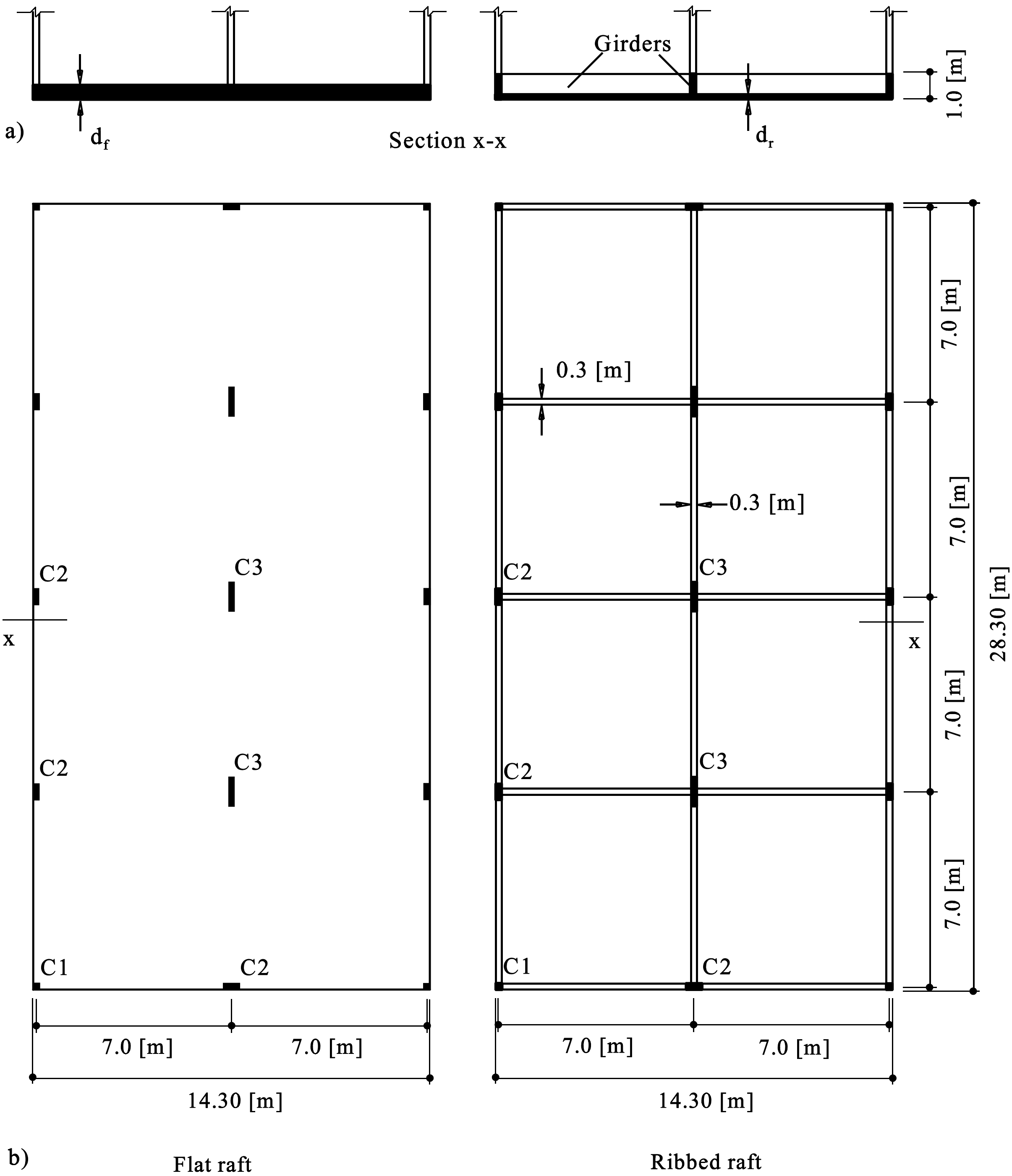


Figure (49) a) Plan of rafts and dimensions
b) Section through the rafts

2 Properties of the raft material

The material of rafts is reinforcement concrete that has the following parameters:

Young's modulus of concrete	E_b	$= 3.2 * 10^7$	[kN/m²]
Poisson's ratio of concrete	ν_b	= 0.20	[1]
Shear modulus of concrete	$G_b = 0.5\ E_b\ (1+\nu_b)$	$=1.3 * 10^7$	[kN/m²]
Unit weight of concrete	γ_b	= 25	[kN/m³]

3 Soil properties

The rafts rest on three soil layers consist of silty sand, silt and clay, respectively. A rigid base of sandstone is found under the clay layer. Figure (50) shows soil layers under rafts while Table (54) shows the soil parameters. Poisson's ratio is constant for all soil layers. The effect of reloading of the soil and limit depth of the soil layers are taken into account. The general soil parameters are:

Poisson's ratio of the soil layers	ν_s	= 0.30	[m]
Level of foundation depth under the ground surface	d_f	= 2.50	[m]
Level of ground water under the ground surface	GW	= 2.20	[m]

Table (54) Soil properties

Layer No. I	Type of soil	Depth of layer under the ground surface z [m]	Modulus of compressibility of the soil for		Unit weight above ground water γ_s [kN/m³]	Unit weight under ground water γ'_s [kN/m³]
			Loading E_s [kN/m²]	Reloading W_s [kN/m²]		
1	Silty sand	4.00	60 000	150 000	19	11
2	Silt	6.00	10 000	20 000	-	8
3	Clay	20.0	5 000	10 000	-	9

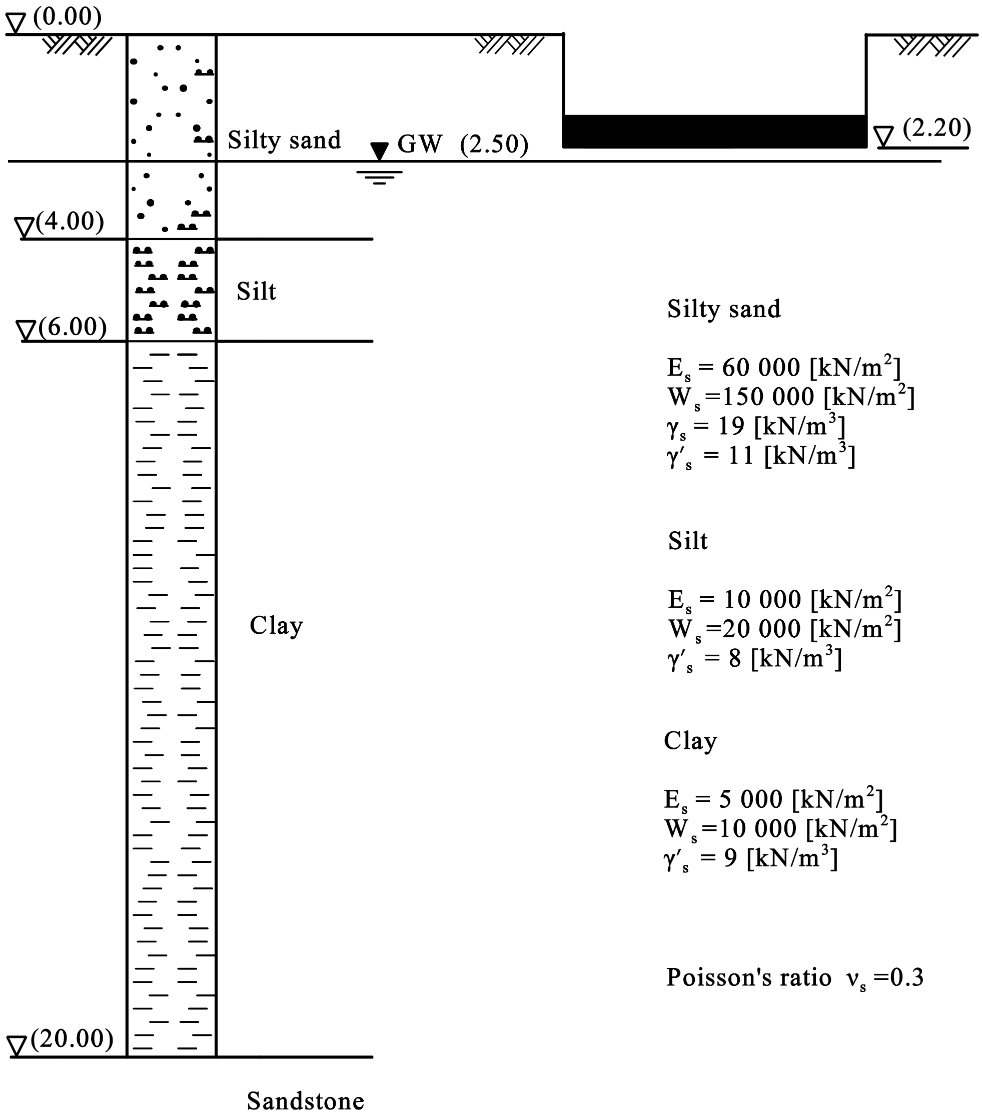


Figure (50) Soil layers and soil parameters under rafts

4 Analysis of the raft

4.1 Modeling of ribs

For modeling of ribs, different possibilities can be applied as follows:

i) The raft is analyzed first separately, by considering the ribs as a non-displaceable or elastic line supports. Then, the obtained support reactions apply to equivalent girders. This mathematical model supposes that the rib has more significant stiffness than that of the raft. In this case, a linear contact pressure under the raft may be assumed in the analysis (Conventional method (1)), where the interaction between the raft and the subsoil is not taken into account.

ii) Using a combination of two types of finite elements represent the system of a ribbed raft. The raft is represented by plate bending elements according to the two-dimensional nature of the raft. Beam elements are used to represent the rib action along the raft.

iii) Using a thicker line of plate elements represent the rib action along the raft. Then, for design of the rib, the required internal forces are determined from the plate element results. This model is reasonable for a wide rib.

iv) Using a three-dimensional shell model of block elements with six degrees of freedom at each node represent the rib and raft together. This model gives an exact representation of the rib behavior but it is complected.

In this example the analysis of the ribbed raft is carried out using a combination of plate and beam elements. Figure (51) shows FE-nets of flat and ribbed rafts. Each raft is subdivided into 312 plate elements. For the ribbed raft, the ribs are considered through inserting additional 138 beam elements along the location of the ribs on the FE-net.

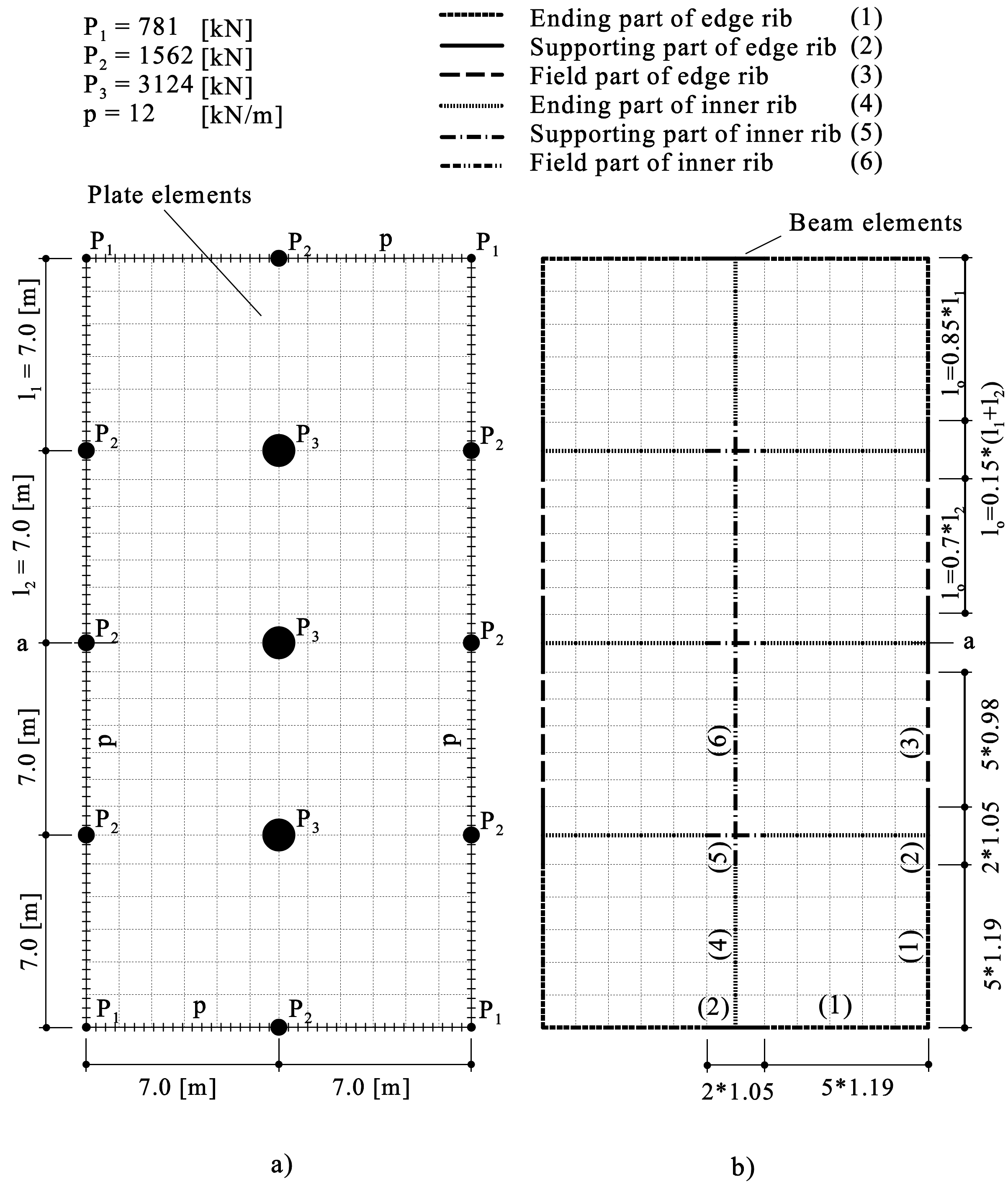


Figure (51) a) Flat raft with loads, dimensions and FE-net
b) Ribbed raft, arrangement of beam elements, dimensions and FE-net

4.2 Determination of replacement rib height h_{Ers}

To simulate the rib stiffness on the FE-net by using additional beam elements, the actual properties of the beam elements must be determined. The stiffness of the rib can be obtained through a replacement beam arranged in the center plane of the plate. The dimensions of the replacement beam can be taken as in DIN 1075 or EC 2. This can be carried out by determining firstly the moment of inertia for the effective section of the rib I_{pb}that contains two parts, flange and web (Figure (52). The rib section may be L-section or inverted T-section. Then, the replacement height of the web h_{Ers} can be determined by equating the section of inertia I_{pb} to two equivalent moments of inertia. The first moment of inertia I_p corresponds a rectangular flange of dimensions b_{eff} and h_f while the second moment of inertia I corresponds a rectangular web of dimensions b_w and h_{Ers}. The replacement height of the web h_{Ers} must be higher than the sum of slab thickness h_f and clear height of the rib h_w. In the finite element model of the ribbed raft, the rib is represented by beam element that has the property of b_w and h_{Ers} while the flange is already included in the plate finite element.

According to EC 2 the rib is defined by different stiffness distribution along its length, depending on the points of zero moment at the rib, where the effective flange width of the rib depends on the position of this point. This stiffness can be determined approximately independent of the load geometry at different spans. Guidelines for calculating effective spans l_o and flange widths b_{eff} are given in Figures (52) and (53) while Table (55) shows effective spans and flange widths of ribs at different rib parts for the raft.

Table (55) Effective span and flange width of the rib

Rib part	Effective rib span l_o [m]	Effective flange width b_{eff} [m]	
		Edge rib $b_{eff}=b_w+l_o/10$	Inner rib $b_{eff}=b_w+l_o/5$
Ending part	$0.85\ l_1 = 5.95$	0.895	1.49
Supporting part	$0.15\ (l_1+l_2) = 2.1$	0.51	0.72
Field part	$0.7\ l_2 = 4.9$	0.79	1.28

where in Table (55) is:

$l_1 = l_2 = 7.0$ [m] Rib span

$b_w = 0.30$ [m] Width of rib

Figures (54) and (55) show the moment of inertia ratios $r = I_{pb}/I$ at different clear heights h_w. From these figures, it can be concluded that the small clear height h_w has a great influence on the ratio *r*. The replacement heights h_{Ers} for different clear heights h_w are plotted as curves in Figures (56) and (57). These curves indicate that the maximum replacement height occurs when the clear height h_w is about 0.75 - 0.80 [m]. At this clear height, the dimensions of the rib are considered optimal.

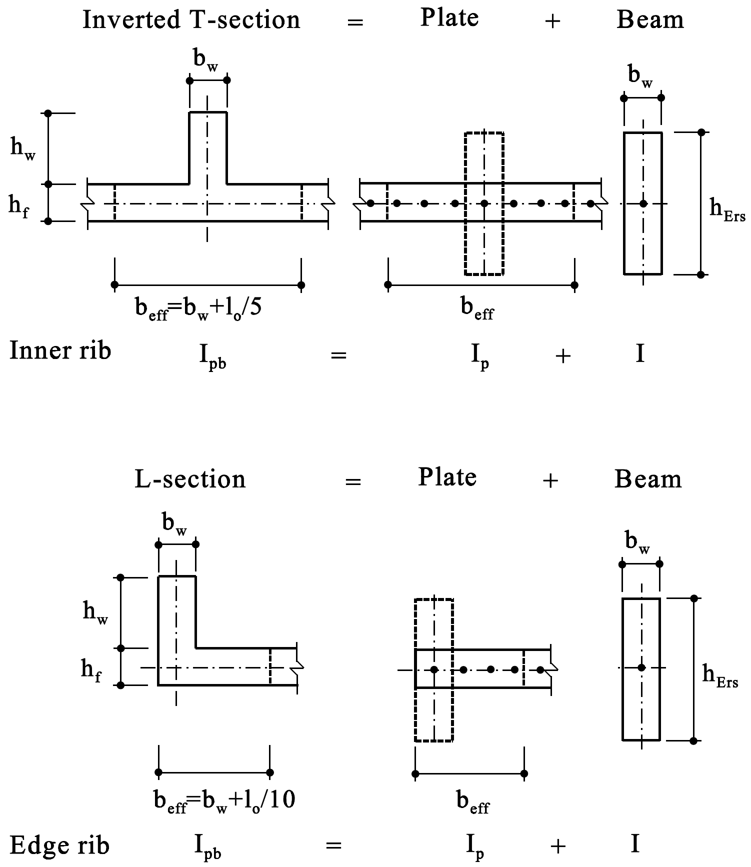


Figure (52) Determination of replacement height h_{Ers}

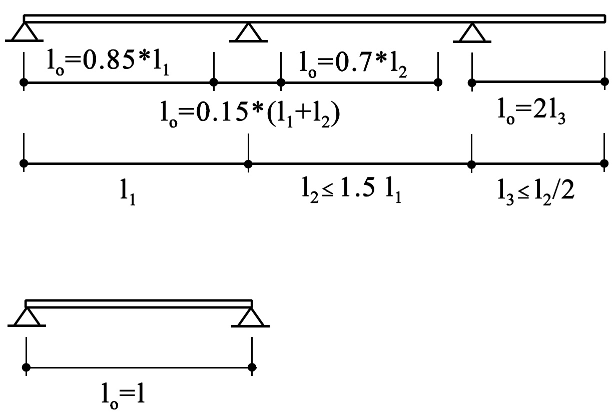


Figure (53) Definition of effective span l_o

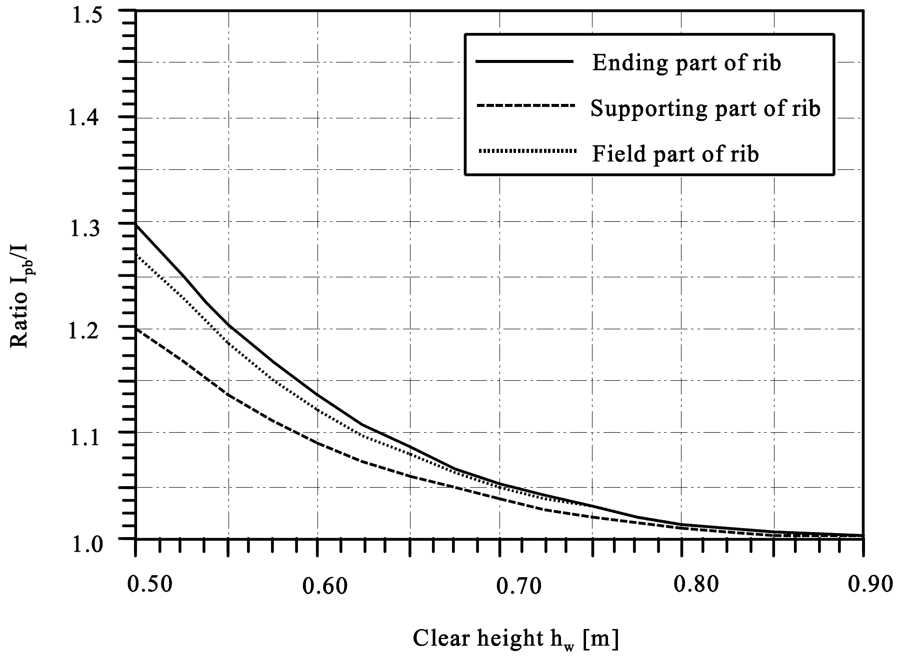


Figure (54) Moment of inertia ratio $r = I_{pb}/I$ for edge ribs

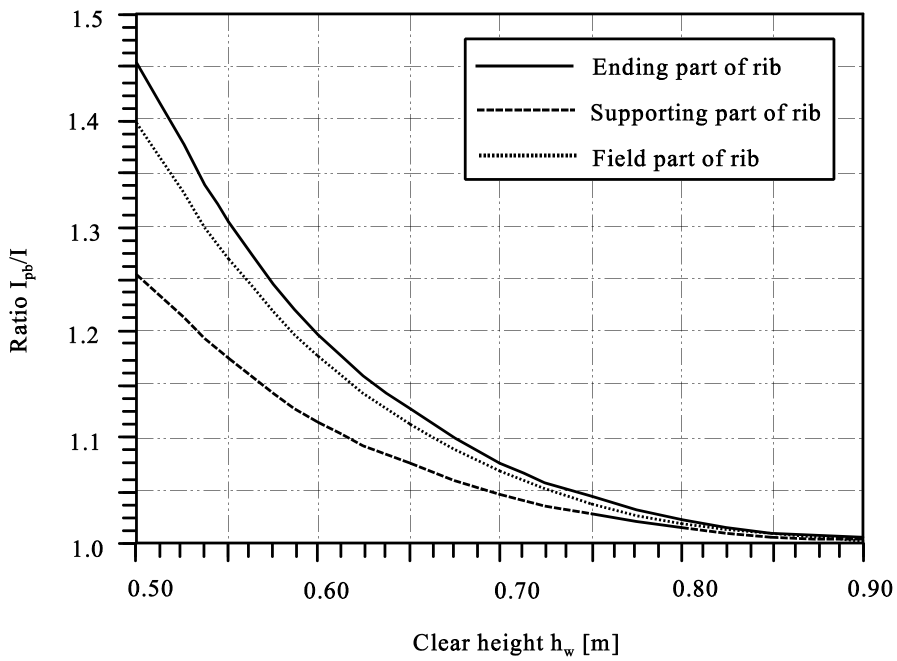


Figure (55) Moment of inertia ratio $r = I_{pb}/I$ for inner ribs

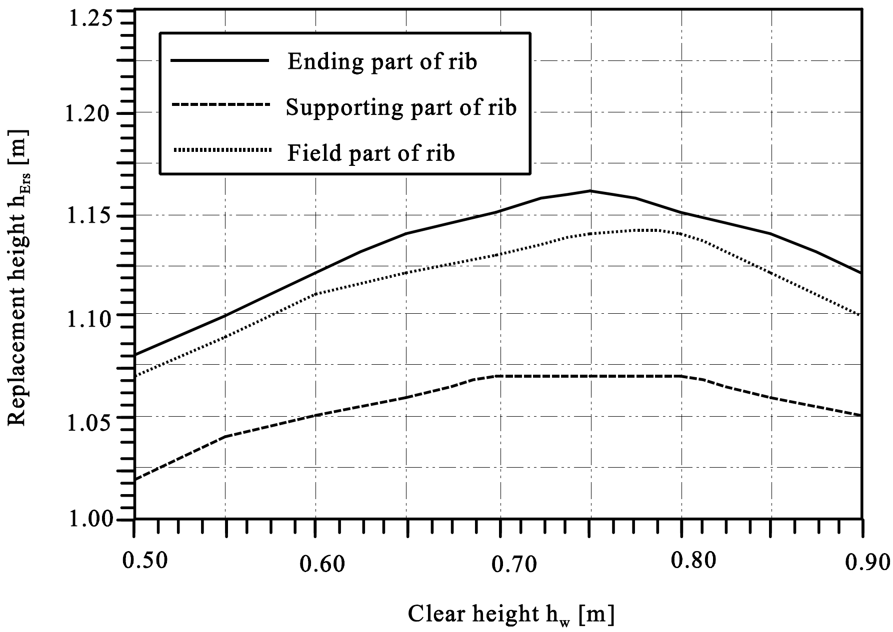


Figure (56) Replacement height h_{Ers} for edge ribs

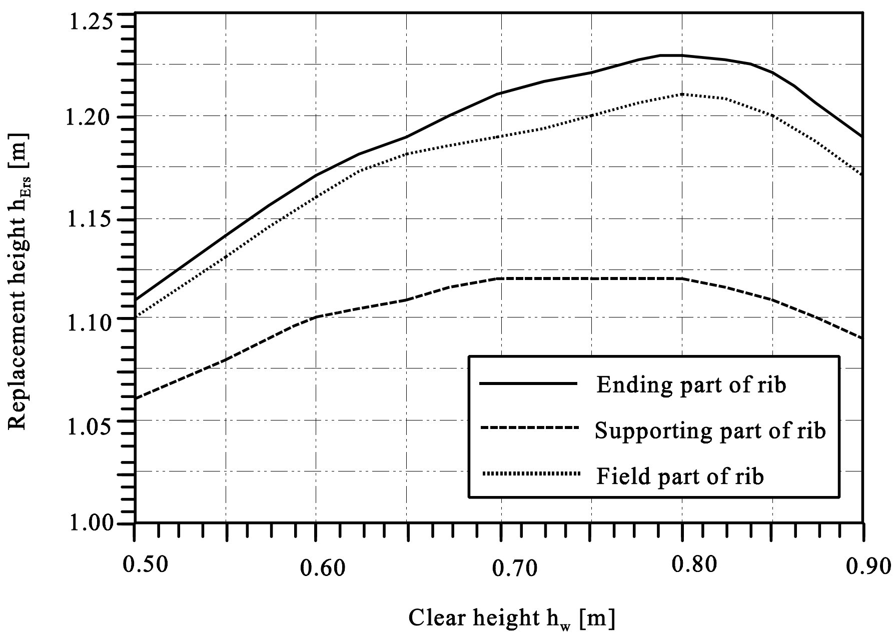


Figure (57) Replacement height h_{Ers} for inner ribs

4.3 Optimal thickness

The optimal thickness is designed to be the minimum thickness of the raft for which the concert section and tensile reinforcement are enough to resist the flexure moments without compressive reinforcement. The optimal thickness is designed according to EC 2 for the following parameters:

Material

Concrete grade C 30/37		
Steel grade BSt 500		
Characteristic compressive cylinder strength of concrete f_{ck}	= 30	[MN/m^2]
Characteristic tensile yield strength of reinforcement f_{yk}	= 500	[MN/m^2]
Partial safety factor for concrete strength γ_c	= 1.5	
Design concrete compressive strength $f_{cd} = f_{ck}/\gamma_c$	= 30/1.5 = 20	[MN/m^2]
Partial safety factor for steel strength γ_s	= 1.15	
Design tensile yield strength of reinforcing steel $f_{yd} = f_{yk}/\gamma_s$	= 500/1.15 = 435	[MN/m^2]

Geometry

Width of the section to be designed	b	= 1.0	[m]
Concrete cover + 1/2 bar diameter	c	= 5	[cm]

Factored moment

The maximum moment m_{max} for the raft is obtained at different raft thicknesses t for flat raft and slab thicknesses h_f for ribbed raft. As soil layers represent the subsoil under the rafts, one of the methods for Continuum model may be used. The considered rafts and system of loads will lead to appearing a negative contact pressure, if the method (6) or (7) is used. Therefore, the Modification of modulus of subgrade reaction by iteration (method 4) with sufficient accuracy $\varepsilon = 0.002$ [m] is used in the analyses. It is found that the maximum moment m_{max} for the flat raft occurs always at its center while for the ribbed raft occurs at different places depending on the slab thickness.

Total load factor for both dead and live loads	$\gamma = 1.5$
Factored moment	$M_{sd} = \gamma m_{max}$

Check for section capacity

The limiting value of the ratio x/d is $\xi_{lim} = 0.45$ for f_{ck} # 35 [MN/m^2]

The normalized concrete moment capacity $\mu_{sd,\,lim}$ as a singly reinforced section is

$$\mu_{sd,\ \lim} = 0.8\ \xi_{\lim}\ (1 - 0.4\ \xi_{\lim})$$

$$\mu_{sd,\ \lim} = 0.8 \times 0.45\ (1 - 0.4 \times 0.45) = 0.295$$

The sustained moment M_a for singly reinforced section will be obtained from

$$\mu_{sd,\ \lim} = \frac{M_a}{bd^2 (0.85 f_{cd})}$$

$$M_a = \mu_{sd,\ \lim} \times bd^2 (0.85 f_{cd}) = 0.295 \times 1.0 \times d^2 \times (0.85 \times 20)$$

$$M_a = 5.015\ d^2$$

where for flat raft:
d = *t* - 5 [cm] cover
t = raft thickness for flat raft
and for flat raft:
d = h_f - 5 [cm] cover
h_f = slab thickness for ribbed raft

The factored moment M_{sd} and the sustained moment M_a for both flat and ribbed rafts are calculated at different thicknesses and plotted in Figures (58) and (59). The minimum thickness is obtained from the condition $M_{sd} = M_a$. From Figures (58) and (59) the minimum thickness for the flat raft is t = 0.58 [m] while for the ribbed raft is h_f = 0.24 [m]. Therefore, the optimal thickness for the flat raft is chosen to be t = 0.60 [m] while for the ribbed raft is chosen to be h_f = 0.25 [m]. Table (56) shows a comparison between flat and ribbed rafts, which indicates that ribbed raft leads to less concrete volume and weight than those of flat raft by 44 [%].

Table (56) Comparison between flat and ribbed rafts

Cases	Concrete volume [m^3]	Concrete weight [kN]	Avarage contact pressure σ_o [kN/m^2]
Flat raft	243	6070	81
Ribbed raft	135	3384	75
Difference [%]	44	44	7

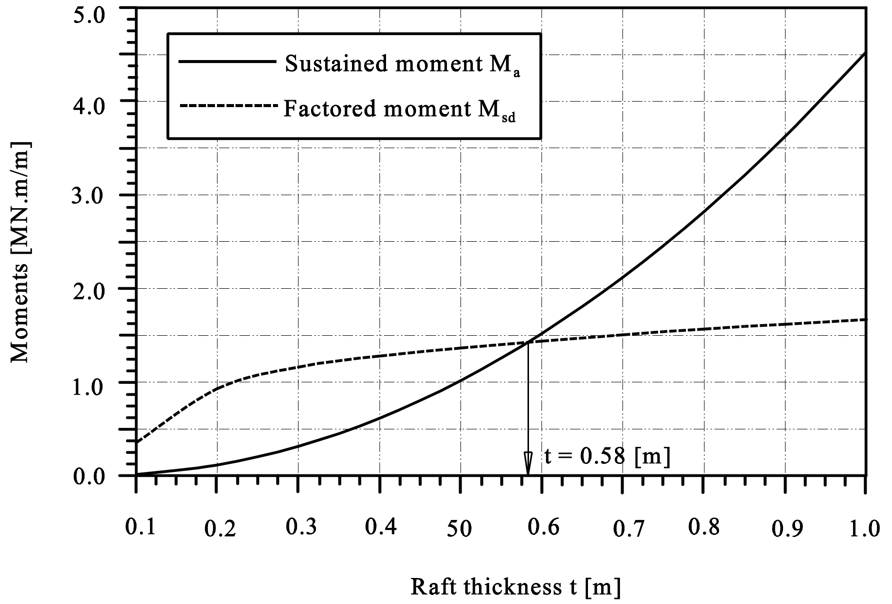


Figure (58) Determination of optimal raft thickness *t* for flat raft

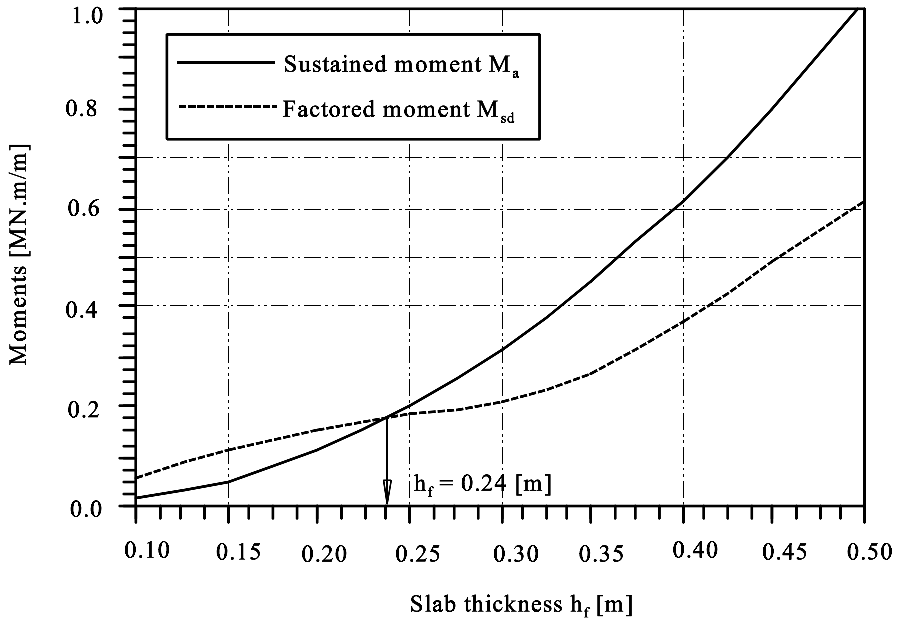


Figure (59) Determination of optimal slab thickness h_f for ribbed raft

Details of rib properties concerning ribbed raft are shown in Table (57).

Table (57) Properties of ribs for slab thickness h_f = 0.25 [m] and clear height h_w = 0.75 [m]

Rib part		Moment of inertia for effective rib section I_{pb} [m^4]	Moment of inertia for rib flange I_b [m^4]	Replacement web		
				Replacement height h_{Ers} [m]	Moment of inertia I [m^4]	Torsional inertia J [m^4]
Edge rib	Ending	0.0398	0.0386	1.16	0.0390	0.0087
	Supporting	0.0316	0.0309	1.07	0.0306	0.0079
	Field	0.0379	0.0368	1.14	0.0370	0.0086
Inner rib	Ending	0.0476	0.0456	1.22	0.0454	0.0093
	Supporting	0.0365	0.0355	1.12	0.0351	0.0084
	Field	0.0452	0.0436	1.20	0.0432	0.0091

where:
Moment of Inertia for rib $I = b_w * h_{Ers}^3/12$
Torsional Inertia for rib $J = h_{Ers} * b_w^3 * (1/3-0.21(b_w/h_{Ers})(1-b_w^4/(12*h_{Ers}^4)))$

4.4 Determination of the limit depth t_s

The level of the soil under the raft in which no settlement occurs or the expected settlement will be very small where can be ignored is determined as a limit depth of the soil. The limit depth in this example is chosen to be the level of which the stress due to the raft σ_E reaches the ratio ξ = 0.2 of the initial vertical stress σ_V. The stress in the soil σ_E is determined at the characteristic point *c* of the rectangular foundation. The stress σ_E is due to the average stress from the raft at the surface σ_O = 81 [kN/m^2] for flat raft and σ_O = 75 [kN/m^2] for ribbed raft. At the characteristic point, from the definition of Graßhoff (1955), the settlement if the raft is full rigid will be identical with that if the raft is full flexible. The characteristic point *c* takes coordinates x = 0.87*L = 12.18 [m] and y = 0.13*B = 3.64 [m] as shown in Figure (60). The results of the limit depth calculation are plotted in a diagram as shown in Figure (60). The limit depth is t_s = 13.55 [m] for flat raft and t_s = 12.93 [m] for ribbed raft under the ground surface.

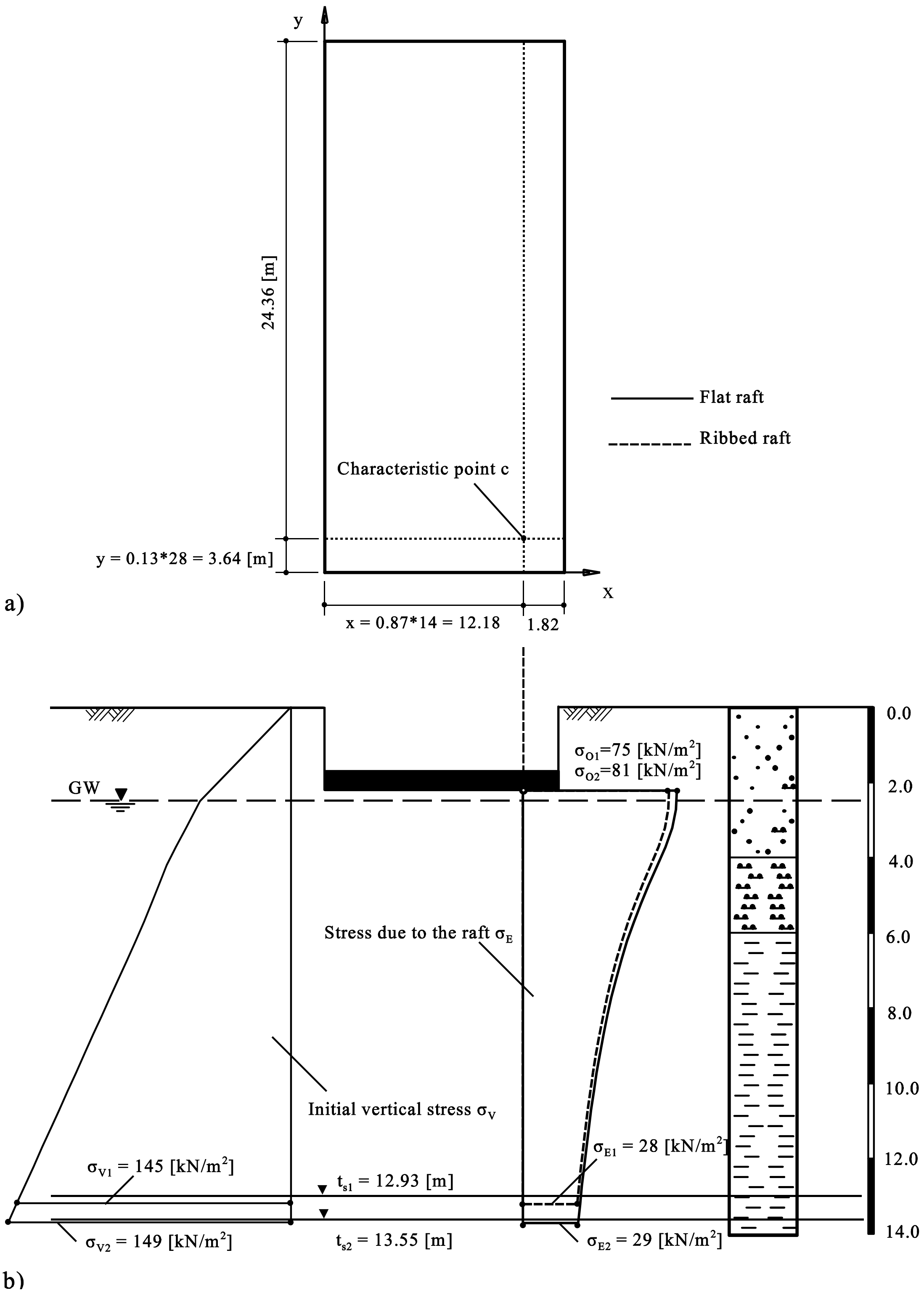


Figure (60) a) Position of characteristic point *c*
b) Limit depth t_s of the soil under the rafts

5 Evaluation and conclusions

To evaluate of analysis results, the results of the two cases of analyses are compared together. The following conclusions are drawn:

Settlements

Table (58) shows the extreme values of settlements for both flat and ribbed rafts. Figure (61) shows the contour lines of settlements *s* while Figure (62) shows the settlements *s* at section a-a under the middle of rafts. From the table and figures, it can be concluded the following:

- The ribs in the raft reduce the differential settlement by 7 [%], if a ribbed raft is used instead of flat raft.

- The settlement of the flat raft is greater than that of the ribbed raft because the flat raft has concrete volume greater than that of the ribbed raft, which leads to an increase in the self weight of the foundation.

Table (58) Extreme values of settlements for both flat and ribbed rafts

Cases of analysis	Maximum settlement s_{max} [cm]	Minimum settlement s_{min} [cm]	Maximum differential settlement Δs [cm]
Flat raft	5.42	3.56	1.86
Ribbed raft	4.80	3.06	1.74

Contact pressures

Figure (63) shows the contact pressures *q* at section a-a for the two cases of analyses.

- The difference in contact pressure for the two cases of analyses is not great along the rafts.

Moments

Figure (64) shows the moments m_x at section a-a for the two cases of analyses.

- The moment in slab of flat raft is greater than that of the ribbed raft by 93 [%]. This means the ribs resist most of the stresses in the ribbed raft.

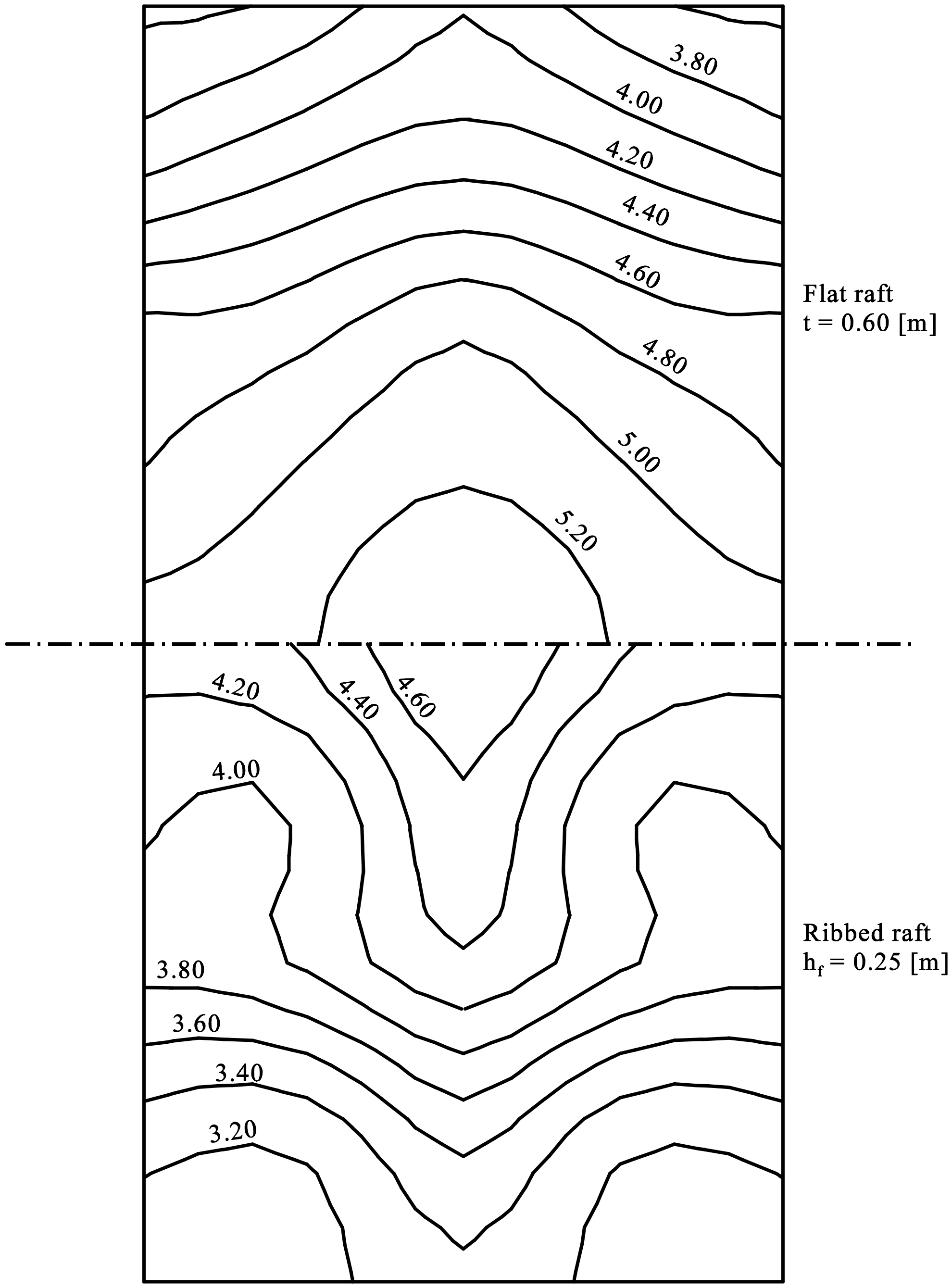


Figure (61) Contour lines of settlements for flat and ribbed rafts

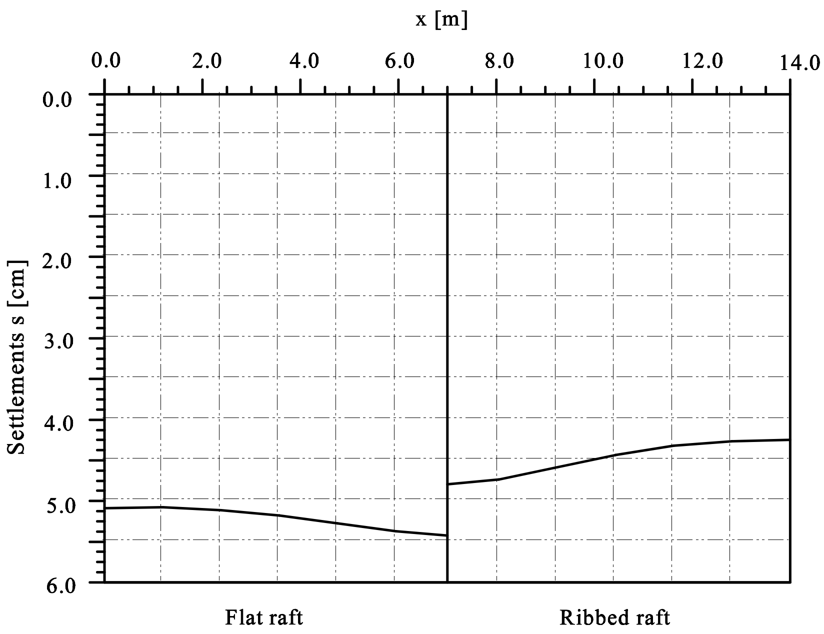


Figure (62) Settlements *s* [cm] at middle section a-a for flat and ribbed rafts

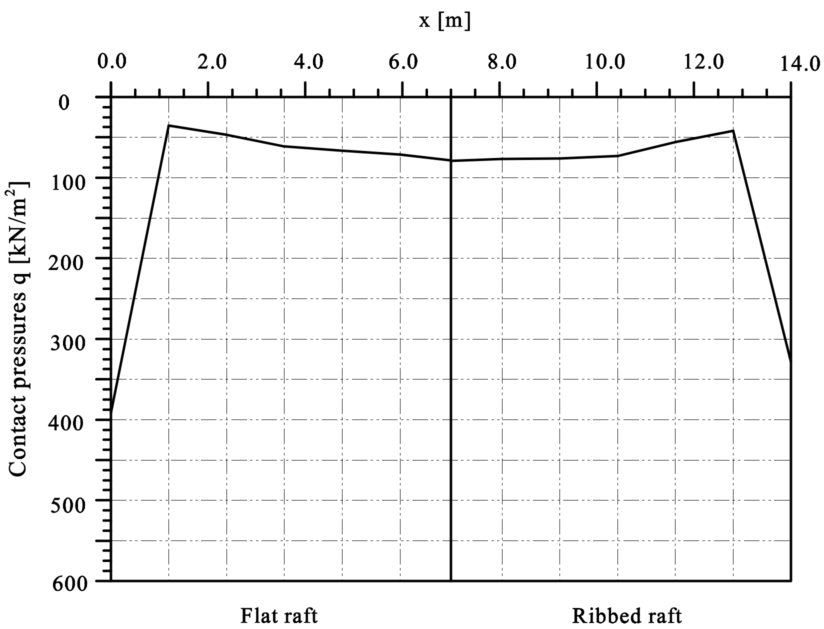


Figure (63) Contact pressures *q* [kN/m^2] at middle section a-a for flat and ribbed rafts

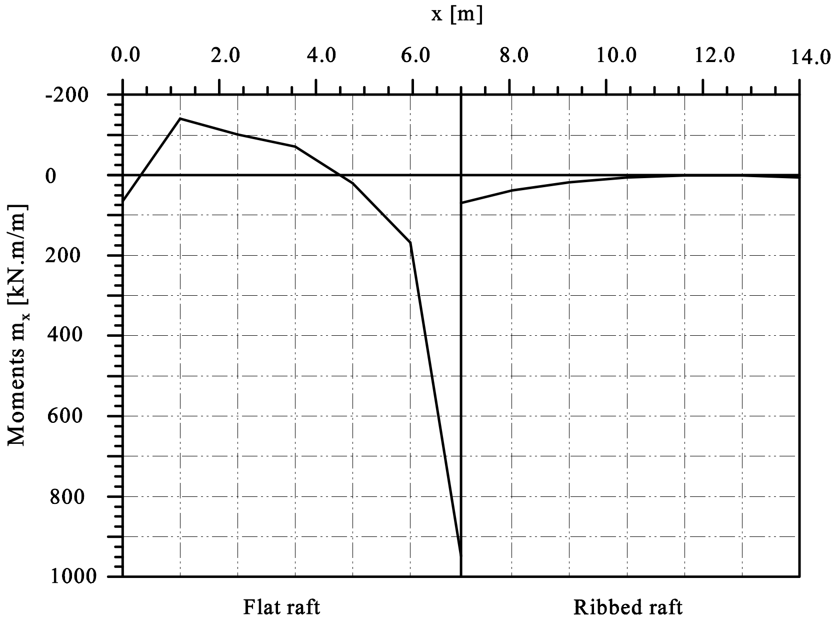


Figure (64) Moments m_x [kN.m/m] at middle section a-a for flat and ribbed rafts

6 Design of the flat raft for flexure moment

6.1 Definition of critical sections

The flat raft is designed for optimal thickness t = 0.60 [m]. Figures (65) and (66) show circular diagrams of moments and critical strips in x- and y-directions. The use of circular diagrams is an effective representation for moments where the critical zones can be quickly identified. Two critical strips are considered for each direction, column strip and field strip. It can be seen from circular diagrams that in each direction either column strips or field strips are nearly the same. Critical strips in x-direction are chosen to be the column strip (III) and the field strip (IV), while in y-direction are the column strip (3) and the field strip (2). Figures (67) to (70) and Table (59) show the extreme values of moments of these strips.

Table (59) Extreme values of moments in critical strips

<table>
<tr><td colspan="4">x-direction</td><td colspan="8">y-direction</td></tr>
<tr><td colspan="4">m_x [kN.m/m]</td><td colspan="8">m_y [kN.m/m]</td></tr>
<tr><td colspan="2">Column strip</td><td colspan="2">Field strip</td><td colspan="4">Column strip</td><td colspan="4">Field strip</td></tr>
<tr><td>Min.</td><td>Max.</td><td>Min.</td><td>Max.</td><td colspan="2">Min.</td><td colspan="2">Max.</td><td colspan="2">Min.</td><td colspan="2">Max.</td></tr>
<tr><td rowspan="2">-143</td><td rowspan="2">939</td><td rowspan="2">-96</td><td rowspan="2">454</td><td>Po1</td><td>Po2</td><td>Po3</td><td>Po4</td><td>Po5</td><td>Po6</td><td>Po7</td><td>Po8</td></tr>
<tr><td>-163</td><td>-45</td><td>897</td><td>918</td><td>-141</td><td>-45</td><td>402</td><td>424</td></tr>
</table>

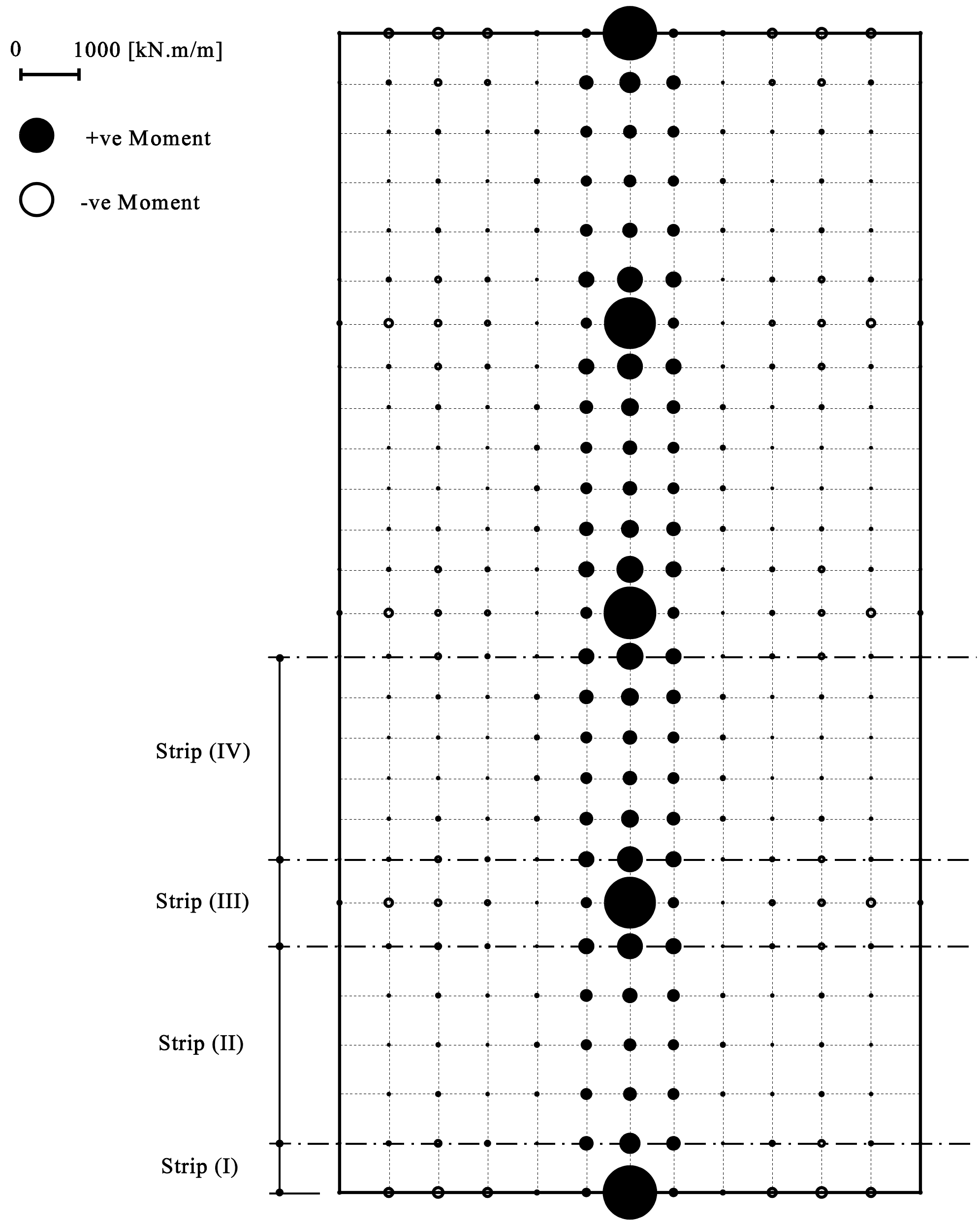


Figure (65) Circular diagrams of moments m_x [kN.m/m] and critical strips in x-direction

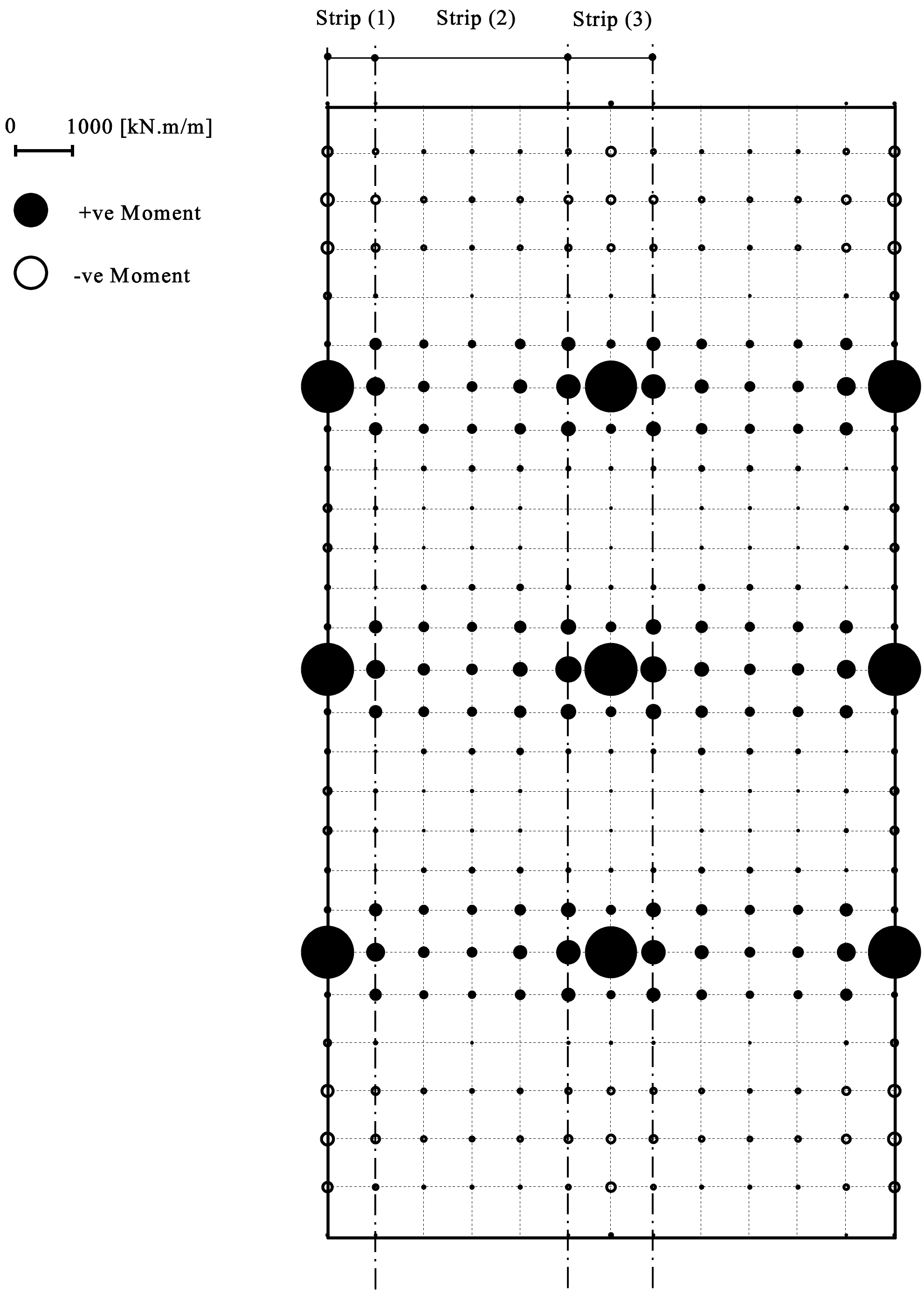


Figure (66) Circular diagrams of moments m_y [kN.m/m] and critical strips in y-direction

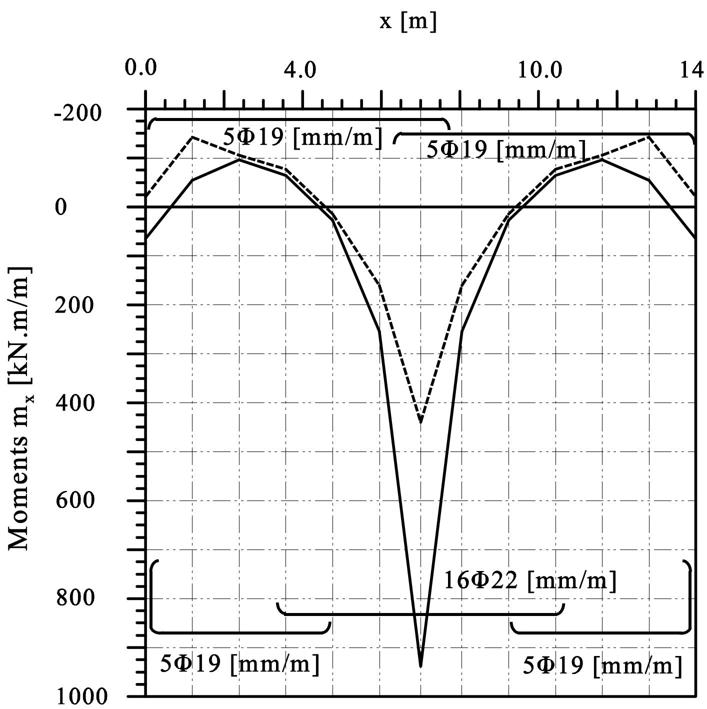


Figure (67) Extreme values of moments m_x [kN.m/m] in column strip (III)

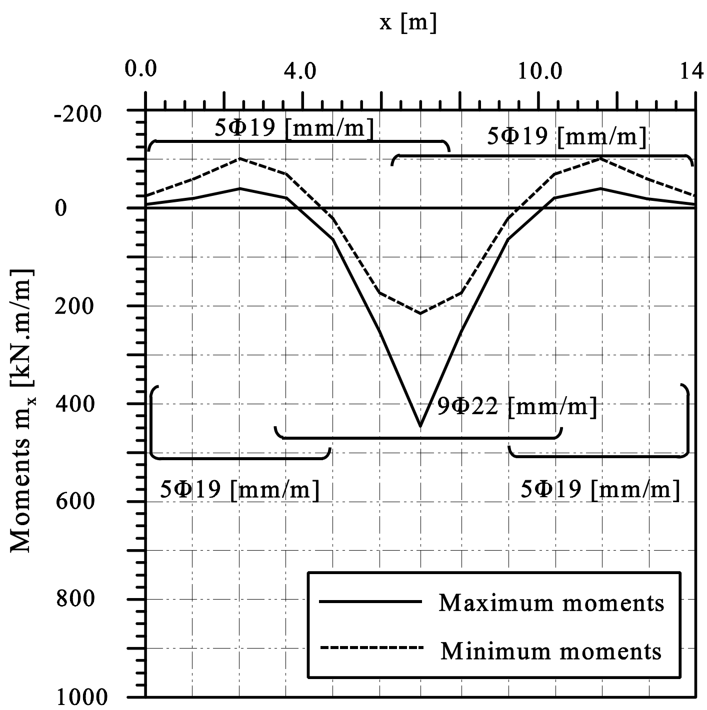


Figure (68) Extreme values of moments m_x [kN.m/m] in field strip (IV)

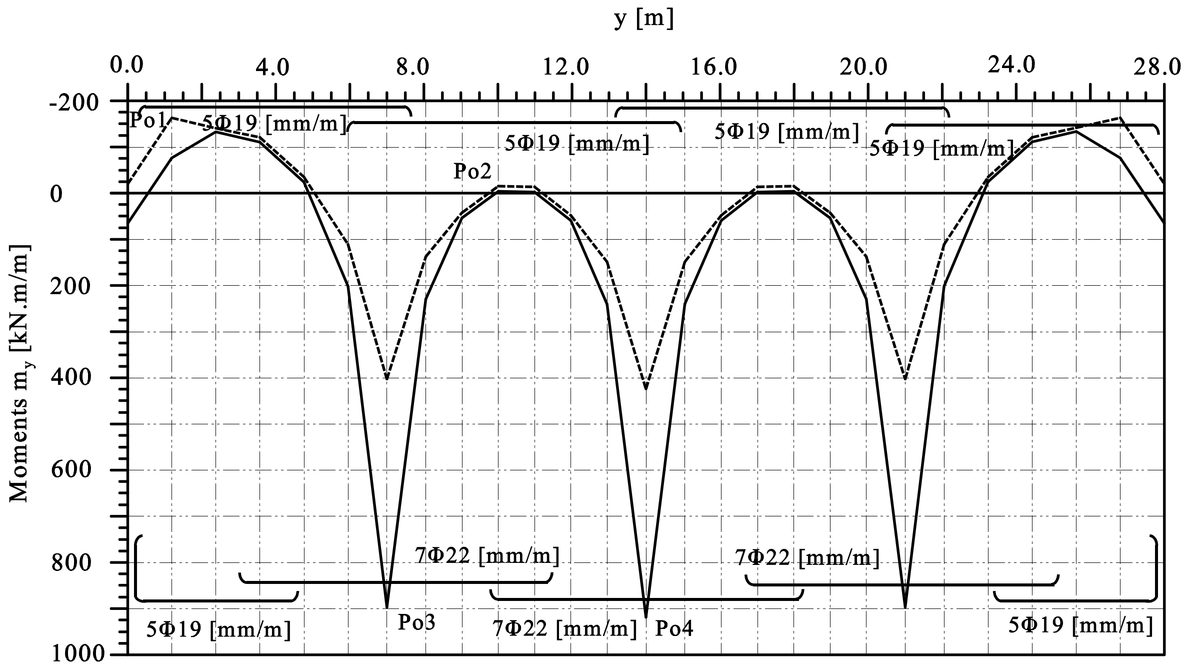


Figure (69) Extreme values of moments m_y [kN.m/m] in column strip (3)

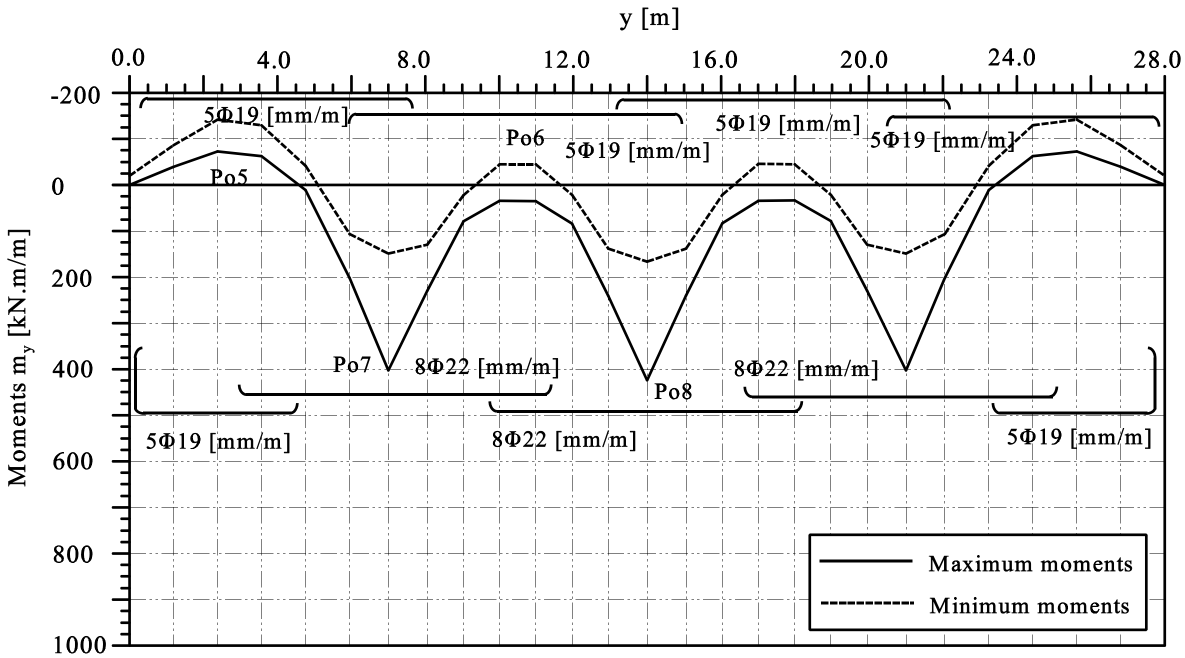


Figure (70) Extreme values of moments m_y [kN.m/m] in field strip (2)

6.2 Modified moments under columns

Because the column load is represented by a point load on the FE-net, the moment under the column will be higher than the real moment. Therefore, to take into account the load distribution through the raft thickness, the obtained moments under columns are modified according to Figure (71) and the following formula (Rombach (1999)):

$$M^{*} = M - \frac{P \cdot a}{8}$$

where:

M^*	=	Modified moment under the column [kN.m/m]
M	=	Calculated moment under the column [kN.m/m]
P	=	Column load as a point load [kN]
a	=	Column width [m]

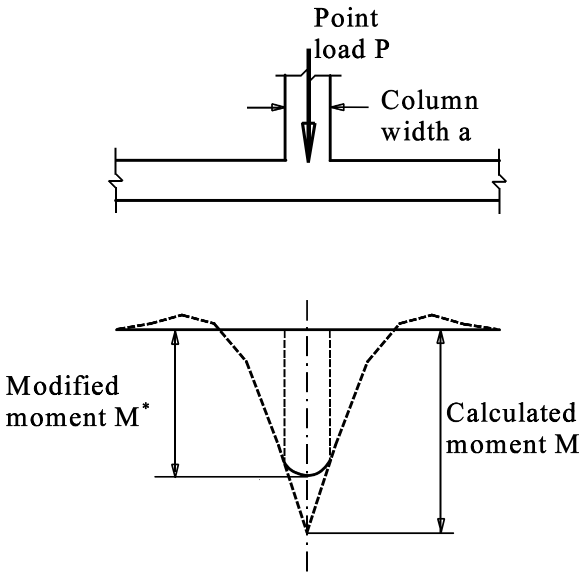


Figure (71) Modified moment under the column

It can be seen from Table (60) that the modified moment in y-direction is less than half the calculated moment. This is due to the wide column side in y-direction (a = 1.4 [m]). The difference between modified and calculated moments in x-direction is small as resulting of the short column side in this direction. However, considering the load distribution through the raft thickness has great influence on the results particularly for wide columns, the modified moment is neglected when determining the optimal thickness of the flat raft in this example.

Table (60) Modified moments under the columns

Direction		Column load P [kN]	Column width a [m]	calculated moment M [kN.m/m]	modified moment $M^{*}=M-P.a/8$ [kN.m/m]	Factored moment $M_{sd}=\gamma\ M^{*}$ [MN.m/m]
x-direction		3124	0.3	939	821.85	1.233
y-direction	Po3	3124	1.4	897	350.30	0.525
	Po4	3124	1.4	918	371.30	0.557

6.3 Geometry

Effective depth of the section d = 0.55 [m]
Width of the section to be designed b = 1.0 [m]

6.4 Determination of tension reinforcement

The design of critical strips is carried out for EC 2 using concrete grade C 30/37 and steel grade Bst 500.

The normalized design moment μ_{sd} is:

$$\mu_{sd} = \frac{M_{sd}}{bd^{2}(0.85f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 0.55^{2} \times (0.85 \times 20)} = 0.1945\ M_{sd}$$

The normalized steel ratio ω is:

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 0.1945\ M_{sd}} = 1 - \sqrt{1 - 0.3889\ M_{sd}}$$

The required area of steel reinforcement per meter A_s is:

$$A_s = \omega \left(\frac{(0.85 f_{cd})\ b\ d}{f_{yd}} \right)$$

$$A_s = \omega \left(\frac{0.85 \times 20 \times 1.0 \times 0.55}{435} \right) = 0.02149\ \omega\ [m^2/m]$$

$$A_s = 214.94\ \omega\ [cm^2/m]$$

Minimum reinforcement per meter, *min* A_s, is assumed as:

min A_s = 0.15 [%] * concrete section = 0.0015 * 100 * 60 = 9 [cm^2/m]
Chosen *min* A_s = 5Φ19 [mm/m] = 14.2 [cm^2/m]

Minimum reinforcement, *min* A_s, can resist factored moment M_{sd} equal to:

$$M_{sd} = \frac{1 - (1 - \omega)^2}{0.3889} = \frac{1 - \left(1 - \frac{14.2}{214.94}\right)^2}{0.3889} = 0.329\ [MN.m/m]$$

It can be seen from Figures (67) to (70) that the negative moments in x- and y-directions are trivial. Therefore, the chosen minimum reinforcement, min A_s = 5Φ19 [mm/m] is sufficient to resist the negative moments in the raft at the top.

The following Tables (61) to (62) show the required bottom reinforcement in critical strips.

Table (61) Required bottom reinforcement in x-direction A_{sxb}

Strip	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{sxb} [cm^2/m]
Column strip	1.233	0.2398	0.2786	59.89
Field strip	0.681	0.1325	0.1426	30.65

Table (62) Required bottom reinforcement in y-direction A_{syb}

Strip		M_{sd} [MN.m/m]	μ_{sd}	ω	A_{syb} [cm^2/m]
Column strip	Po3	0.525	0.1021	0.1079	23.20
	Po4	0.557	0.1083	0.1149	24.70
Field strip	Po7	0.603	0.1183	0.1251	26.89
	Po8	0.636	0.1237	0.1325	28.48

Chosen reinforcement

Table (63) shows the number of bottom steel bars in critical strips. The chosen main diameter of bottom steel bars is Φ = 22 [mm].

Table (63) Chosen main bottom reinforcement in critical strips

Direction	Strip		Chosen reinforcement A_s
x-direction	Column strip		16Φ22 = 60.80 [cm^2/m]
	Field strip		9Φ22 = 34.2 [cm^2/m]
y-direction	Column strip	Po3	7Φ22 = 26.6 [cm^2/m]
		Po4	7Φ22 = 26.6 [cm^2/m]
	Field strip	Po7	8Φ22 = 30.40 [cm^2/m]
		Po8	8Φ22 = 30.40 [cm^2/m]

Check of punching shear are to be done for corner column *C1*, edge column *C2* and interior column *C3* according to EC 2.

The details of reinforcement in plan and cross section through the raft are shown in Figure (72). Arrangement of reinforcement with moments are shown in details also in Figures (67) to (70).

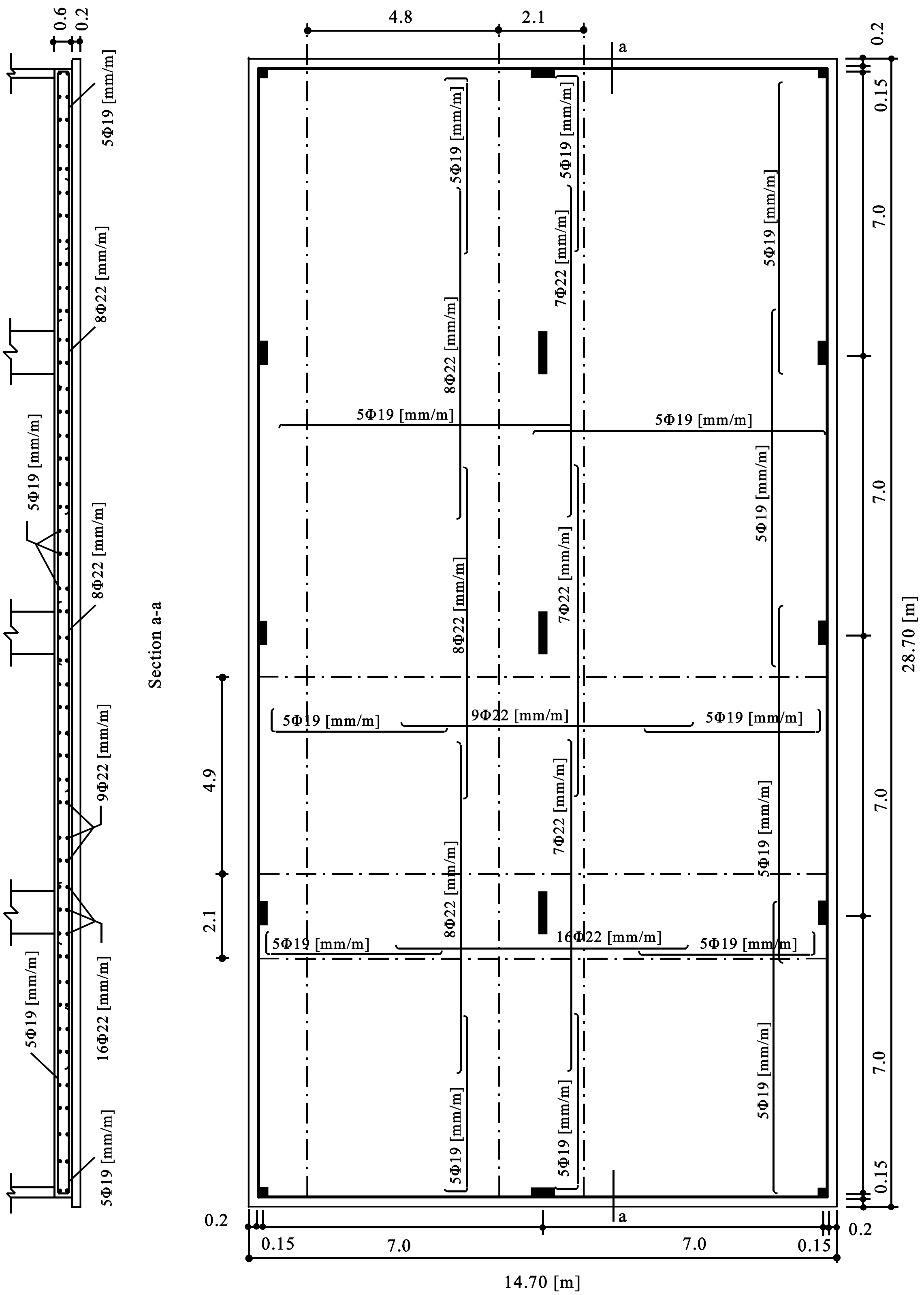


Figure (72) Details of reinforcement in plan and section a-a through the raft

7 Design of the ribbed raft for flexure moment

7.1 Definition of critical sections

The ribbed raft is designed for optimal slab thickness *of* h_f = 0.25 [m], while the ribs are designed for rib width of b_w = 0.3 [m] and total height *of* $h_w + h_f$ = 1.0 [m]. Figures (73) and (74) show circular diagrams of moments for slabs in x- and y-directions. It can be seen from these diagrams that in either x- or y-direction the moments along the slab are nearly constant. Therefore, only one critical strip in each direction is required to design. Figures (75) to (76) and Table (64) show the extreme values of moments of these strips. Figure (77) shows the bending moments m_b in the ribs, while Figure (78) shows the shear forces Q_s in the ribs.

Table (64) Extreme values of moments in critical strips

x-direction		y-direction			
m_x [kN.m/m]		m_y [kN.m/m]			
Min.	Max.	Min.		Max.	
-39	123	Po1	Po2	Po3	Po4
		-38	-36	96	101

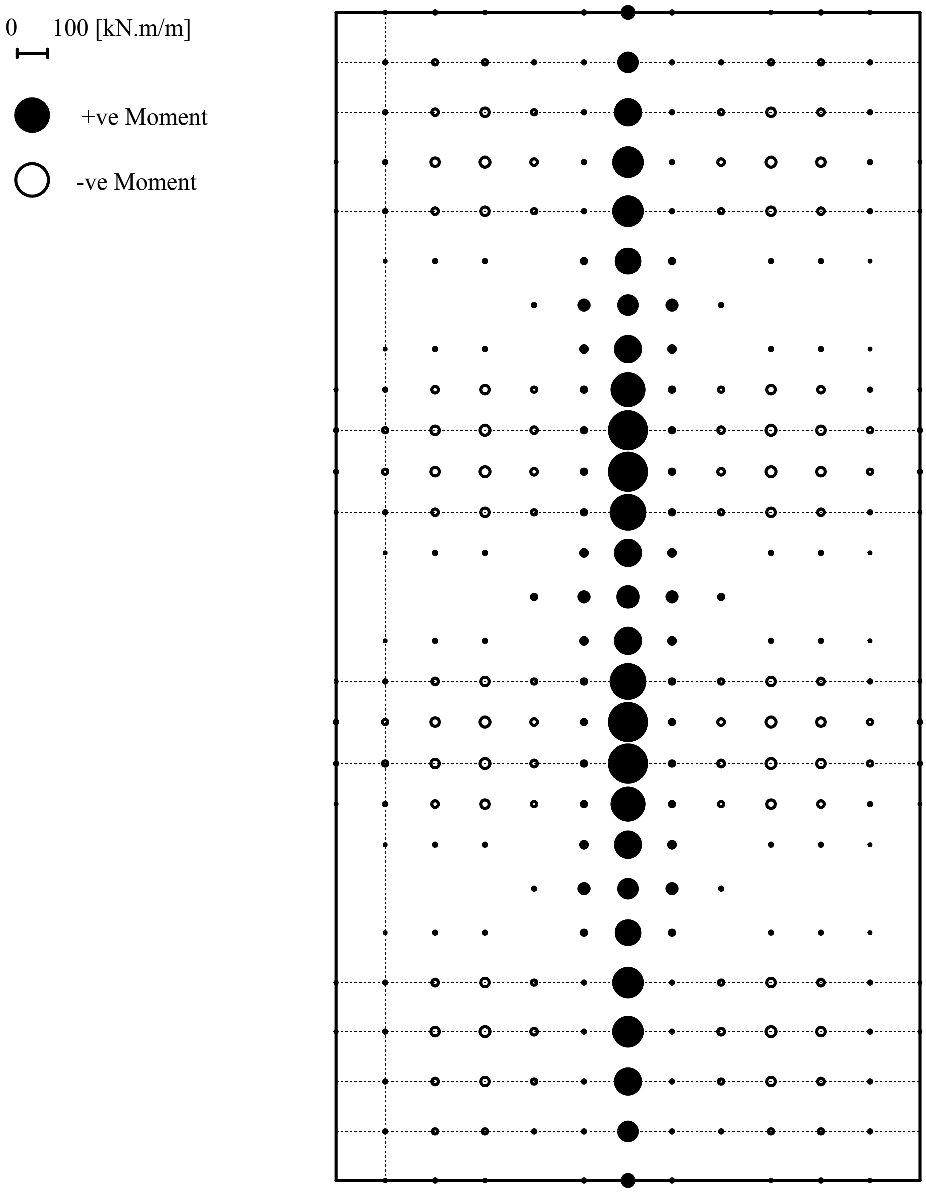


Figure (73) Circular diagrams of moments m_x [kN.m/m] for the slab in x-direction

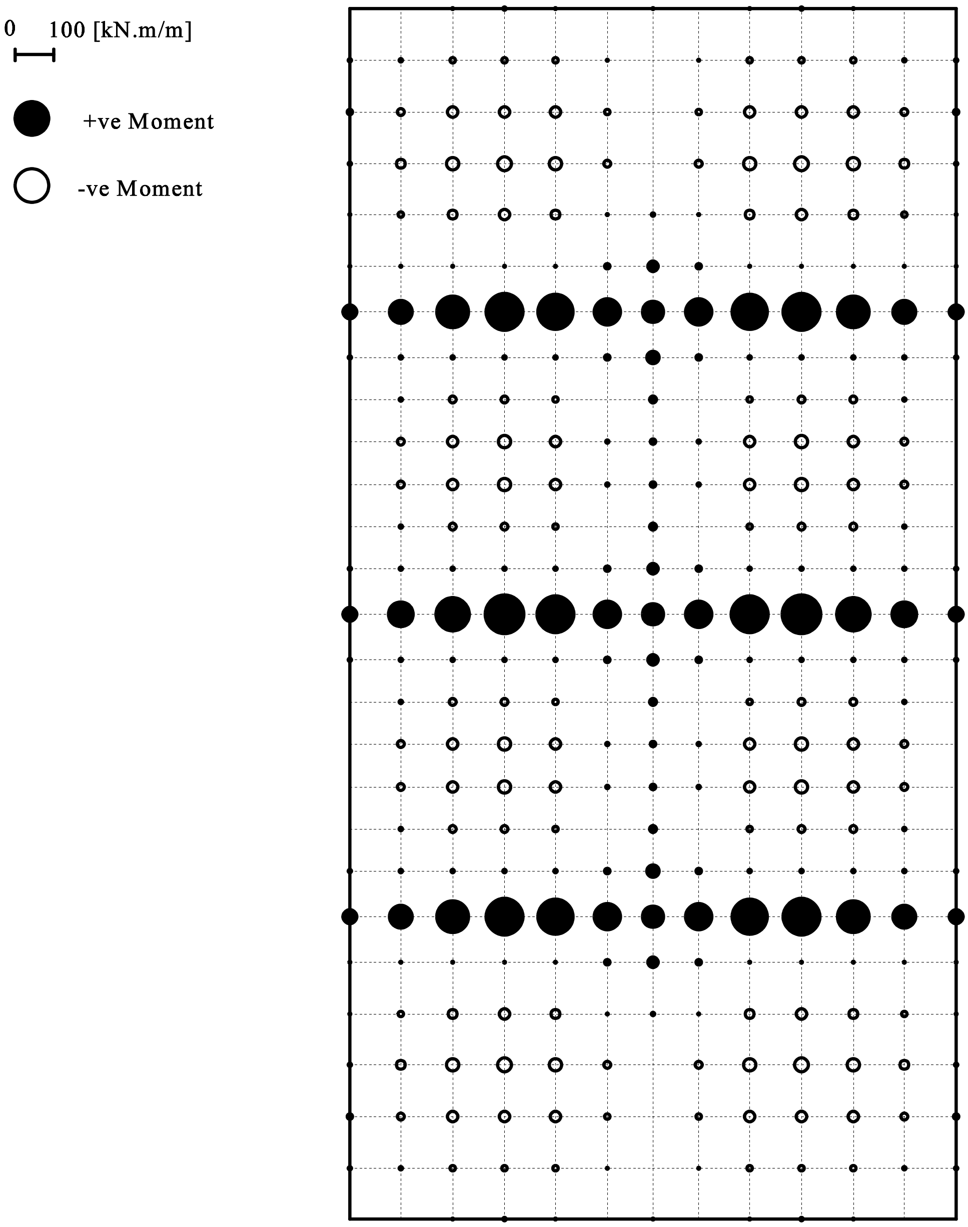


Figure (74) Circular diagrams of moments m_y [kN.m/m] for the slabs in y-direction

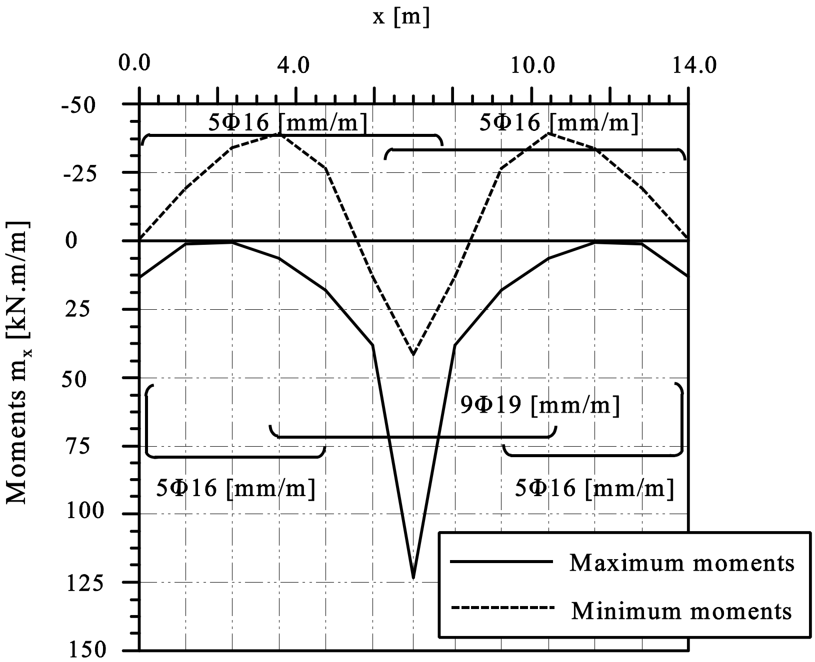


Figure (75) Extreme values of moments m_x [kN.m/m] in the slab

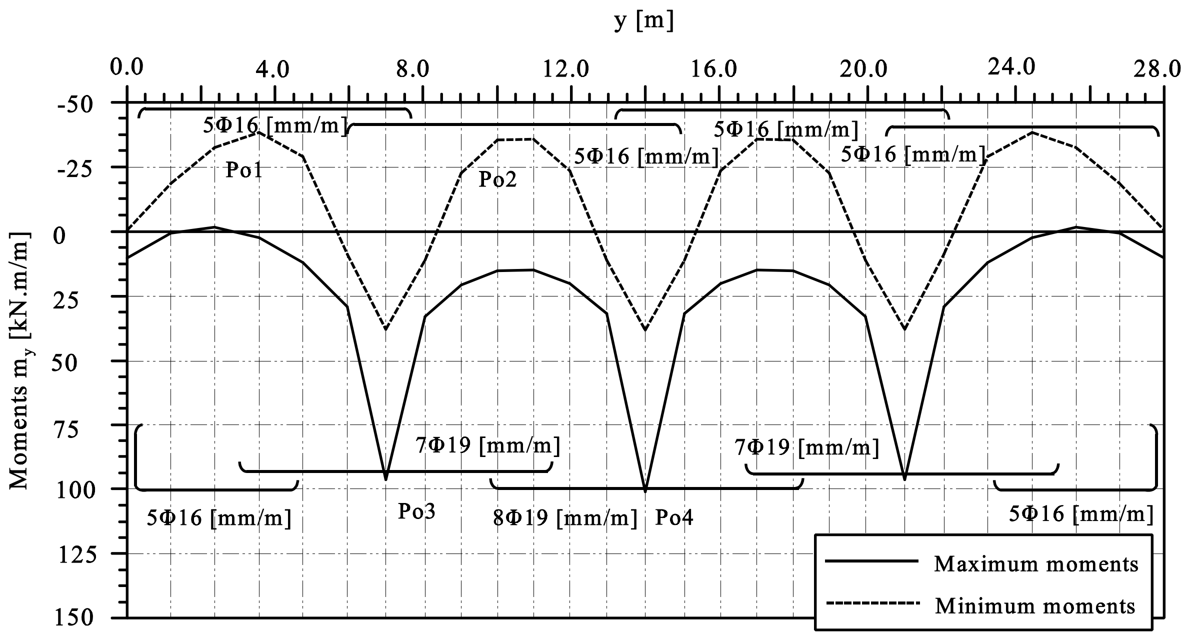


Figure (76) Extreme values of moments m_y [kN.m/m] in the slab

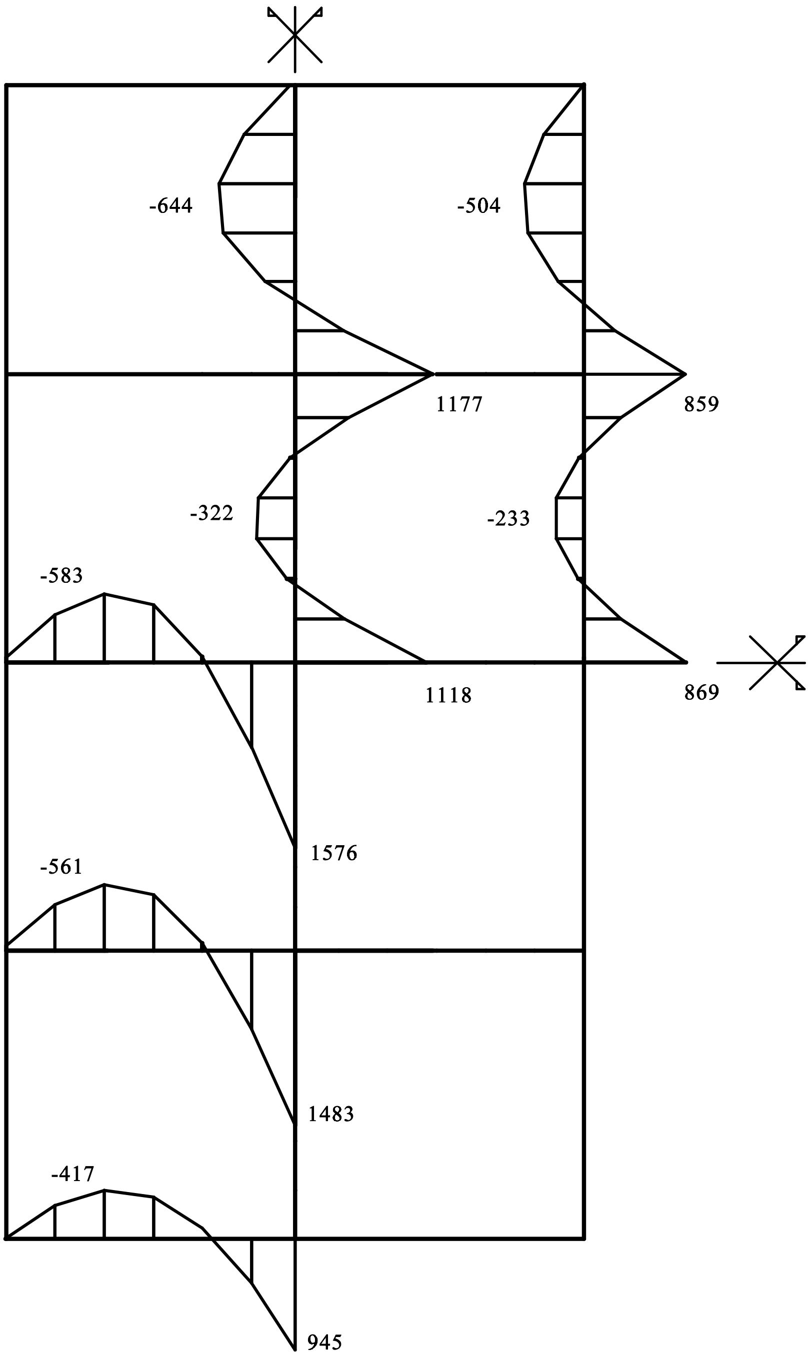


Figure (77) Bending moments m_b [kN.m] in the ribs

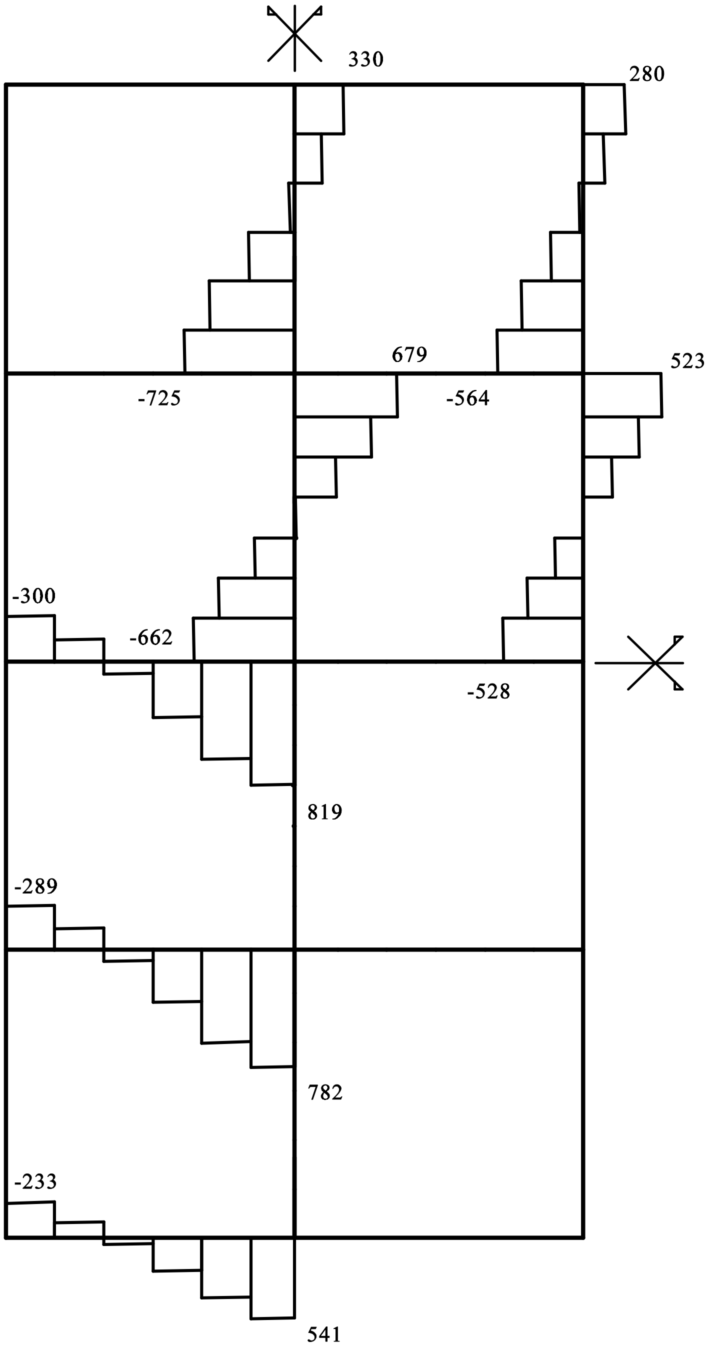


Figure (78) Shear forces Q_s [kN] in the ribs

7.2 Factored moments in the slab

Total load factor for both dead and live loads is taken as γ = 1.5. Table (64) shows the factored positive moments at critical sections.

Table (64) Factored positive moments at critical sections

Direction		calculated moment M [kN.m/m]	Factored moment $M_{sd}=\gamma\ M^*$ [MN.m/m]
x-direction		123	0.185
y-direction	Po3	96	0.144
	Po4	101	0.152

7.3 Geometry of the slab

Effective depth of the section d = 0.20 [m]
Width of the section to be designed b = 1.0 [m]

7.4 Determination of tension reinforcement in the slab

The design of critical strips is carried out for EC 2 using concrete grade C 30/37 and steel grade Bst 500.

The normalized design moment μ_{sd} is:

$$\mu_{sd} = \frac{M_{sd}}{bd^2(0.85 f_{cd})}$$

$$\mu_{sd} = \frac{M_{sd}}{1.0 \times 0.20^2 \times 0.85 \times 20} = 1.4706\ M_{sd}$$

The normalized steel ratio ω is:

$$\omega = 1 - \sqrt{1 - 2\mu_{sd}}$$

$$\omega = 1 - \sqrt{1 - 2 \times 1.4706\ M_{sd}} = 1 - \sqrt{1 - 2.9412\ M_{sd}}$$

The required area of steel reinforcement per meter A_s is:

$$A_s = \omega \left(\frac{(0.85 f_{cd})\ b\ d}{f_{yd}} \right)$$

$$A_s = \omega \left(\frac{0.85 \times 20 \times 1.0 \times 0.20}{435} \right) = 0.007816\ \omega\ [m^2/m]$$

$$A_s = 78.16\ \omega\ [cm^2/m]$$

Minimum reinforcement per meter, *min* A_s, is assumed as:

min A_s = 0.15 [%] * concrete section = 0.0015 * 100 * 25 = 3.75 [cm^2/m]
Chosen *min* A_s = 5Φ16 [mm/m] = 8.04 [cm^2/m]

Minimum reinforcement, *min* A_s, can resist factored moment M_{sd} equal to:

$$M_{sd} = \frac{1 - (1 - \omega)^2}{2.9412} = \frac{1 - \left(1 - \frac{8.04}{78.16}\right)^2}{2.9412} = 0.066\ [MN.m/m]$$

It can be seen from Figures (75) to (76) that the negative moments in x- and y-directions are trivial. Therefore, the chosen minimum reinforcement, *min* A_s = 5Φ16 [mm/m] is sufficient to resist the negative moments in the slab at the top.

The following Tables (65) and (66) show the required bottom reinforcement in critical strips in both directions.

Table (65) Required bottom reinforcement in x-direction A_{sxb}

M_{sd} [MN.m/m]	μ_{sd}	ω	A_{sxb} [cm^2/m]
0.185	0.2721	0.3248	25.39

Table (66) Required bottom reinforcement in y-direction A_{syb}

Position	M_{sd} [MN.m/m]	μ_{sd}	ω	A_{syb} [cm^2/m]
Po3	0.144	0.2118	0.2408	18.82
Po4	0.152	0.2235	0.2564	20.04

Chosen reinforcement

Table (67) shows the number of bottom steel bars in critical strips. The chosen main diameter of bottom steel bars is Φ = 19 [mm].

Table (67) Chosen main bottom reinforcement in critical strips

Direction		Chosen reinforcement A_s
x-direction		9Φ19 = 25.5 [cm^2/m]
y-direction	Po3	7Φ19 = 19.9 [cm^2/m]
	Po4	8Φ19 = 22.7 [cm^2/m]

Design of the rib sections are to be done according to EC 2. The design of section maybe as L-section for edge ribs or inverted T-section for inner ribs or rectangular section depending on the compression side of the rib. The effective flange width of the ribs can be taken from Table (55).

The details of reinforcement in plan are shown in Figure (79). Arrangement of reinforcement with moments are shown in details also in Figures (75) to (76).

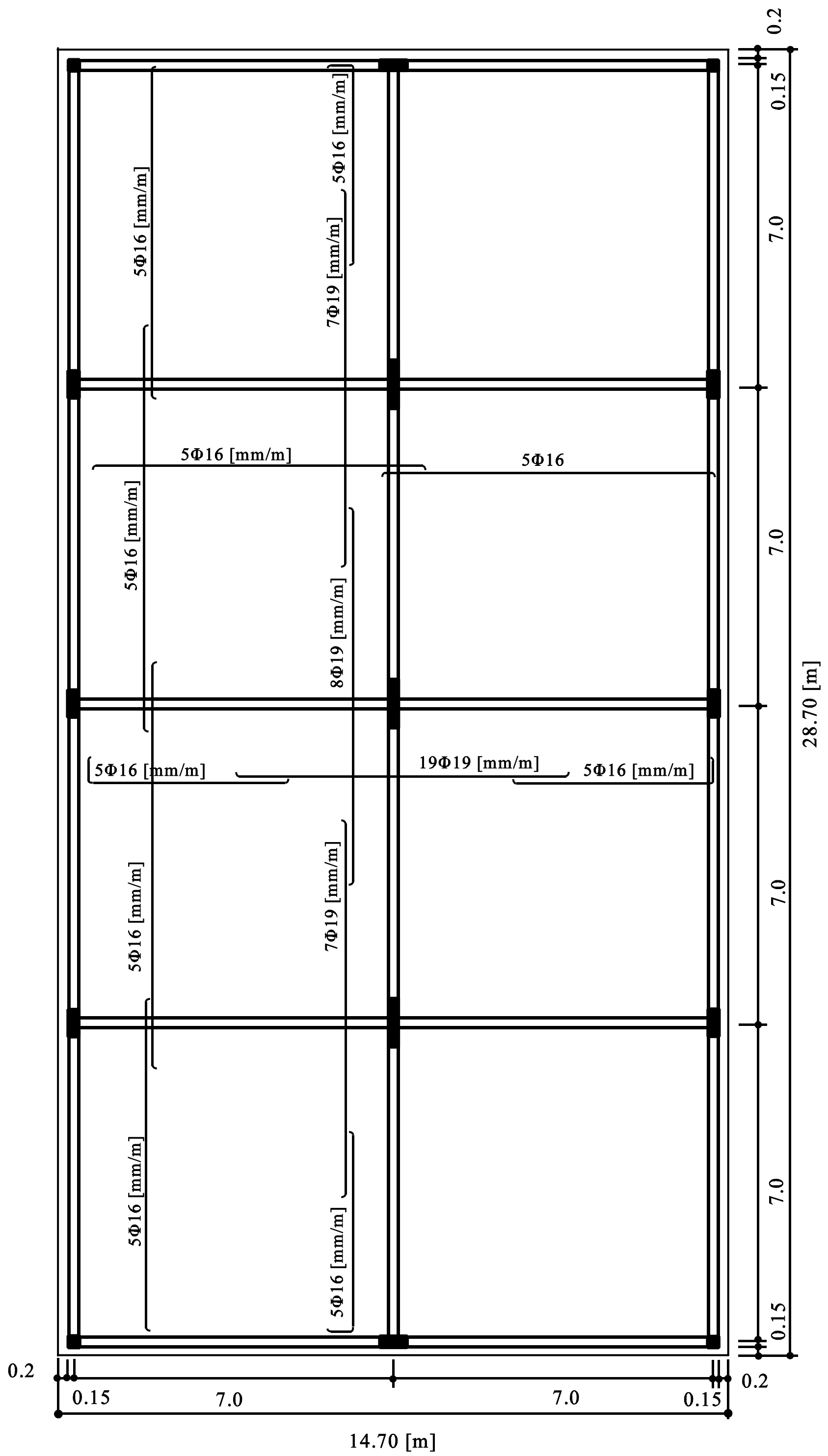


Figure (79) Details of reinforcement in plan

8 References

[1] DIN 1075 Betonbrücken; “Bemessung und Ausführung (ausgabe 04.81)”

[2] DIN 1045 (1988): “Stahlbeton- und Spannbetonbau. Beton und Stahlbeton”, Bemessung und Ausführung. Ausgabe Juli 1988.

[3] EUROCODE 2 (1993): “Design of Concrete Structures”. Deutsche Fassung: DIN V 18932 Teil 1 Beuth-Verlag GmbH Berlin und Beton-Kalender Oktober 1991

[4] Graßhoff, H. (1955): “Setzungsberechnungen starrer Fundamente mit Hilfe des kennzeichnenden Punktes”, Der Bauingenieur, S. 53 bis 54.

[5] Rombach, G. (1999): “Anwendung der Finite-Elemente-Methode im Betonbau”, Ernst & Sohn, Berlin.

Example (6): Design of trapezoidal footing

1 Description of the problem

In the primary design of footings or rafts, it is generally assumed that the contact pressure distribution is planar, whatever the type of model used in the analysis of the footing. Therefore, to achieve a desirable uniform contact stress distribution beneath the footing it is necessary to arrange the center of area of the footing directly beneath the center of gravity of the external loads. This may lead to irregular-shaped footing. If equal column loads are symmetrically disposed about the center of the footing, the contact pressure distribution will be uniform. In order to achieve a theoretically uniform contact pressure distribution, the footing can be extended so that the center of area of the footing coincides with the center of gravity of the external loads. This is easy to be done by rectangular footing.

A special case of footings is the trapezoidal footing, which may be used to carry two columns of unequal loads when distance outside the column of the heaviest load is limited. In such case, using a rectangular footing may lead to the resultant of loads dos not fall at the middle length of the footing. To overcome this difficulty, a trapezoidal footing is used in such a way that the center of gravity of the footing lies under the resultant of the loads. Correspondingly, the distribution of contact pressure will be uniform.

As a design example for trapezoidal footing, consider the trapezoidal combined footing of 0.60 [m] thickness shown in Figure (80). The footing is support to two columns *C1* and *C2* spaced at 4.80 [m] apart. Due to the site conditions, the projections of the footing beyond the centers of columns *C1* and *C2* are limited to 0.90 [m] and 1.30 [m], respectively. Column *C1* is 0.50 [m] * 0.50 [m], reinforced by 8Φ16 [mm] and carries a load of 1200 [kN]. Column *C2* is 0.60 [m] * 0.60 [m], reinforced by 12Φ19 [mm] and carries a load of 2000 [kN]. The allowable net soil pressure is $(q_{net})_{all}$ = 240 [kN/m^2]. The subsoil model used in the analysis of the footing is represented by isolated springs, which have a modulus of subgrade reaction of k_s = 50 000 [kN/m^3]. A thin plain concrete of thickness 0.15 [m] is chosen under the footing and is not considered in any calculation.

2 Footing section and material

The footing section and material are supposed to have the following parameters:

Section properties

Width of the section to be designed	b = 1.0	[m]
Section thickness	t = 0.60	[m]
Concrete cover + 1/2 bar diameter	c = 5	[cm]
Effective depth of the section	d = t - c = 0.55	[m]
Steel bar diameter	Φ = 25	[mm]

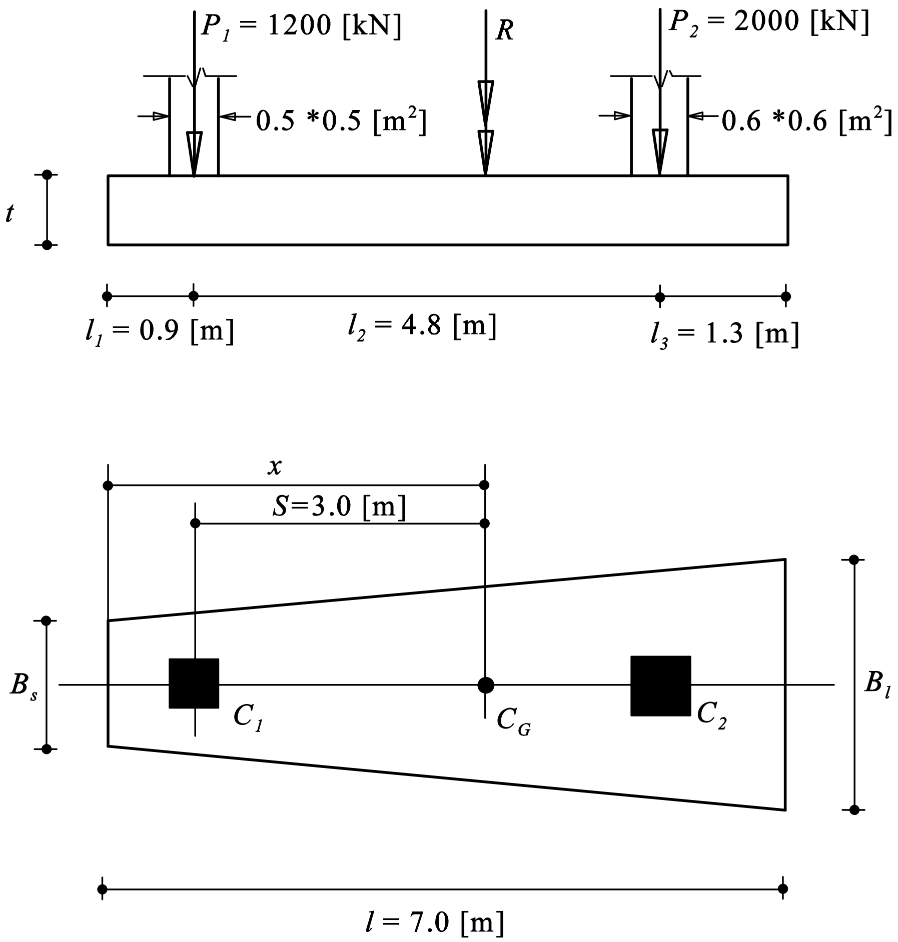


Figure (80) Combined trapezoidal footing

Material properties

Concrete grade according to ECP	*C 250*			
Steel grade according to ECP	*S 36/52*			
Compressive stress of concrete	$f_c = 95$	[kg/cm^2]	= 9.5	[MN/m^2]
Tensile stress of steel	$f_s = 2000$	[kg/cm^2]	= 200	[MN/m^2]
Young's modulus of concrete	$E_b = 3*10^7$	[kN/m^2]	= 30000	[MN/m^2]
Poisson's ratio of concrete	$\nu_b = 0.20$	[1]		
Unit weight of concrete	$\gamma_b = 0.0$	[kN/m^3]		

Unit weight of concrete is chosen $\gamma_b = 0.0$ to neglect the self weight of the footing.

3 Analysis of the footing

3.1 Determination of footing sides B_s and B_l

The primary design required to establish the area of footing so that the center of area of the footing coincides with the center of gravity of the resultant. This will be conducted as follows:

Resultant of loads R is given by:

$$R = P_1 + P_2 = 1200 + 2000 = 3200 \text{ [kN]}$$

Area of footing A_f is obtained from:

$$A_f = \frac{R}{q_{(all)_{net}}} = \frac{3200}{240} = 13.33 \text{ [m}^2\text{]}$$

Referring to Figure (80), area of footing A_f is given by:

$$A_f = \frac{l}{2}\,(B_s + B_l)$$

$$13.33 = \frac{7.0}{2}\,(B_s + B_l)$$

Simplifying,

$$B_s + B_l = 3.81 \qquad \text{(i)}$$

Taking the moment of the column loads about the center of the column *C1*, the distance S between the point of application of the resultant and the center of column *C1* is obtained from:

$$S \times R = P_2 \times l_2$$

$$S \times 3200 = 2000 \times 4.8$$

$$S = 3.0 \text{ [m]}$$

Hence, the point of application of the resultant is also the centroid of the footing area. Therefore, it can be shown from the geometry of the footing that the distance x from the small side B_s to the center of area is given by:

$$x = \frac{l}{3}\,\frac{B_s + 2B_l}{B_s + B_l}$$

$$l_1 + S = \frac{l}{3}\,\frac{B_s + 2B_l}{B_s + B_l}$$

$$0.9 + 3.0 = \frac{7.0}{3}\,\frac{B_s + 2B_l}{B_s + B_l}$$

Simplifying,

$$2.04\ B_s - B_l = 0.0 \qquad \text{(ii)}$$

Solving Equation (i) and (ii) yields the required dimensions of B_s and B_l as follows:

$$B_s = 1.25\ [\text{m}] \ \text{and}\ B_l = 2.56\ [\text{m}]$$

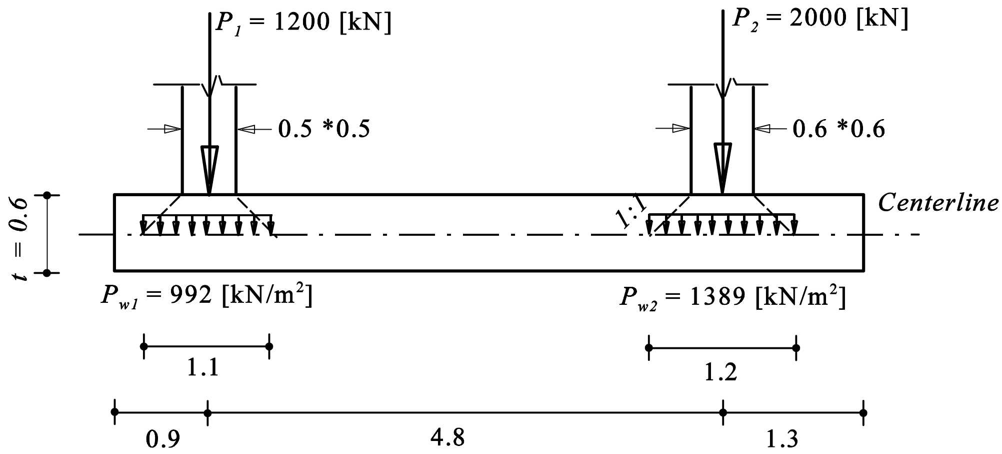
Chosen dimensions of B_s and B_l are:

$$B_s = 1.30\ [\text{m}] \ \text{and}\ B_l = 2.60\ [\text{m}]$$

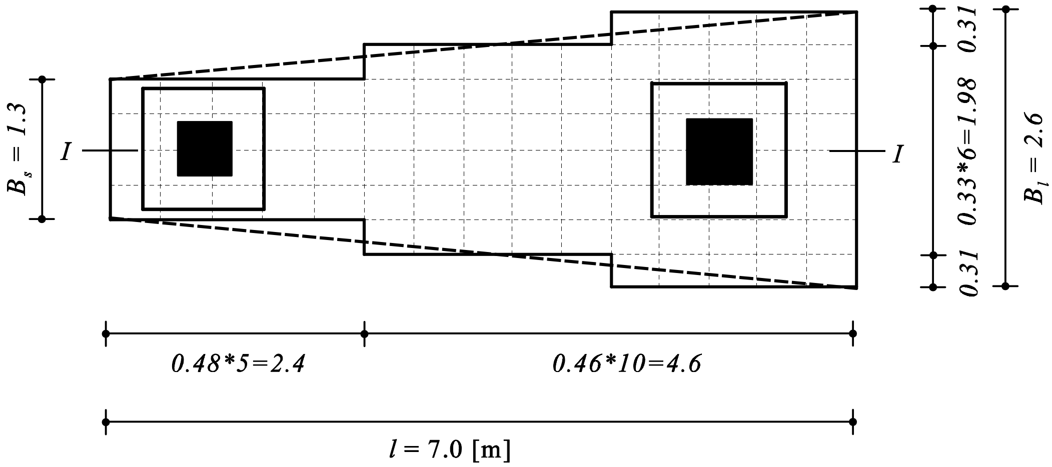
3.2 Finite element analysis

As the type of finite elements used to analysis of the footing is a rectangular element, the total area of the elements from the FE-Net must be chosen nearly as equal as to the actual area of the footing. Hear in this example the footing is subdivided into 75 rectangular elements as shown in Figure (81). It is found that the total area from FE-Net is 13.70 [m^2] while the actual area of the footing is 13.65 [m^2] with difference of 0.37 [%].

If a point load represents the column load on the mesh of fine finite elements, the moment under the column will be higher than the real moment. Therefore, the column load is distributed at the centerline of the footing on an area of $(a+d)^2$ as shown in Figure (81). Figure (82) shows the calculated contact pressure q [kN/m^2] while Figure (83) shows the moment m_x [kN.m/m] at the critical section I-I of the footing. Figure (84) shows the distribution of the moment m_y [kN.m/m] in the plan. For ECP codes, the footing is designed to resist the bending moment and punching shear. Then, the required reinforcement is obtained.

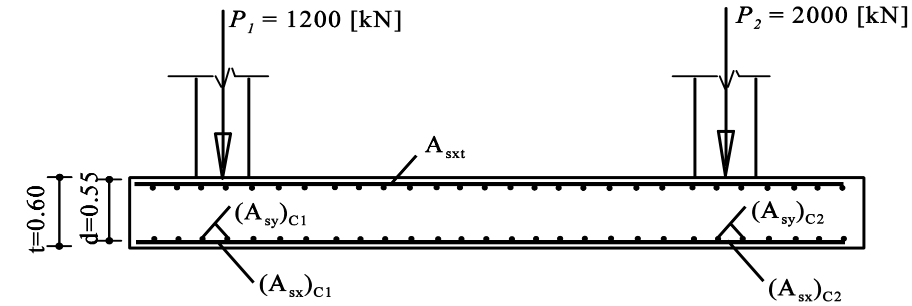


a) Section I-I



b) Plan

Figure (81) FE-Net and distribution of column loads through the footing



Section I-I

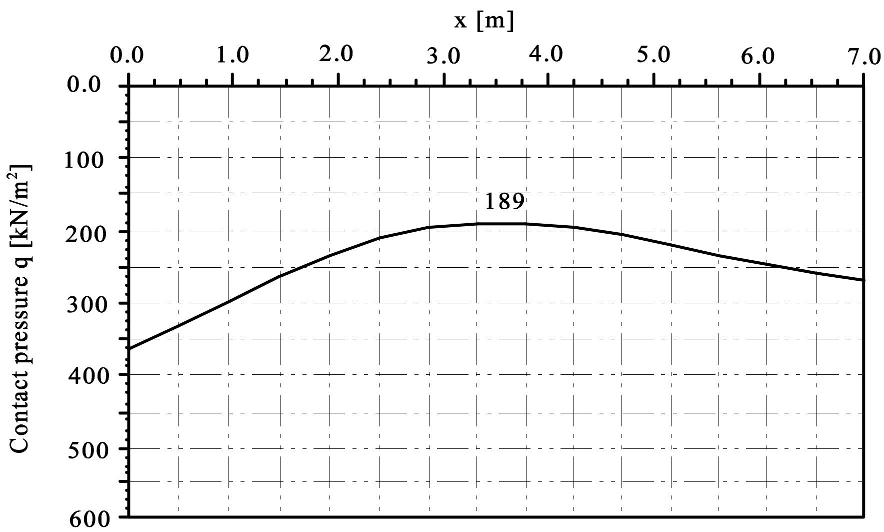


Figure (82) Contact pressure q [kN/m^2] at section I-I

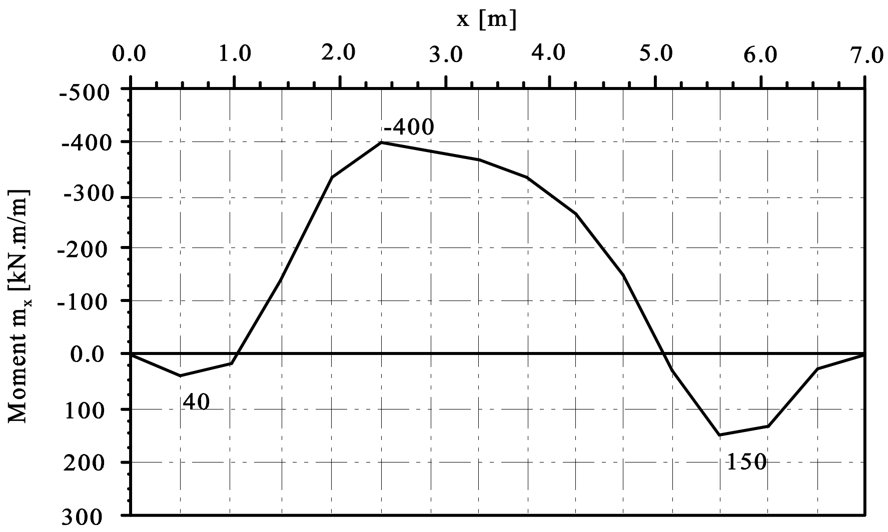


Figure (83) Moment m_x [kN.m/m] at section I-I

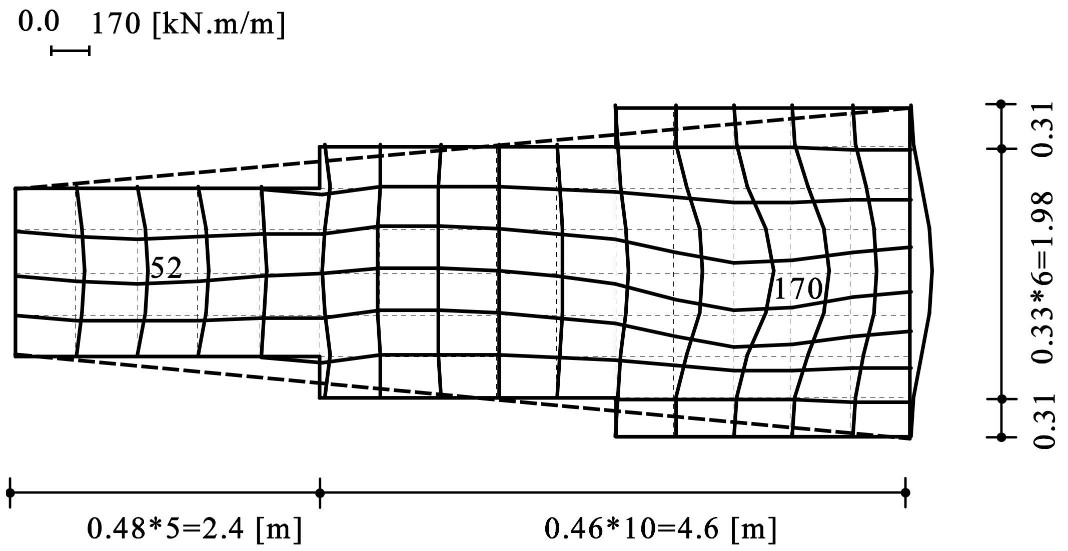


Figure (84) Distribution of the moment m_y [kN.m/m] in the plan

4 Design for ECP (working stress method)

Material

Concrete grade C 250
Steel grade S 36/52
Compressive stress of concrete $f_c = 95$ [kg/cm^2] = 9.5 [MN/m^2]
Tensile stress of steel $f_s = 2000$ [kg/cm^2] = 200 [MN/m^2]

Maximum moment

Maximum moment per meter at critical section obtained from analysis M =400 [kN.m] = 0.4 [MN.m]

Geometry

Effective depth of the section $d = 0.55$ [m]
Width of the section to be designed $b = 1.0$ [m]

Determination of depth required to resist moment d_m

From Table (68) for $f_c = 9.5$ [MN/m^2] and $f_s = 200$ [MN/m^2], the coefficient k_1 to obtain the section depth at balanced condition is $k_1 = 0.766$, while the coefficient k_2 [MN/m^2] to obtain the tensile reinforcement for singly reinforced section is $k_2 = 172$ [MN/m^2]

The maximum depth d_m as a singly reinforced section is given by:

$$d_m = k_1 \sqrt{\frac{M}{b}}$$

$$d_m = 0.766 \sqrt{\frac{0.4}{1.0}} = 0.48 \text{ [m]}$$

Take $d = 0.55$ [m] > $d_m = 0.48$ [m], then the section is designed as singly reinforced section.

Check for punching shear

The critical punching shear section on a perimeter at a distance $d/2 = 0.275$ [m] from the face of the column as shown in Figure (85). The check for punching shear under columns *C1* and *C2* is shown in Table (68).

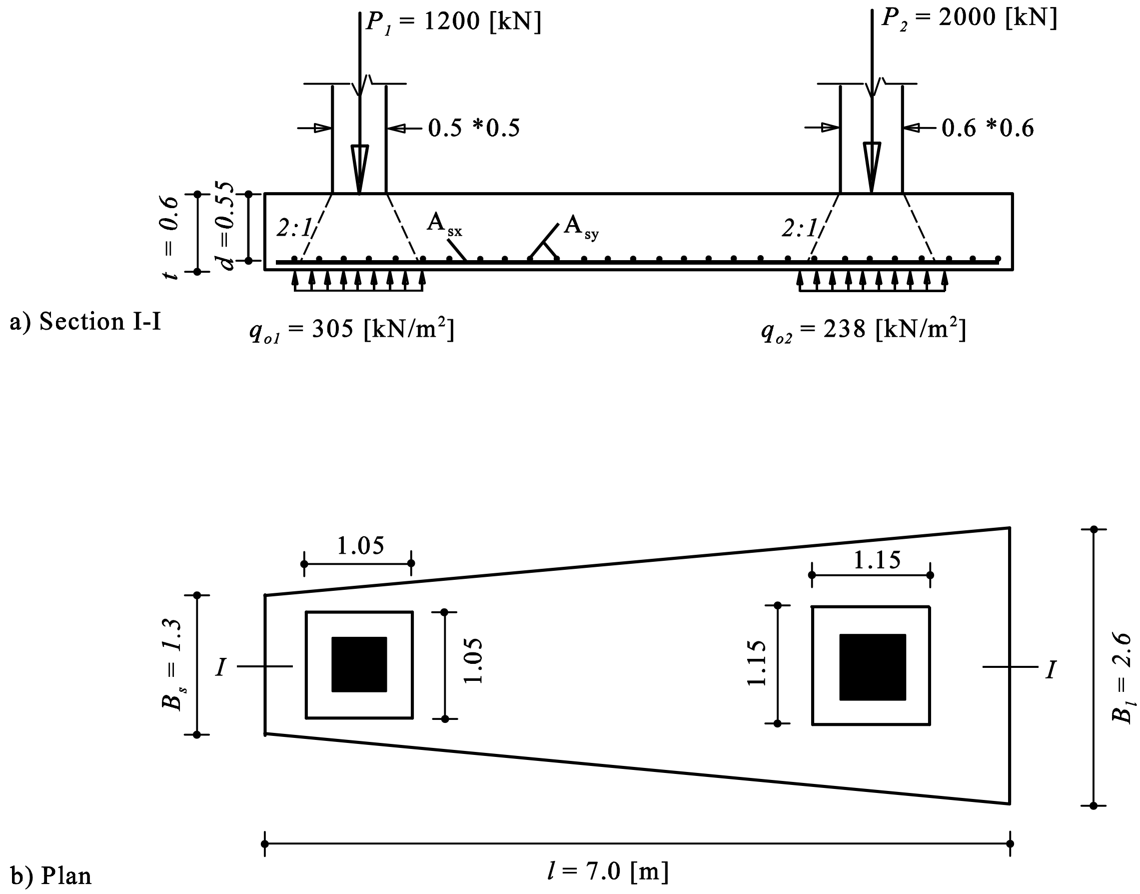


Figure (85) Critical section for punching shear according to ECP

Table (68) Check for punching shear

Load, stress and geometry	Column *C1*	Column *C1*
Column load P [MN]	1.2	2.0
Contact pressure q_o [MN/m^2]	0.305	0.238
Column sides $a*b$ [m^2]	0.5*0.5	0.6*0.6
Footing thickness d [m]	0.55	0.55
Critical perimeter $b_o = 4\ (a+b)$ [m]	4.2	4.6
Critical area $A_p = (a+d)^2$ [m^2]	1.1025	1.3225
Punching load $Q_p = P - q_o.A_p$ [MN]	0.86	1.69
Punching shear stress $q_p = Q_p/(b_o.d)$ [MN/m^2]	0.37	0.67

The allowable concrete punching strength q_{pall} [MN/m^2] is given by

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp}$$

$$q_{pall} = (0.5 + 1)\ 0.9 \leq 0.9$$

$$q_{pall} = 0.9\ [MN/m^2]$$

For both columns $q_{pall} > q_p$, the footing section is safe for punching shear.

Determination of tension reinforcement

Minimum area of steel reinforcement $A_{s.min}$ = 0.15 % A_c =0.0015 *60*100 = 9 [cm^2/m]
Take $A_{s.min}$ = 5Φ 16/m = 10.1 [cm^2/m]

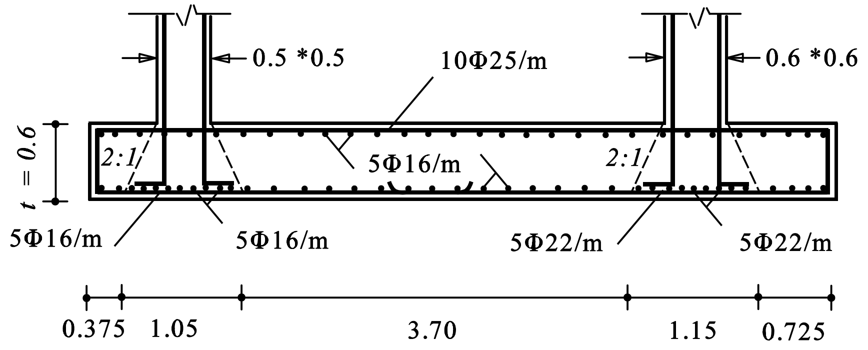
The determination of the required area of steel reinforcement in both x- and y-directions is shown in Tables (69) and (70). The details of reinforcement in plan and section a-a through the footing are shown in Figure (86).

Table (69) Determination of tension reinforcement for x-direction

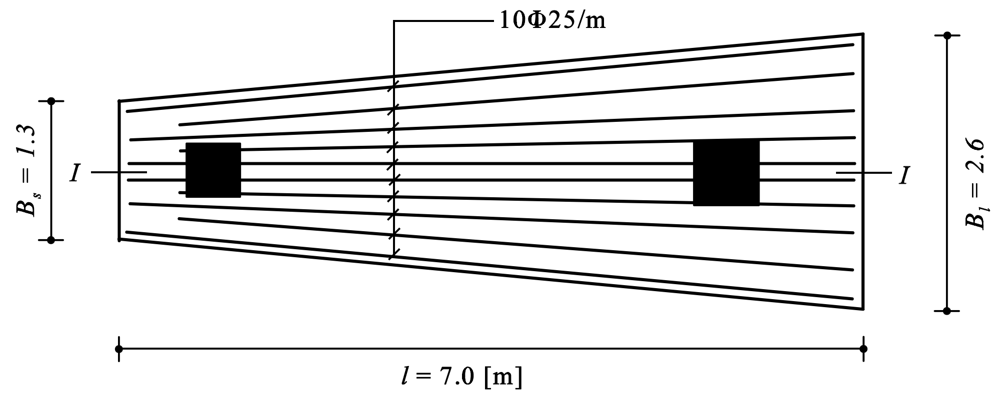
Position	Moment M [MN.m/m]	Calculated A_s $A_s = M/(k_2.d)$ [cm^2/m]	Chosen reinforcement A_s
A_{sxt}	0.4	42.28	10Φ25/m
$(A_{sxb})_{C1}$	0.15	15.86	5Φ22/m
$(A_{sxb})_{C2}$	0.04	4.23	5Φ16/m = $A_{s.min}$

Table (70) Determination of tension reinforcement for y-direction

Position	Moment M [MN.m/m]	Calculated A_s $A_s = M/(k_2.d)$ [cm^2/m]	Chosen reinforcement A_s
$(A_{syb})_{C1}$	0.052	5.50	5Φ16/m = $A_{s.min}$
$(A_{syb})_{C2}$	0.17	17.97	5Φ22/m



a) Section I-I



b) Plan

Figure (86) Details of reinforcement in plan and section a-a through the footing

Example (7): Design of a group of footings with and without tie beams

1 Description of the problem

This example shows the analysis and design of a group of footings resting on an elastic foundation by two different structural systems. In the first one, the group of footings has no connections while in the second one, the group of footings is connected together by stiff tie beams considering the interaction effect among footings, tie beams and the subsoil as one unit. Finally, a comparison is carried out between the two structural systems.

It is obviously that, if there is no accurate method to determine the stress due to the interaction between the footings and tie beams, the purpose of the presence of the tie beams in this case will be only carrying the walls of the ground floor. Where it is impossible to construct the walls directly on the soil. In the other case, the presence of the tie beams is unnecessary when walls for the ground floor are not required. It is impossible in any way to depend on the tie beams for reducing the deferential settlements for footing or footing rotations without perfect knowledge about the extent of their effect in the structural analysis accurately.

The program ELPLA has the possibility to composite two types of finite elements in the same net. In which, the footings are represented by plate elements while the tie beams are represented by beam elements. Thus, footings and tie beams can be analyzed correctly.

Figure (87) shows a layout of columns for a multi-storey building. The columns are designed to carry five floors. The dimensions of columns, reinforcement and column loads are shown in the same Figure (87).

It is required to design the building footings considering property lines at the west and south sides of the building (a neighbor building). The design must be carried out twice. In the first one, the footings are designed as isolated footings without connection among them, while in the second, the footings are designed as connected footings with tie beams to reduce the differential settlements among them and footing rotations.

2 Soil properties

The soil under the foundation level till the end of the boring up to 10 [m] consists of homogeneous middle sand, which has the following parameters:

Allowable net bearing capacity of the soil	$(q_{net})_{all}$	= 200	[kN/m^2]
Modulus of subgrade reaction	k_s	= 40 000	[kN/m^3]
The proposal foundation level	d_f	= 1.5	[m]

The level of the groundwater is G_w = 3.0 [m] under the ground surface. The groundwater effect is neglected in the analysis of footings, because the groundwater level is lower than the foundation level,

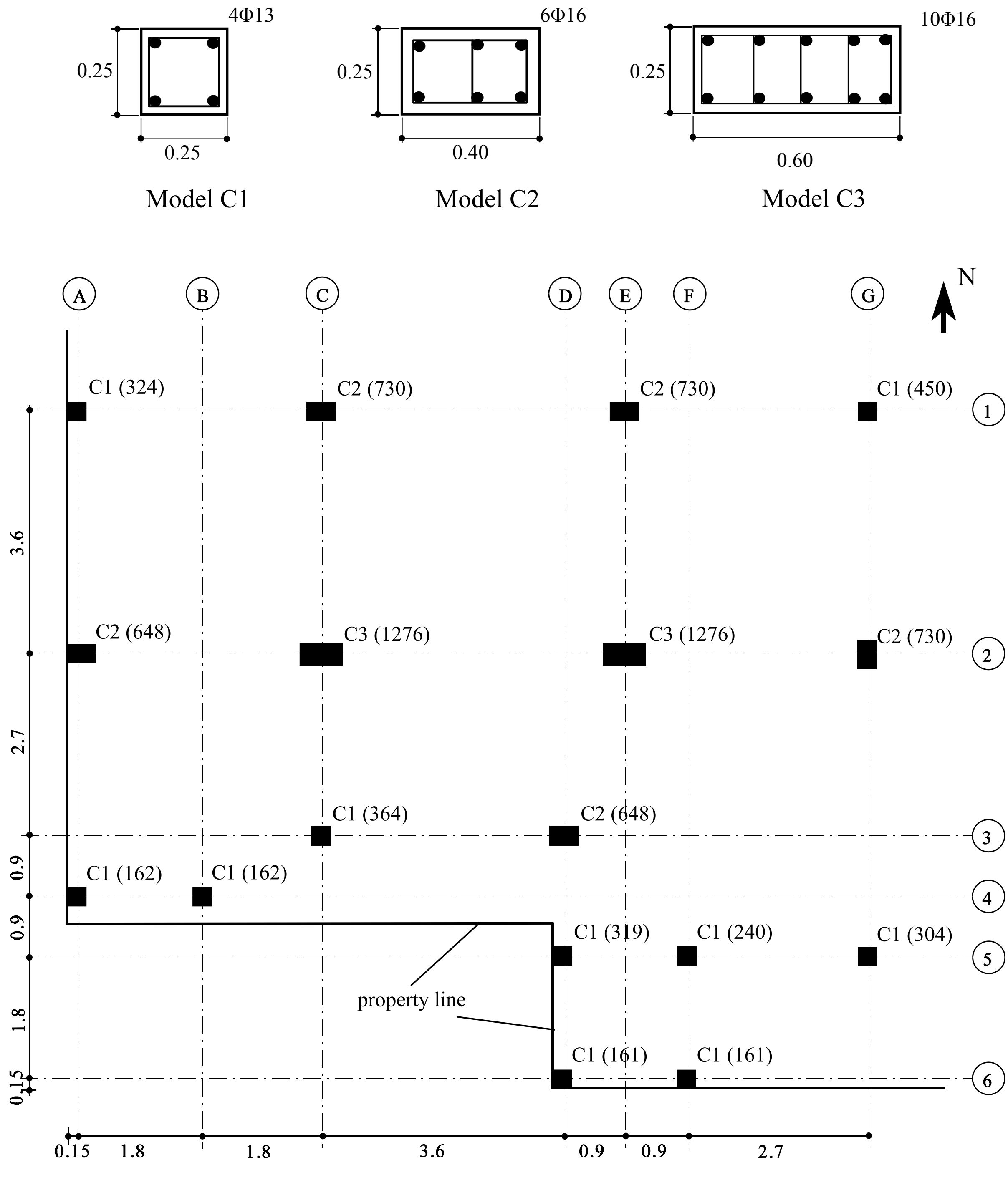


Figure (87) Layout of columns, column loads [kN] and models

3 Choice of the mathematical soil model

The system of the suggested foundation for the building is a group of footings, which have limited areas. This system of foundation doesn't require a complicated soil model, because the contact pressure between the soil and the footing in most cases will be nearly uniform. Especially, if the footing area is chosen carefully so that the center of gravity of the footing area lies on the point of application of the resultant force. In this case, the choice of the simple assumption model that considers a linear contact pressure under the footing is acceptable. But the disadvantage of this model is the neglect of the interaction between the footing and the soil. In this example, a mathematical soil model, which is more accurate than the simple assumption model, is used. The mathematical model is Winkler's model, which represents the soil by a group of elastic springs.

4 Footing material and section properties

The design of footings and tie beams is carried out by the working method according to ECP. The footing material and section are supposed to have the following parameters:

Material properties

Concrete grade according to ECP	C 250			
Steel grade according to ECP	S 36/52			
Concert cube strength	f_{cu} = 250	[kg/cm^2]	= 25	[MN/m^2]
Concert cylinder strength	f'_c = 0.8 f_{cu}		= 20	[MN/m^2]
Compressive stress of concrete	f_c = 95	[kg/cm^2]	= 9.5	[MN/m^2]
The main value of shear strength	q_{cp} = 9	[kg/cm^2]	= 0.9	[MN/m^2]
Allowable shear stress of concrete	q_c = 9	[kg/cm^2]	= 0.9	[MN/m^2]
Allowable bond stress	q_b = 12	[kg/cm^2]	= 1.2	[MN/m^2]
Tensile stress of steel	f_s = 2000	[kg/cm^2]	= 200	[MN/m^2]
Reinforcement yield strength	f_y = 3600	[kg/cm^2]	= 360	[MN/m^2]
Young's modulus of concrete	E_b =3*10^7	[kN/m^2]	= 30000	[MN/m^2]
Shear modulus of concrete	G_b =1.3*10^7	[kN/m^2]	= 13000	[MN/m^2]
Poisson's ratio of concrete	ν_b = 0.15	[kN/m^3]		
Unit weight of concrete	γ_b = 0.0			

Unit weight of concrete is chosen γ_b = 0.0 to neglect the own weight of the footing.

The minimum section properties and reinforcement are:

Concrete cover + 1/2 bar diameter	c = 5	[cm]
Minimum steel bar diameter in footings and tie beams	Φ = 18	[mm]
Minimum number of steel bars	n = 5	[bars]
Minimum footing thickness	t = 0.3	[m]
Minimum footing length	l = 1.1	[m]

The allowable minimum area of steel in the footings and tie beams is 0.15% from the concrete section (min A_s = 0.0015 A_c).

5 Plain concrete properties

The reinforcement concrete cannot be constructed directly on the ground. Therefore, a thin plain concrete of thickness 15 [cm] under the footings and tie beams is used. The plain concrete is not considered in any calculation because of its weakness.

6 Structural analysis and design

6.1 Footing areas

The area of each footing is determined so that the contact pressure between the footing and the soil does not exceed than the net allowable capacity of the soil $(q_{net})_{all}$ = 200 [kN/m^2]. To avoid the footing rotation, isolated footings are chosen to be support for interior columns while combined footings are chosen to be support for exterior columns. It must be considered that the point of application of the force P for the isolated footing or the resultant forces ΣP for the combined footing lies as far as possible on the center of gravity of the footing. Then, the footing area A_f is determined from $A_f = \Sigma P /(q_{net})_{all}$. It must be considered also that the contact pressure is uniform for all footings and nearly is the same. Table (71) shows the load on the footing P, footing area A_f and net contact pressure f_n between the footing and the soil.

6.2 Dimensions of tie beams

Tie beams are chosen so that their axes coincide with those of columns to avoid the torsion. The width of the tie beam is chosen to be not longer than the smallest column side, d_g = 0.30 [m], while the depth of the tie beam is chosen to be at least double of its width to make it stiff enough, d_g = 0.6 [m]. Tie beams for all footings have a constant rectangular section of 0.3 [m] * 0.6 [m]. It is considered that footings and tie beams are resting on the soil and there is no looseness of the contact pressure between them and the soil.

Table (71) Load ΣP, footing areas A_f and the net contact pressures f_n

Footing	Total load on footing ΣP [kN]	Footing area A_f [m^2]	Net contact Pressure f_n [kN/m^2]
F1	1054	5.0*1.1	192
F2	730	2.0*2.0	183
F3	450	1.5*1.5	200
F4	1924	5.0*2.0	192
F5	1276	2.6*2.6	189
F6	730	2.0*2.0	183
F7	364	1.4*1.4	186
F8	648	1.8*1.8	200
F9	324	2.1*0.8	193
F10	881	2.1*2.1	200
F11	304	1.3*1.3	180

6.3 Thickness of footings

The footing thickness and reinforcement are designed according to the Egyptian code of practice ECP, working stress method. In this case, the reinforcement concrete section must resist the working stress that acting on it safely such as the shear stress, punching stress, bond stress and bending moment. It is expected that, the stresses of shear, punching and bond for the footings connected with tie beams are strong enough to resist the permissible stresses. Consequently, there is no requirement to check on these stresses and it is sufficient only to check on the bending moment to determine the thickness of the footings, tie beams and reinforcement.

The thickness of the footing in this example is chosen to fulfill the safety conditions at the analysis of the footing whether they are connected with or without tie beams excepting the reinforcement, which is chosen for every structural system separately.

The first step in the design is determination of the primary footing thickness from the depth d_p that resists the punching stress. This depth is mostly the critical depth for the isolated footing. The depth to resist punching shear d_p [m] is given by:

$$d_p = \frac{Q_p}{b_o \; q_{pall}} \qquad \text{(i)}$$

Where:

b_o Perimeter of critical punching shear section around the column considering the position of the column wherever the column at the edge, corner or inside [m].

Q_p Punching force [kN], $Q_p = P_{col} - A_p \cdot f_n$

A_p Punching area [m^2], for simplicity $A_p = A_{col}$

A_{col} Cross section of the column [m^2].

P_{col} Column load [kN].

f_n Average contact pressure between the footing und the soil under the column [kN/m^2].

q_{pall} Allowable concrete punching strength [MN/m^2].

The allowable concrete punching strength q_{pall} [MN/m^2] is given by:

$$q_{pall} = \left(0.5 + \frac{a}{b}\right) q_{cp} \leq q_{cp} \qquad \text{(ii)}$$

where:

q_{cp} The main value of shear strength [MN/m^2], $q_{cp} = 0.9$ [MN/m^2]

b, a Column sides [m].

The allowable concrete punching strength for the columns those have the greatest cross section (a * b = 0.25 * 0.6 [m^2]) will be $q_{pall} = 0.825$ [MN/m^2] while for the other columns will be q_{pall} = 0.9 [MN/m^2].

Substituting the value of allowable punching shear strength q_{pall} in Equation (i) leads to an equation of second order in the unknown d_p. Solving this equation, gives the depth d_p that is required to resist the punching shear as shown in Table (72). This depth, addition to the concrete cover for the nearest 10 cm, is chosen as a primary data for the footing thickness, considering that the minimum footing thickness is 30 [cm]. After carrying out the analysis, this depth may be modified when necessary to fulfill the condition of safety against the remaining shear, bond, bending moment stresses.

Table (72) Determination of the footing depth d_d to resist the punching shear

Footing	Load P [kN]	Net contact pressure f_n [kN/m^2]	Column section $A_{col} = a*b$ [m^2]	Punching load Q_p=P-f_n*A_{col} [kN]	Punching depth d_p [m]	Chosen depth d_d [m]
F1	730	192	0.25*0.40	711	0.31	0.35
F2	730	183	0.25*0.40	712	0.31	0.35
F3	450	200	0.25*0.25	437	0.25	0.25
F4	1276	192	0.25*0.60	1247	0.44	0.45
F5	1276	189	0.25*0.60	1248	0.44	0.45
F6	730	183	0.25*0.40	712	0.31	0.35
F7	364	186	0.25*0.25	352	0.21	0.25
F8	648	200	0.25*0.40	628	0.29	0.35
F9	162	193	0.25*0.25	150	0.16	0.25
F10	319	200	0.25*0.25	307	0.39	0.45
F11	304	180	0.25*0.25	293	0.19	0.25

Generation of the finite elements-net

In regard to the narrowness of the distance between some axes and design dimensions of the footings, columns and tie beams, a refined net of finite elements is used. It is become necessary to consider the following notes when generating the finite elements-net:

- Generate a homogenous mesh over the whole foundation area as possible as you could.

- Element size is chosen to be equal the foundation thickness if possible.

- Switching from a small element to a large one must be done gradually so that the difference between the side of the element and that of its neighboring element is not larger than the double in both directions.

- The net of the finite elements is generated for the entire area firstly, and then the unnecessary elements are removed to define the foundation shape. The footings are represented by plate elements while the tie beams are represented by beam elements. It is not allowed to leave a beam element separately without connection with a plate element because the mean element used in the program ELPLA is the plate element.

- In the program ELPLA, loads may be applied to the net of the finite elements outside nodes at any position independently from the element sizes.

- As the tie beam is represented by beam elements, the width of the plate element adjacent to the beam element is chosen to be half the width of the tie beam. Consequently, the soil effect on the area around beam nodes will be equivalent to that on the actual contact area of the tie beam.

- In spite of the plate elements must join with beam elements in free places among footings, but it is easy to cancel its effect quietly. This can be done by assuming that the modulus of elasticity or the thickness of the plate elements equal to zero, then the program will cancel their effect automatically.

- Beam elements may be placed in *x*- or *y*-direction on the net connected to plate elements at their nodes to represent tie beams in x- or y-direction. Diagonal tie beams are represented by diagonal beam elements. Each diagonal beam element may be paced on the nearest two diagonal nodes.

- Using the advantage of generating all footings on one net, it is easy to take a combined section for a group of footings indicating the internal forces, settlements and contact pressures.

- It was possibly at the analysis of isolated footings without tie beams to generate an independent net for each footing, but it is preferred to generate one net for the whole foundation area considering all footings to save the effort for constructing another net at the analysis of a group of footings connected with tie beams.

6.5 Creation of loads on the net

Creation of loads on the net may be carried out by one of the following two ways:

- Considering the column load as a point load on the net to simplify the editing of the input data. Consequently, the critical positive moment under the column will be calculated at the column face. Due to the small size of the finite element, it is expected that the moment under the point load will be so heigh.

- Converting the concentrated column load *P* to an equivalent distributed load P_w acting on the centerline of the footing thickness with slope 1:1 such as:

$$p_w = \frac{p}{(a + t)\,(b + t)} \qquad \text{(iii)}$$

where:

a, *b* Column sides [m].
t Footing thickness [m].

In this case the critical positive moment and area of reinforcement steel must be determined under the column directly. The second way for creation the loads are considered in this example. Figure (88) shows the net of the finite elements for the isolated footings without tie beams while Figure (89) shows that of a group of footing connected with tie beams. Table (73) shows the conversion of the concentrated column load *P* to an equivalent distributed load P_w

Table (73) Conversion of the concentrated column load *P* to an equivalent distributed load P_w

Column	Load	Coordinate		Column section	Footing thickness	Distributed area	Distributed load
	P [KN]	x [m]	y [m]	$A_{col}=a*b$ [m^2]	t [m]	$A_w=(a+t)*(b+t)$ [m^2]	$P_w=P/A_w$ [kN/m^2]
A-1	324	0.125	10.05	0.25*0.25	0.40	0.45*0.65	1108
C-1	730	3.75	10.05	0.25*0.40	0.40	0.65*0.80	1404
E-1	730	8.25	10.05	0.25*0.40	0.40	0.65*0.80	1404
G-1	450	11.85	10.05	0.25*0.25	0.30	0.55*0.55	1488
A-2	648	0.20	6.45	0.25*0.40	0.50	0.65*0.75	1329
C-2	1276	3.75	6.45	0.25*0.60	0.50	0.75*1.10	1547
E-2	1276	8.25	6.45	0.25*0.60	0.50	0.75*1.10	1547
G-2	730	11.85	6.45	0.25*0.40	0.40	0.65*0.80	1404
C-3	364	3.75	3.75	0.25*0.25	0.30	0.55*0.55	1203
D-3	648	7.35	3.75	0.25*0.40	0.40	0.65*0.80	1246
A-4	162	0.125	2.85	0.25*0.25	0.30	0.40*0.55	736
B-4	162	1.95	2.85	0.25*0.25	0.30	0.40*0.55	736
D-5	319	7.35	1.95	0.25*0.25	0.50	0.50*0.50	1276
F-5	340	9.15	1.95	0.25*0.25	0.50	0.50*0.50	960
G-5	304	11.85	1.95	0.25*0.25	0.30	0.55*0.55	1005
D-6	161	7.35	0.125	0.25*0.25	0.50	0.50*0.50	644
F-6	161	9.15	0.125	0.25*0.25	0.50	0.50*0.50	644

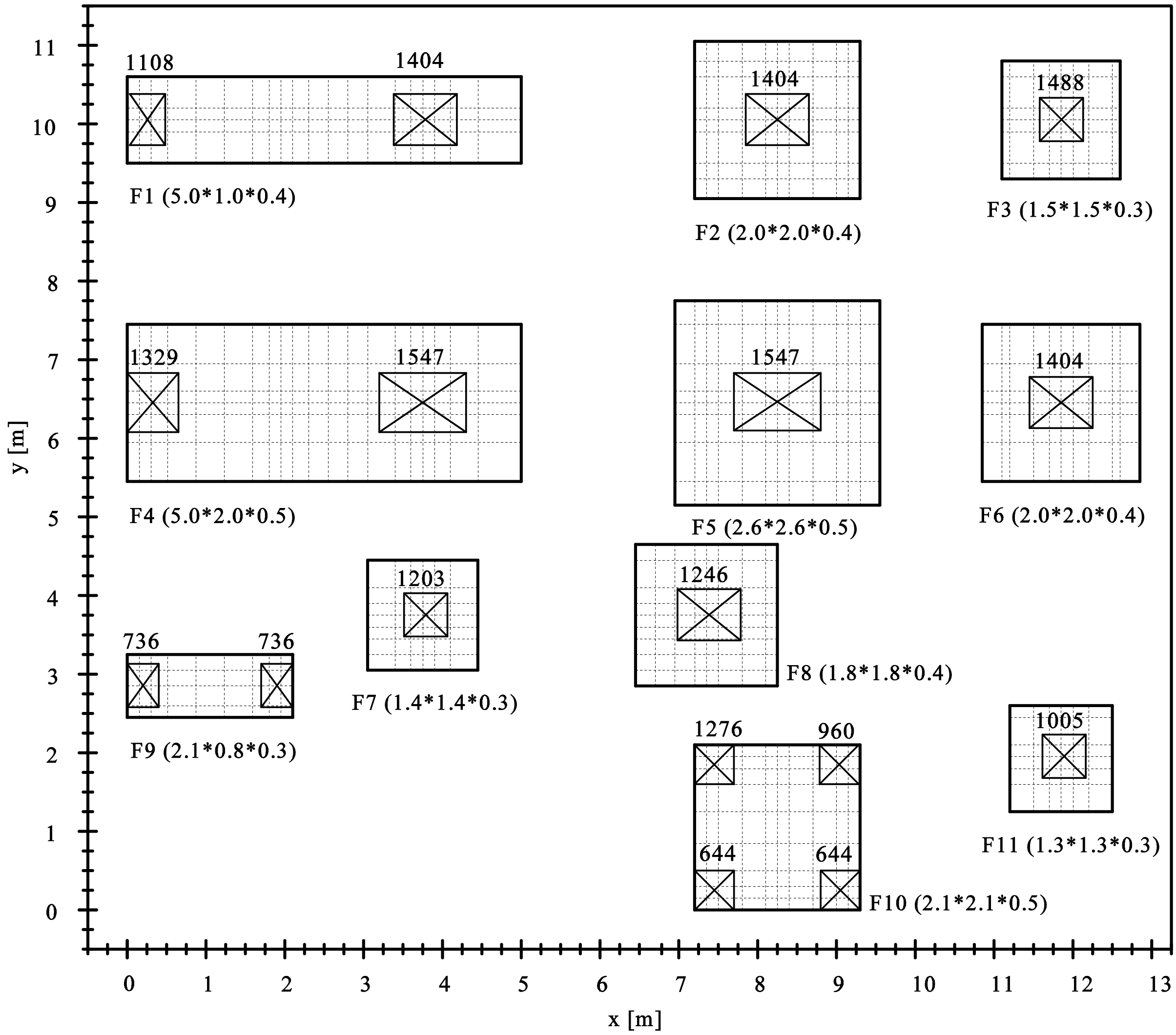


Figure (88) FE-Net of footings without tie beams, loads [kN/m^2] and footing dimensions [m]

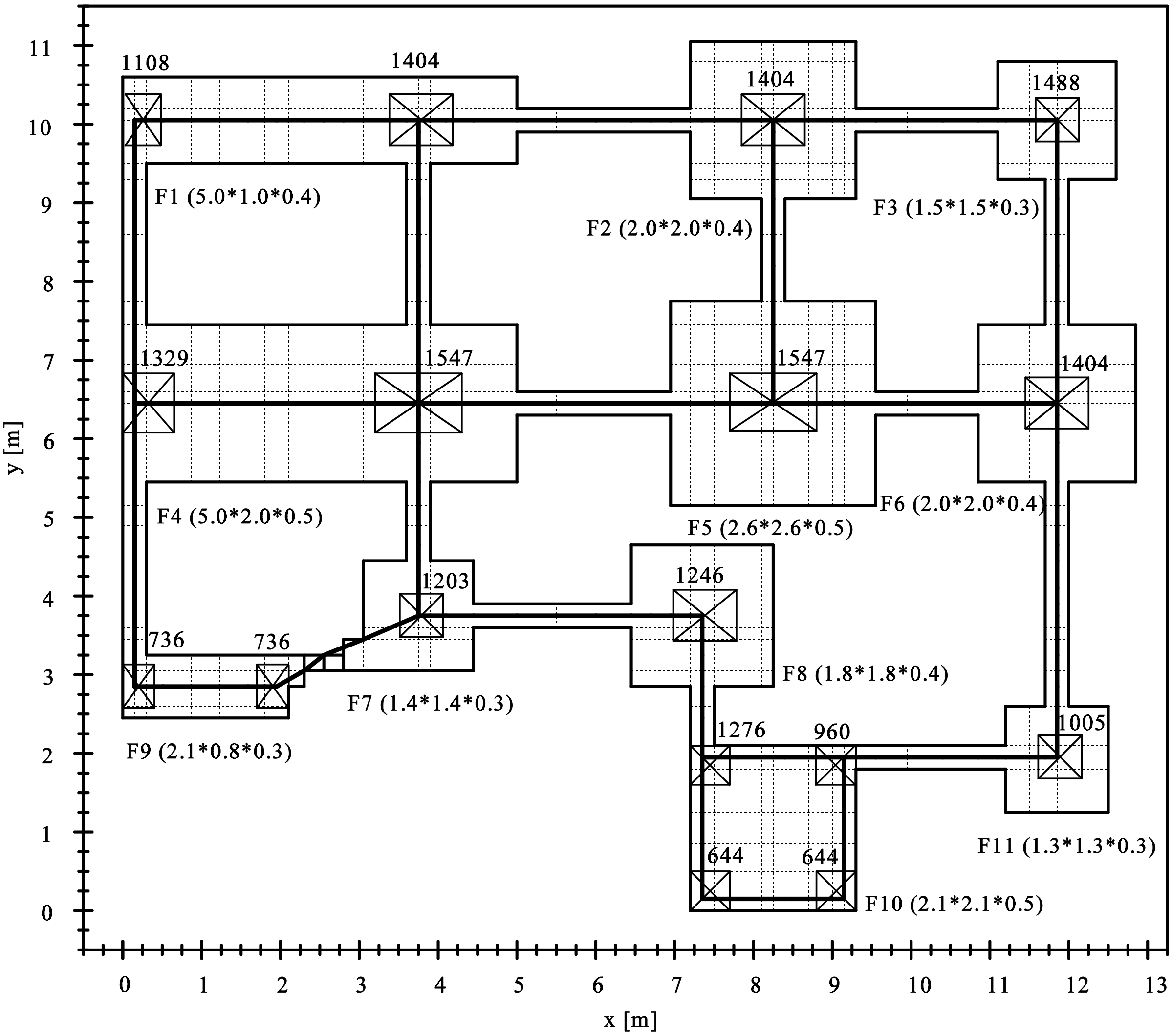


Figure (89) FE-Net of footings with tie beams, loads [kN/m^2] and footing dimensions [m]

6.6 Reinforcement steel for isolated footings

The area of reinforcement steel A_s is given by:

$$A_s = \frac{M}{k_2 \, d_m} \qquad \text{(iv)}$$

It is required firstly to check if the footing depth to resist punching shear is also sufficient to resist the bending moment at the critical section according to Equation (v).

$$d_m = k_1 \sqrt{\frac{M}{b}} \qquad \text{(v)}$$

where:

M	Moment at a section obtained from analysis [MN.m]
b	Width of the section to be designed [m], b = 1.0 [m]
d_m	Depth required to resist the moment [m]
k_1 and k_2	Coefficients according to ECP

The program ELPLA gives the results of the bending moments per meter in both directions x and y and also the values of areas of reinforcement steel at all nodes of the net of finite elements. Figure (90) shows bending moments m_x while Figure (91) shows bending moments m_y for the critical sections in directions x and y, respectively. Tables (74) and (75) shows check depth required to resist the bending moment and also the area of reinforcement steel that is required for the critical section in case of analysis of isolated footings.

Table (74) Check depth required to resist the bending moment and determination of the area of reinforcement steel in x-direction

Footing	Moment m_x [kN.m/m]		f_c [kg/cm²]	Required area of steel A_s [cm²/m]		Chosen steel [Rft/m]	
	-ve m_x	+ve m_x		A_{sx1} Top	A_{sx2} Bottom	A_{sx1} Top	A_{sx2} Bottom
F1	175	124	85	28.77	19.94	10Φ19	10 Φ 16
F2	-	82	55	-	12.98	-	7 Φ 16
F3	-	50	60	-	11.15	-	6 Φ 16
F4	181	112	65	22.55	13.64	8 Φ 19	8 Φ 16
F5	-	126	50	-	15.38	-	8 Φ 16
F6	-	76	50	-	11.93	-	6 Φ 16
F7	-	39	60	-	6.68	-	*min* A_s
F8	-	62	45	-	9.64	-	*min* A_s
F9	61	2	70	13.76	-	7 Φ 16	*min* A_s
F10	76	11	50	9.12	1.25	*min* A_s	*min* A_s
F11	-	29	50	-	6.37	-	*min* A_s

It must be considered the following notes when reinforcing the footings:

S Suitable reinforcement is to be placed at the places of maximum moments wherever in *x*- or *y*- direction. The reinforcement is chosen to be enough to resist the bending moment. It is not required to determine additional reinforcement to resist the punching shear where it is supposed that the concrete section can resist the punching stress without reinforcement.

S The top and bottom reinforcement in both *x*- and *y*-directions at the sections of minimum moments are empirically taken as 0.15% of the concrete cross section. The considered minimum area of reinforcement steel for all footings is $min\ A_s = 6\ \Phi\ 16 = 10.1\ [cm^2/m]$.

S For a combined footing for two columns, the calculated reinforcement under the column in the transversal direction is distributed under the column to a distance *d* from the face of the column.

S For an isolated footing for a column, it is enough to consider only the bottom reinforcement in both *x*- and *y*- directions.

Table (75) Check depth required to resist the bending moment and determination of the area of reinforcement steel in *y*-direction

Footing	Moment m_y [kN.m/m]		f_c [kg/cm^2]	Required area of steel A_s [cm^2/m]		Chosen steel [Rft/m]	
	-ve m_y	+ve m_y		A_{sy1} Top	A_{sy2} Bottom	A_{sy1} Top	A_{yx2} Bottom
F1	-	42	45	-	6.53	*min* A_s	*min* A_s
F2	-	88	55	-	13.97	-	7 Φ 16
F3	-	50	55	-	11.08	-	6 Φ 16
F4	-	117	50	-	14.30	*min* A_s	8 Φ 16
F5	-	153	55	-	18.83	-	10 Φ 16
F6	-	85	60	-	13.50	-	7 Φ 16
F7	-	38	50	-	8.36	-	*min* A_s
F8	-	72	50	-	11.30	-	6 Φ 16
F9	-	13	35	-	2.79	*min* A_s	*min* A_s
F10	65	10	30	7.70	1.14	*min* A_s	*min* A_s
F11	-	31	45	-	6.75	-	*min* A_s

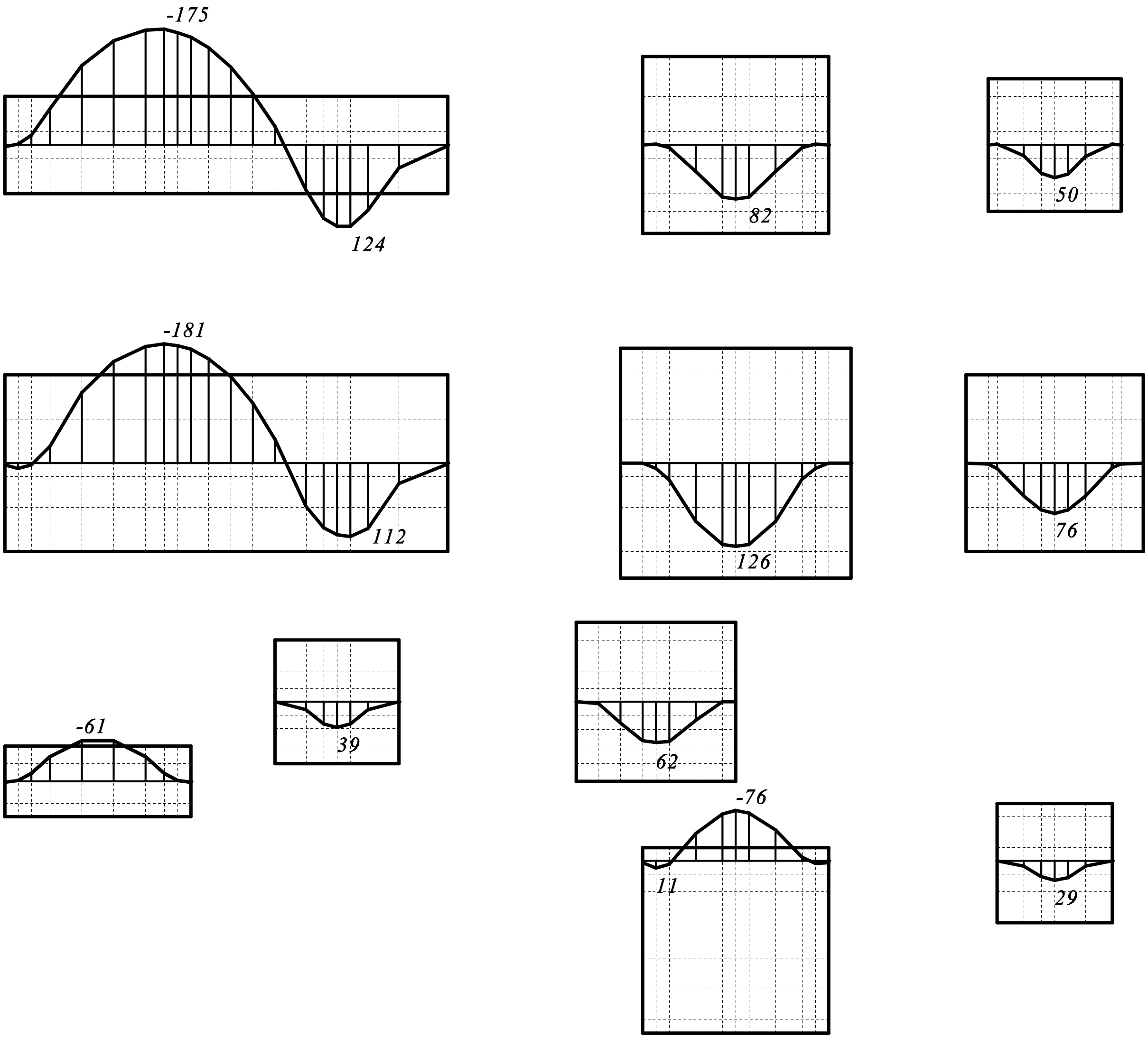


Figure (90) Moment m_x [kN.m/m] at critical sections on the footings

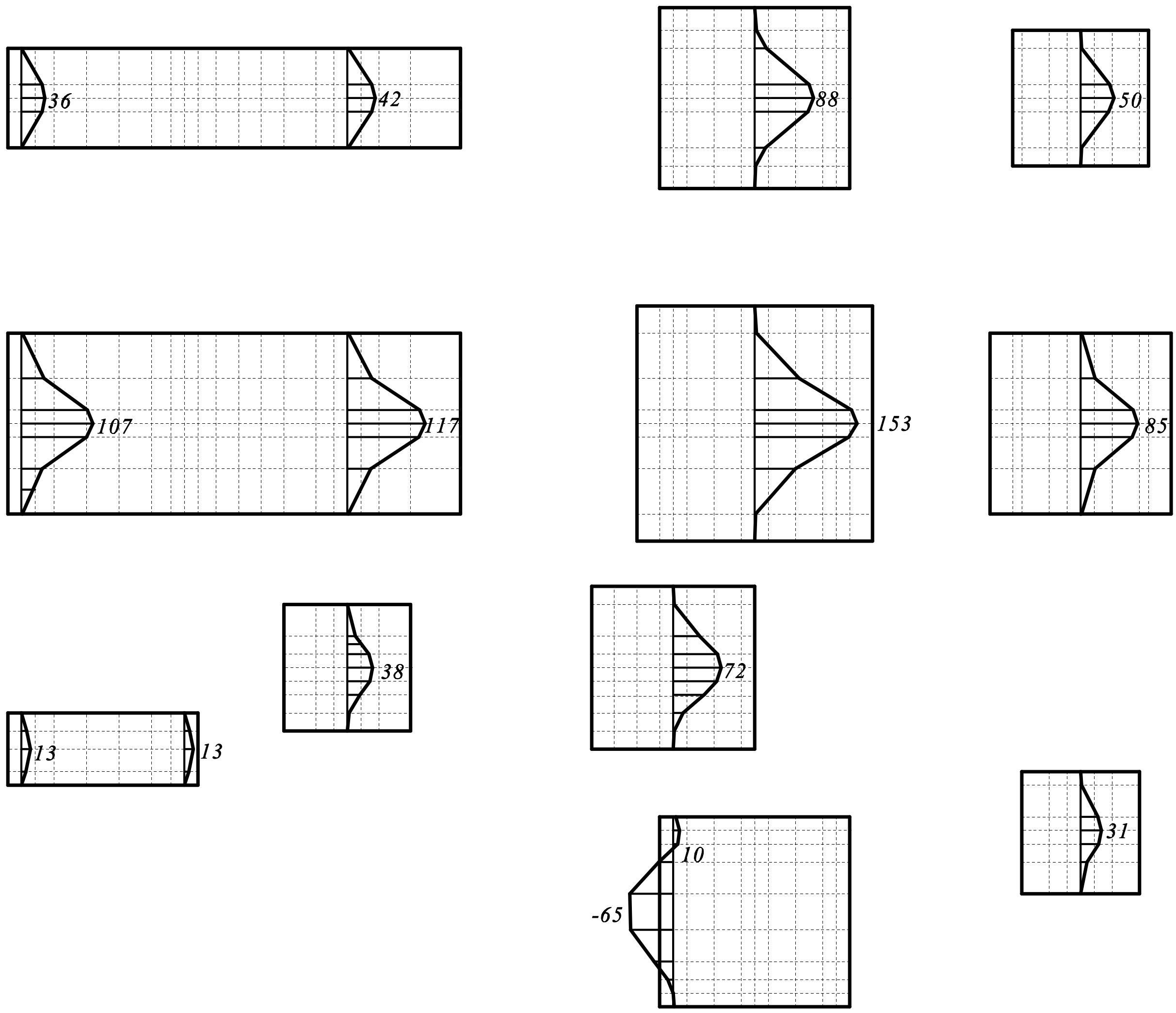


Figure (91) Moment m_y [kN.m/m] at critical sections on the footings

6.7 Check shear stress for isolated footings

It is required for isolated footings to check if the shear stress q_{sh} in the footing does not exceed than the allowable shear stress of concrete q_c = 0.9 [MN/m²]. The shear stress q_{sh} [MN/m²] is given by:

$$q_{sh} = \frac{Q_{sh}}{b\ d_{sh}} \qquad \text{(vi)}$$

where

- Q_{sh} Shearing force at critical section of shear. The program ELPLA gives Q_{sh} per meter at all nodes of the net in both *x*- and *y*- directions [MN/m].
- d_{sh} Depth required to resist shear stress [m]
- b Width of critical section of shear. b = 1.0 [m] where Q_{sh} is per meter.

Figure (92) shows the shearing force Q_{sh} in *x*-direction while Figure (93) shows that in *y*-direction at the critical sections. Table (76) shows check depth required to resist shear stress. The depths for all footings are save in shear stress.

Table (76) Check depth required to resist shear stress

Footing	Footing depth	*x*-direction		*y*-direction	
	d_{sh} [m]	Q_x [MN/m]	$q_{sh} = \frac{Q_x}{b\ d_{sh}}$ [MN/m²]	Q_y [MN/m]	$q_{sh} = \frac{Q_y}{b\ d_{sh}}$ [MN/m²]
F1	0.35	0.252	0.72	0.980	0.28
F2	0.35	0.157	0.45	0.148	0.42
F3	0.25	0.120	0.48	0.910	0.36
F4	0.45	0.308	0.68	0.170	0.38
F5	0.45	0.214	0.48	0.211	0.47
F6	0.35	0.158	0.45	0.129	0.37
F7	0.25	0.900	0.36	0.109	0.44
F8	0.35	0.137	0.39	0.160	0.46
F9	0.25	0.100	0.40	0.370	0.15
F10	0.45	0.133	0.30	0.135	0.30
F11	0.25	0.700	0.24	0.720	0.29

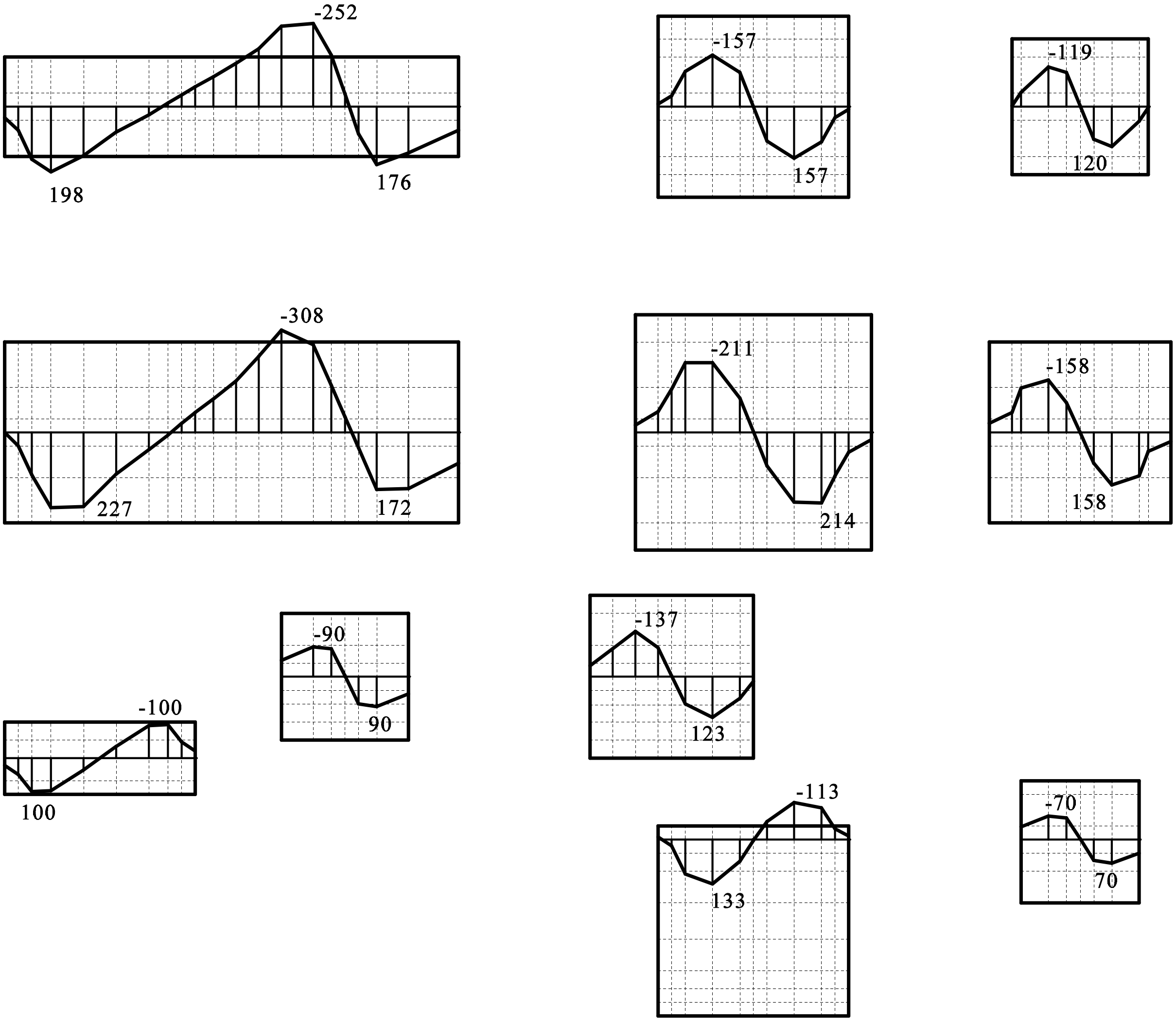


Figure (92) Shearing force Q_x [kN/m] at critical sections on the footings

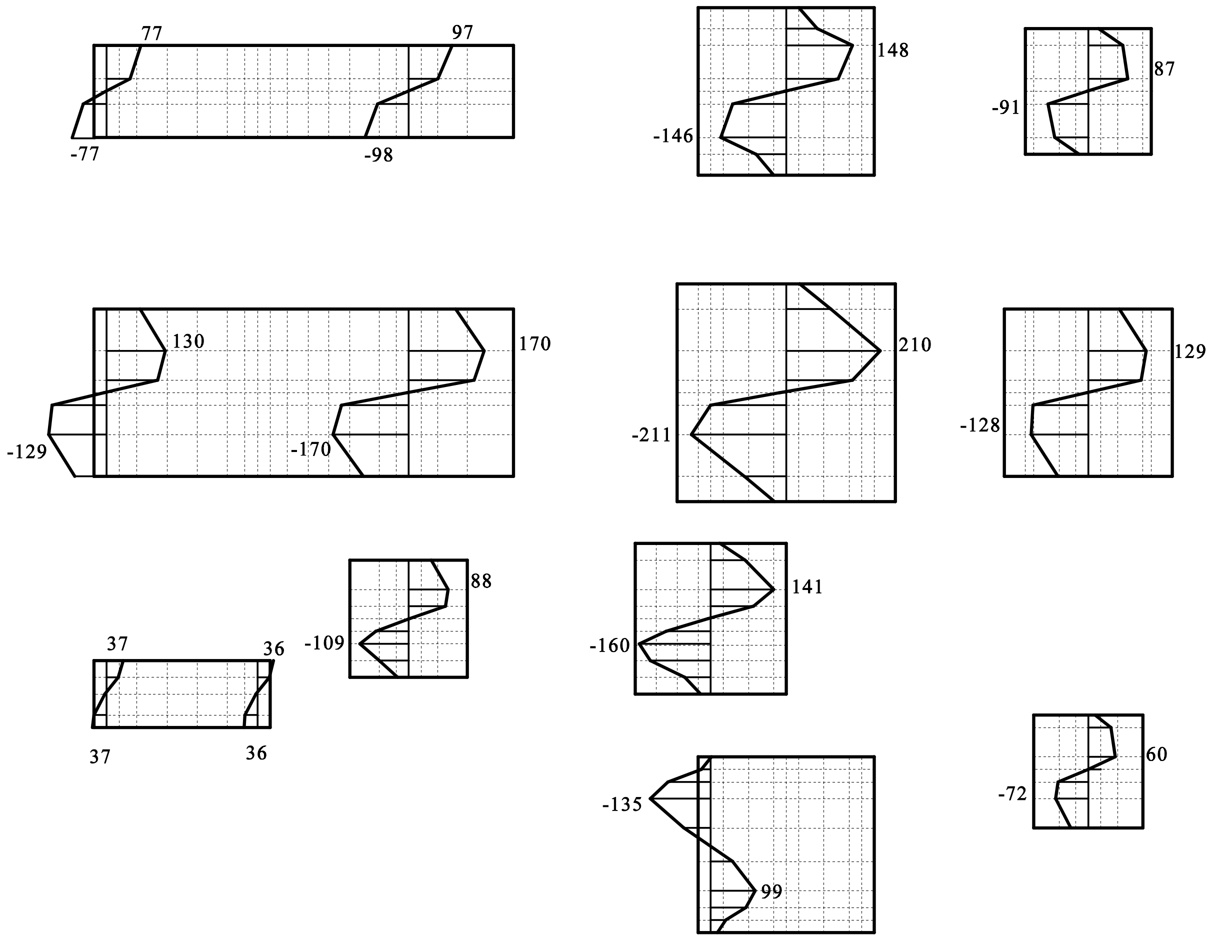


Figure (93) Shearing force Q_y [kN/m] at critical sections on the footings

6.8 Check bond stress for isolated footings

It is required also to check if the bond stress between the reinforcement steel and the concrete does not exceed than the allowable bond stress q_b = 1.2 [MN/m^2]. The bond stress q_{bo} [MN/m^2] is given by:

$$q_{bo} = \frac{Q_p}{0.87\ d\ \Sigma O} \qquad \text{(vii)}$$

where

Q_b Shearing force at section of maximum bending moment [MN].
Shearing force for an isolated footing of a column is Q_b = 0.25 (P_{col} - f_n . A_{col}). It is not necessary to check bond stress for the combined footing of two columns or more, because the critical bending moment in this case lies at the point of zero shear. Here, the zero shearing force is also the bond force. For simplicity, the bond forces Q_b in both *x*- and *y*-directions for all footings are considered equal where the difference in Q_b in both directions is small.

d_b Depth at that section [m]

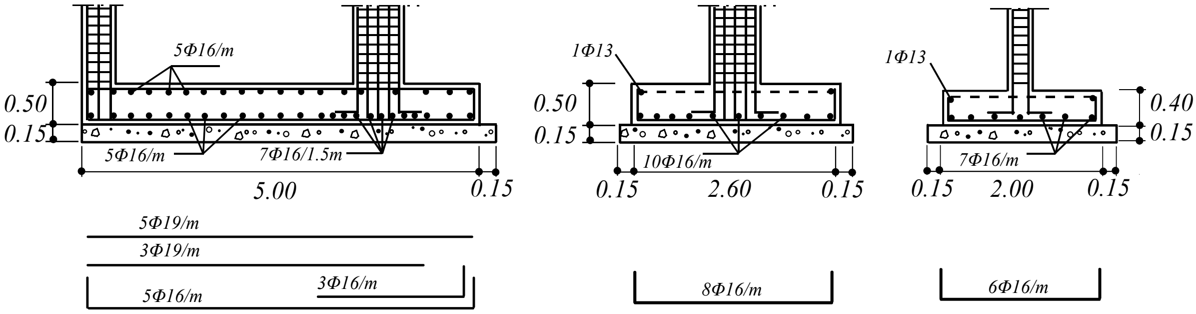
ΣO Sum of the perimeter of main reinforcement steel [m]

The allowable bond stress in this example is for steel bars teak L-shape at their ends. Table (77) shows the bond forces for the isolated footings for a column and also check bond stress. Bond stress for all footings lies within the permissible values.

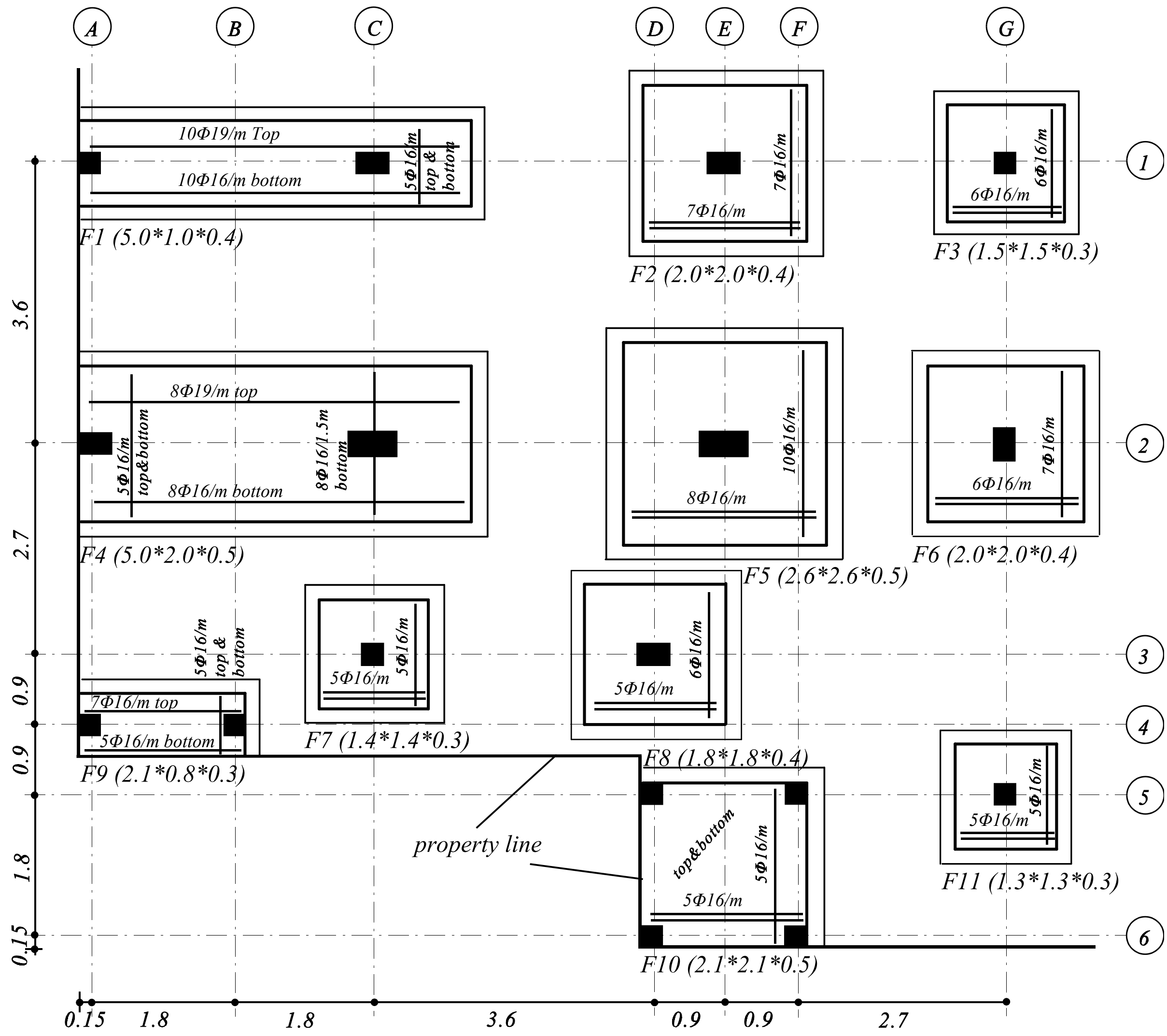
Table (77) Check bond stress for the isolated footings

Footing	Bond force Q_b [MN]	Rft A_{sy2} [/L]	Sum of perimeter of main Rft ΣO [m]	Footing depth d_b [m]	Bond stress q_{bo} [MN/m^2]
F2	0.178	14 Φ 16	0.704	0.35	0.83
F3	0.109	9 Φ 16	0.452	0.25	1.11
F5	0.312	21 Φ 16	0.106	0.45	0.75
F6	0.178	12 Φ 16	0.603	0.35	0.97
F7	0.880	9 Φ 13	0.368	0.25	1.10
F8	0.157	9 Φ 16	0.452	0.35	1.14
F11	0.730	8 Φ 13	0.327	0.25	1.03

Figure (94) shows a plan for the isolated footings indicating the footing dimensions and reinforcement with a section at the axis 2-2 after carrying out all processes of the analysis and design for the isolated footings.



b) section 2-2



a) plan

Figure (94) Footing dimensions [m] and reinforcement

6.9 Reinforcement steel for footings connected with tie beams

As mentioned before the thickness of the footing in this example is chosen to fulfill the safety conditions at the analysis of the footing whether they are connected with or without tie beams excepting the reinforcement, which is chosen for every structural system separately. Therefore, the analysis is carried out again for the footings with the same data of the pervious footings but with considering the tie beams.

Figure (95) shows the bending moment m_x while Figure (96) shows the bending moment m_y for footings connected with tie beams at the critical sections in *x*- and *y*-directions, respectively. Tables (78) and (79) show check depth required to resist the bending moment and also the required reinforcement for the critical sections in case of the structural design of the footings connected with tie beams.

Table (78) Check depth required to resist bending moment and determination of reinforcement steel in *x*-direction

Footing	Moment m_x [kN.m/m]		f_c [kg/cm²]	Required area of steel A_s [cm²/m]		Chosen steel [Rft/m]	
	-ve m_x	+ve m_x		A_{sx1} Top	A_{sx2} Bottom	A_{sx1} Top	A_{sx2} Bottom
F1	79	79	50	12.51	12.45	7Φ16	7Φ16
F2	-	45	35	-	6.94	-	*min* A_s
F3	-	42	55	-	9.36	-	*min* A_s
F4	116	105	50	14.12	12.70	*min* A_s	*min* A_s
F5	-	97	45	-	11.70	-	*min* A_s
F6	-	59	45	-	9.17	-	*min* A_s
F7	-	16	30	-	3.51	-	*min* A_s
F8	-	75	50	-	11.86	-	*min* A_s
F9	12	2	25	2.51	-	*min* A_s	*min* A_s
F10	25	19	20	2.93	2.19	*min* A_s	*min* A_s
F11	-	20	35	-	4.40	-	-

Table (79) Check depth required to resist bending moment and determination of reinforcement steel in y-direction

Footing	Moment m_y [kN.m/m]		f_c [kg/cm^2]	Required area of steel A_s [cm^2/m]		Chosen steel [Rft/m]	
	-ve m_y	+ve m_y		A_{sy1} Top	A_{sy2} Bottom	A_{sy1} Top	A_{sy2} Bottom
F1	-	24	25	-	3.56	*min* A_s	*min* A_s
F2	-	68	45	-	10.68	-	6Φ16
F3	-	43	55	-	9.58	-	*min* A_s
F4	-	82	40	-	9.82	*min* A_s	*min* A_s
F5	-	141	55	-	17.38	-	9Φ16
F6	-	52	40	-	8.05	-	*min* A_s
F7	-	23	35	-	5.04	-	*min* A_s
F8	-	78	50	-	12.21	-	7Φ16
F9	-	6	20	-	1.28	*min* A_s	*min* A_s
F10	17	58	30	2.01	6.89	*min* A_s	*min* A_s
F11	-	29	40	-	6.34	-	*min* A_s

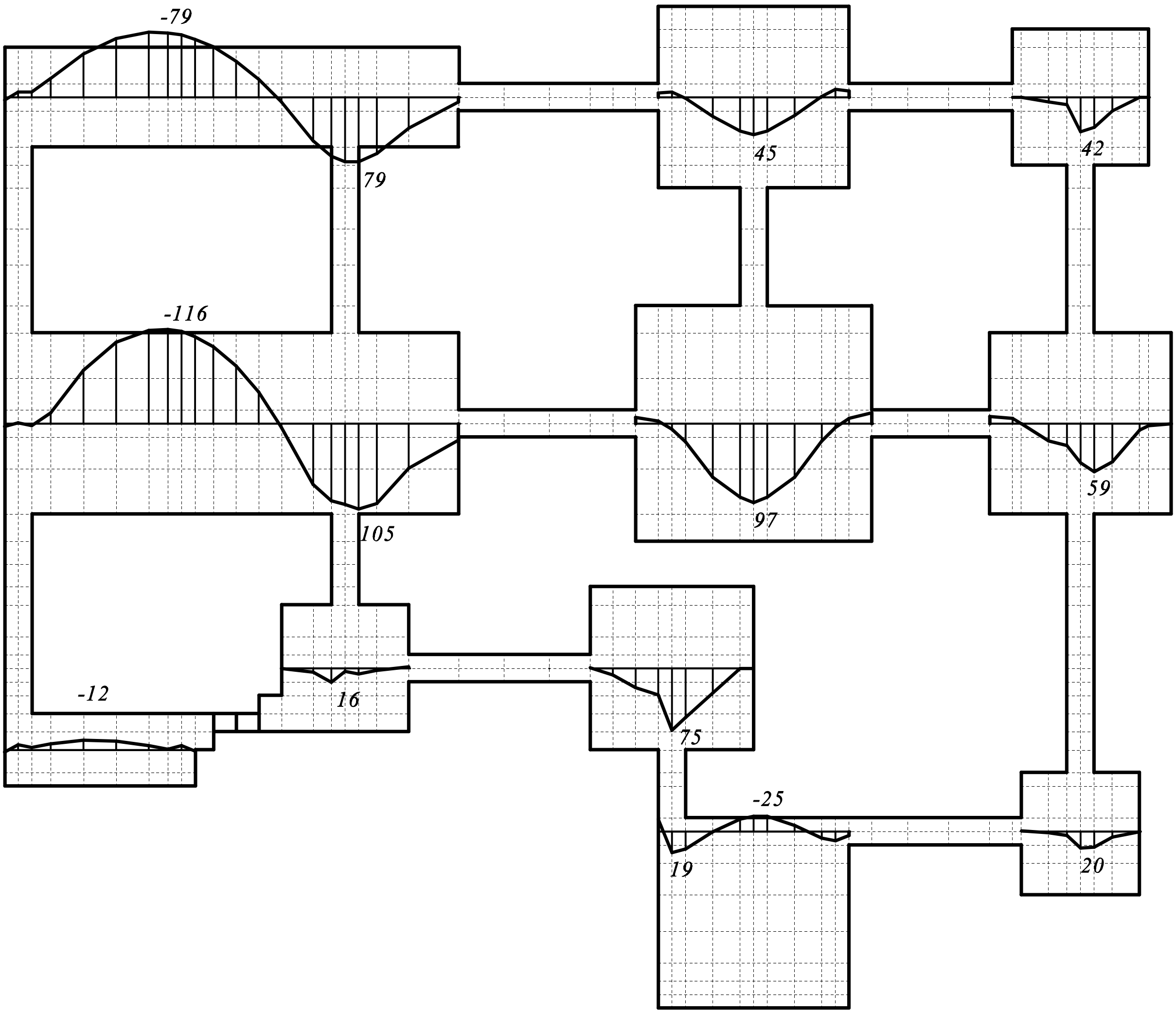


Figure (95) Moment m_x [kN.m/m] at critical sections on the footings

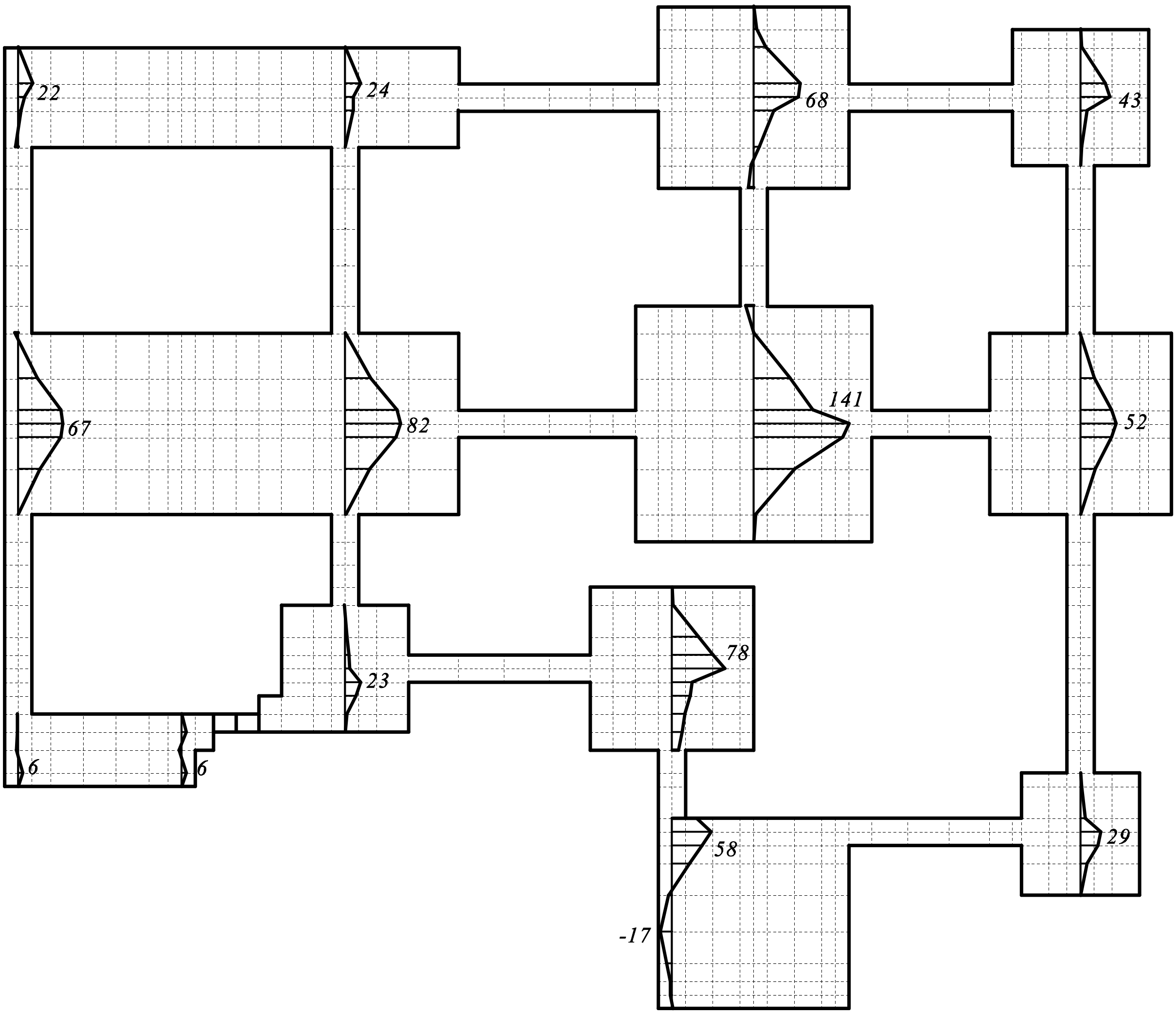


Figure (96) Moment m_y [kN.m/m] at critical sections on the footings

6.10 Reinforcement steel for tie beams

The cross section of the tie beams is a constant rectangular with the dimension of 0.3 [m] * 0.6 [m]. To simplify the analysis of the tie beams, it will be considered that this section is rectangular also inside the footings and constant at either the compression or tension places. The properties of reinforced concrete and the reinforcement for the tie beams and footings are the same as mentioned before.
The minimum area of top or bottom reinforcement steel in the tie beam is taken as 0.15 [%] of the concrete cross section of the tie beam such as:

$min\ A_s = 0.0015\ A_c = 2.7\ [cm^2]$ Y chosen 2Φ16 /m = 4.02 [cm^2]

This area of reinforcement steel is sufficient to resist a bending moment of 40 [kN.m]. This minimum area of reinforcement steel will be generally considered for all cross sections of the tie beams besides another additional steel if required at the sections that have bending moments greater than 40 [kN.m].

Figure (97) shows the bending moments M_b for the tie beams in x-direction, while Figure (98) shows those in y-direction. Table (80) shows the values of bending moments that are greater than 40 [kN.m] and the corresponding additional steel to resist them. Besides, the amount of the additional steel that is required to resist each moment with the definition of its place.

Table (80) Additional reinforcement steel for the tie beams

Moment M_b [kN.m]	Required area of steel [cm^2]	Chosen additional steel [*Rft*]	Footing	Direction
82	8.35	3Φ16	*F1*	Top/longitudinal
79	8.13	3Φ16	*F1*	Bottom/longitudinal
62	6.23	2Φ16	*F4*	Top/longitudinal
49	4.97	1Φ16	*F4*	Bottom/longitudinal
45	4.60	1Φ16	*F6*	Bottom/transversal
51	5.18	1Φ16	*F10*	Bottom/transversal

Figure (99) shows a group of footings connected with tie beams after completion of its design with a plan for reinforcement and a cross section in the tie beams.

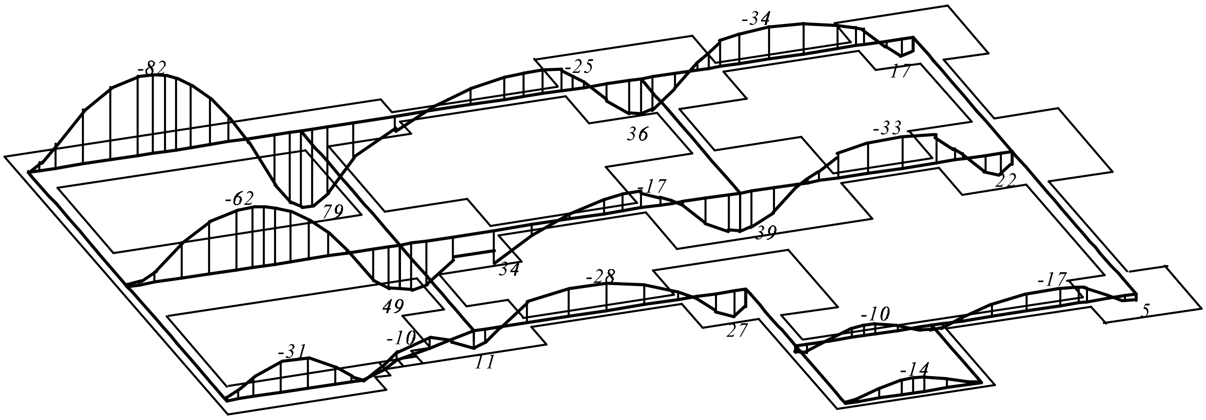


Figure (97) Moment M_b [kN.m] in girders at longitudinal and diagonal directions

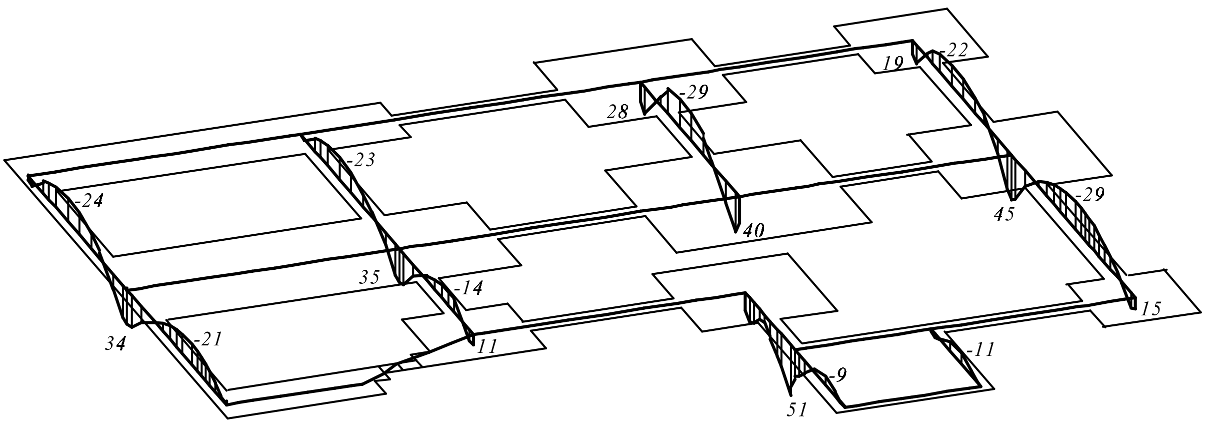
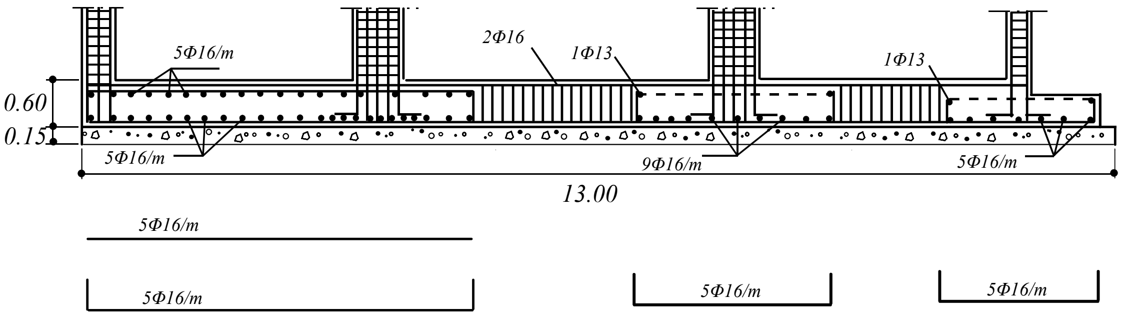
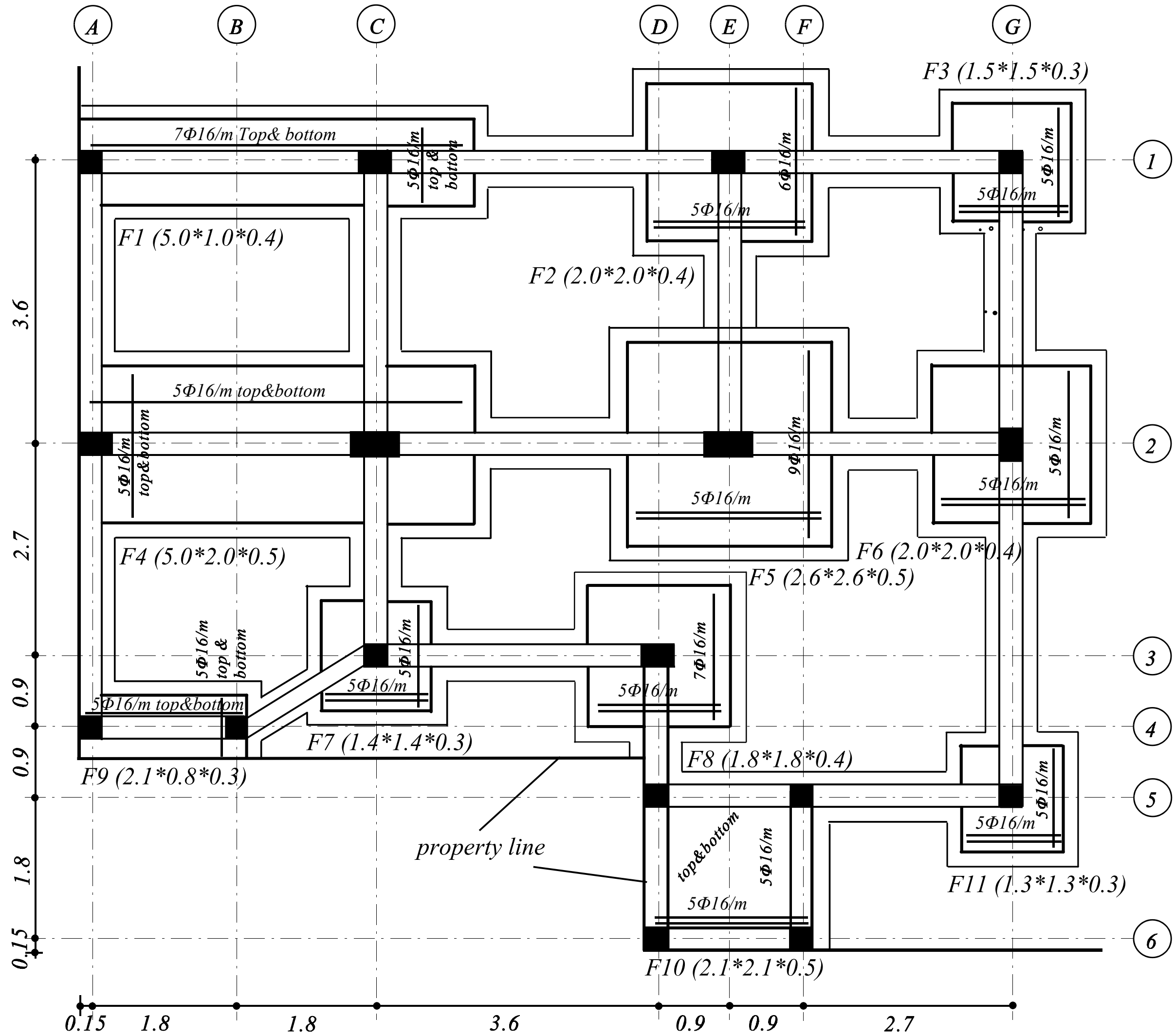


Figure (98) Moment M_b [kN.m] in girders at transversal direction



b) section 2-2



a) plan

Figure (99) Footing dimensions [m], tie beams and reinforcement

7 Comparison between the two structural systems of the footings

7.1 Settlement

Figure (100) shows the settlement at the axis 2-2 for either isolated footings or footings connected with tie beams. It can be observed from this figure that the settlement curve was sharp before connecting the footings, then it becomes much uniform after connecting the footings. Furthermore, the values of settlements decrease greatly, where the maximum settlement at this axis decreases from 0.57 [cm] to 0.47 [cm] with percentage 21 [%].

7.2 Contact pressure

Due to the presence of tie beams those are also resting on the soil, the contact area between the foundation and the soil increases from 45.75 [m^2] to 53.59 [m^2] with percentage 17 [%]. This contact area will perform certainly to reduce the contact pressure between the soil and the foundation. Consequently, the main contact pressure becomes q_o = 164 [kN/m^2] instead of 192 [kN/m^2].

From the assumption of Winkler's model that the contact pressure between the soil and the foundation is proportionally at any node with the settlement at that node ($q = k_s.s$), therefore Figure (100), which represents the settlement at axis 2-2, represents also the contact pressure between the soil and foundation at that axis if the value of settlement is multiplied by the modulus of subgrade reaction k_s. It is clear from this figure that the contact pressure between the foundation and the soil, which represents soil reaction became more uniform due to the presence of tie beams. The maximum contact pressure at this axis decreases with percentage 21 [%] as in case of the settlement.

7.3 Bending moment

The amount of reinforcement steel in the footings is determined according to the bending moment. It can be found from the comparison between the design of footings with and without tie beams that the amount of reinforcement steel decreases to minimum reinforcement due to the presence of the tie beams at the most sections. This is clear in Figure (101), which represents the bending moment at the axis 2-2 where the maximum bending moment m_x decreases from 181 [kN.m/m] to 116 [kN.m/m] with a great percentage 56 [%].

7.4 Shearing force

There is no need to check shear stress for footings connected with tie beams where the presence of the tie beams and their reinforcement steel inside the footings resist greatly the shear stress. It is observed that the shearing force decreases greatly as it is indicated in Figure (102), which shows the shearing force at the axis 2-2. Due to the presence of the tie beams the maximum shearing force Q_x decreases from 308 [kN/m] to 199 [kN/m] with percentage 55 [%]

8 Conclusion

From the pervious analyses, it can be concluded that the design of a group of footings connected with stiff tie beams had improved greatly the behavior of these footings toward deformation and rotation. Besides, it decreases the amount of reinforcement steel at the most sections.

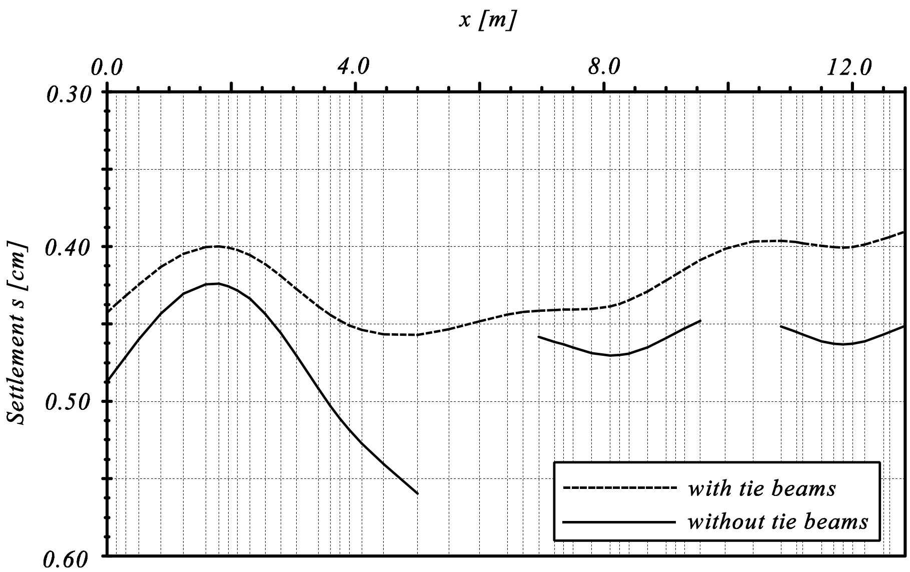


Figure (100) Settlement *s* [cm] at section 2-2

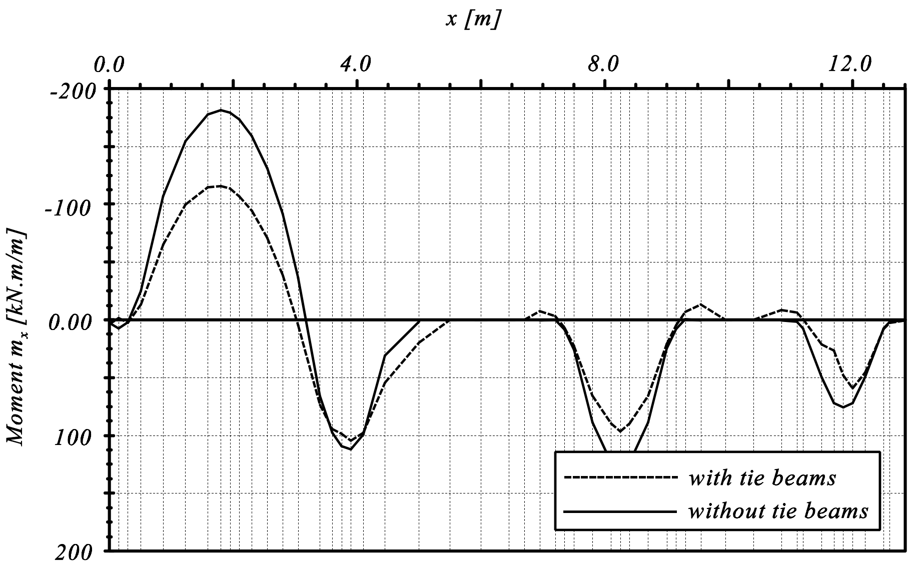


Figure (101) Moment m_x [kN.m/m] at section 2-2

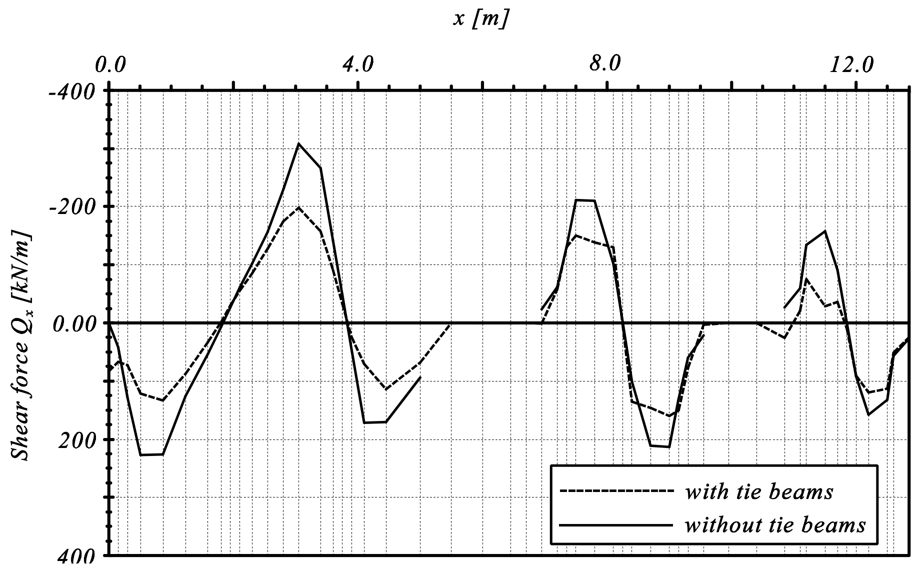


Figure (102) Shearing force Q_x [kN/m] at section 2-2