Example 11: Verifying ultimate bearing capacity for a footing on layered subsoil

1 Description of the problem

To verify the ultimate bearing capacity calculated by *ELPLA*, the results of Example 2, page 9 in DIN 4017 for determining the ultimate bearing capacity of a footing on layered subsoil are compared with those obtained by *ELPLA*.

A rectangular footing of 4.0 [m] × 5.0 [m] on layered subsoil is considered. Footing dimensions and soil layers under the footing with soil constants are shown in Figure 15. It is required to determine the ultimate bearing capacity of the soil under the footing.

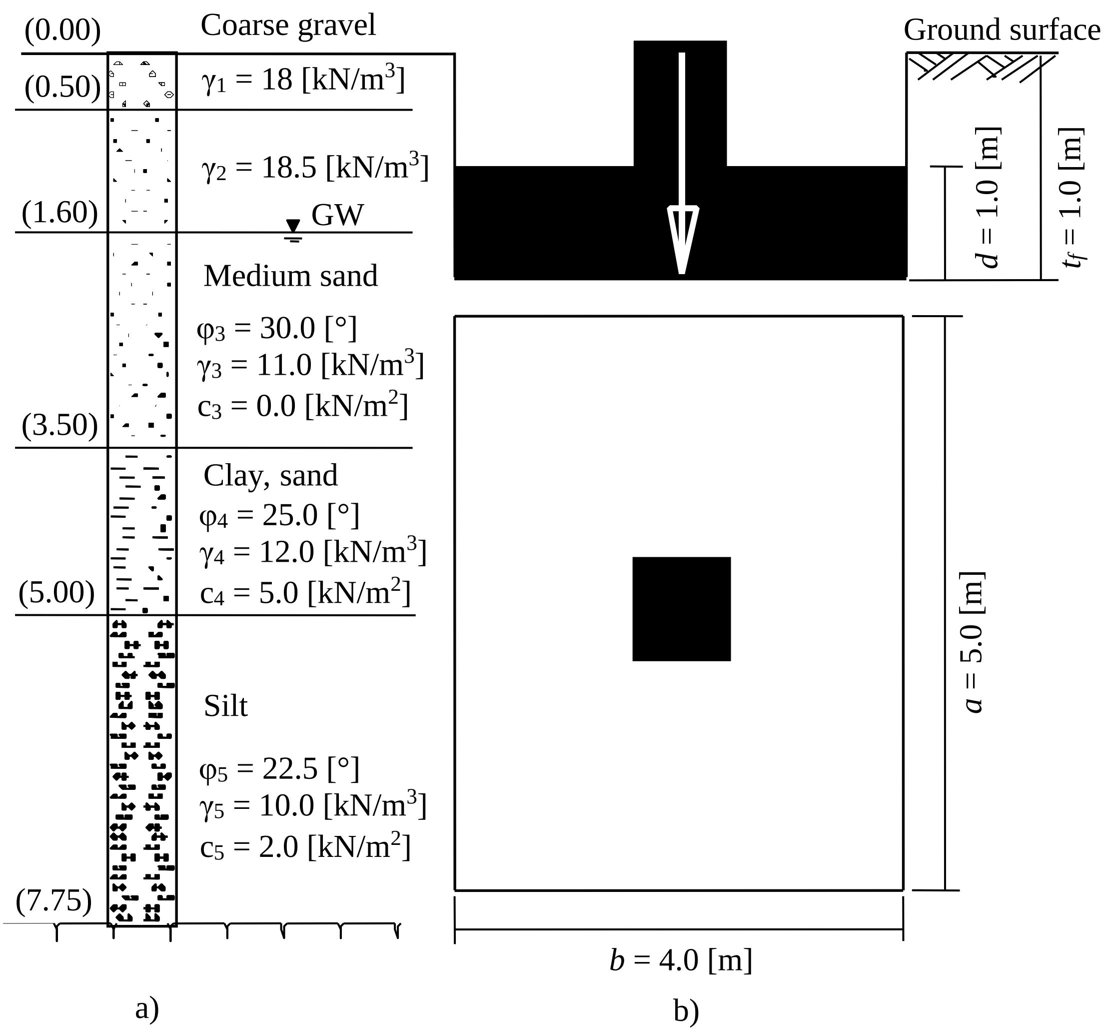

Figure 15 a) Cross section through the soil under the footing

b) Plan of the footing with dimensions

2 Hand calculation of ultimate bearing capacity

According to DIN 4017, the ultimate bearing capacity can be obtained by hand calculation as follows:

Iterative determination of the soil constant φ_m

According to DIN 4017, the mean values of the soil constants are only accepted, if the angle of internal friction for each individual layer φ_i does not exceed the average value of the internal friction $\varphi_{av.}$ by 5 [°].

The average value $\varphi_{av.}$ for the three layers is given by

$$\varphi_{av.} = \frac{30 + 25 + 22.5}{3} = 25.83 \text{ [°]}$$

The difference between each individual value φ_i and the average value φ_{av} is less than 5 [°]. The iteration begins with the angle of internal friction φ_{m0} of the first layer, which lies directly under the footing.

1st Iteration step

The first step is determining the failure shape of the soil under the footing for φ_{m0} = 30 [°]. The failure shape is described in Figure 16. The geometry of the failure shape can be described by the angles β, α, and ω, which are given by

$$\beta = 45 - \frac{\varphi_{m0}}{2} = 45 - \frac{30}{2} = 30 \text{ [°]}$$

$$\alpha = \square = 45 + \frac{\varphi_{m0}}{2} = 45 + \frac{30}{2} = 60 \text{ [°]}$$

Therefore

$$\omega = 90 \text{ [°]}$$

The triangular side r_0 is given by

$$r_0 = \frac{b}{\sin\left(90 - \varphi_{m0}\right)} \sin \square = \frac{4}{\sin\left(90 - 30\right)} \sin 60 = 4 \text{ [m]}$$

The triangular side r_1 is given by

$$r_1 = r_0 e^{(\text{arc}\,\omega\ \tan\varphi_{m0})} = 4e^{\left(\frac{90\pi}{180} \tan 30\right)} = 9.91 \text{ [m]}$$

The length of the slide shape l is given by

$$l = 2r_1 \cos\beta = 2 \times 9.91 \cos 30 = 17.16 \text{ [m]}$$

The depth of the slide shape $max\ T_s$ under the footing is given by

$$\max T_s = r_0 \cos\varphi_0\, e^{(\text{arc}\ \omega\ \tan\varphi_{m0})} = 4\cos 30\, e^{\left(\frac{60\pi}{180}\tan 30\right)} = 6.34 \text{ [m]}$$

The depth of failure shape z under the ground surface is given by

$$z = maxT_s + t_f = 6.34 + 2 = 8.34 \text{ [m]}$$

β

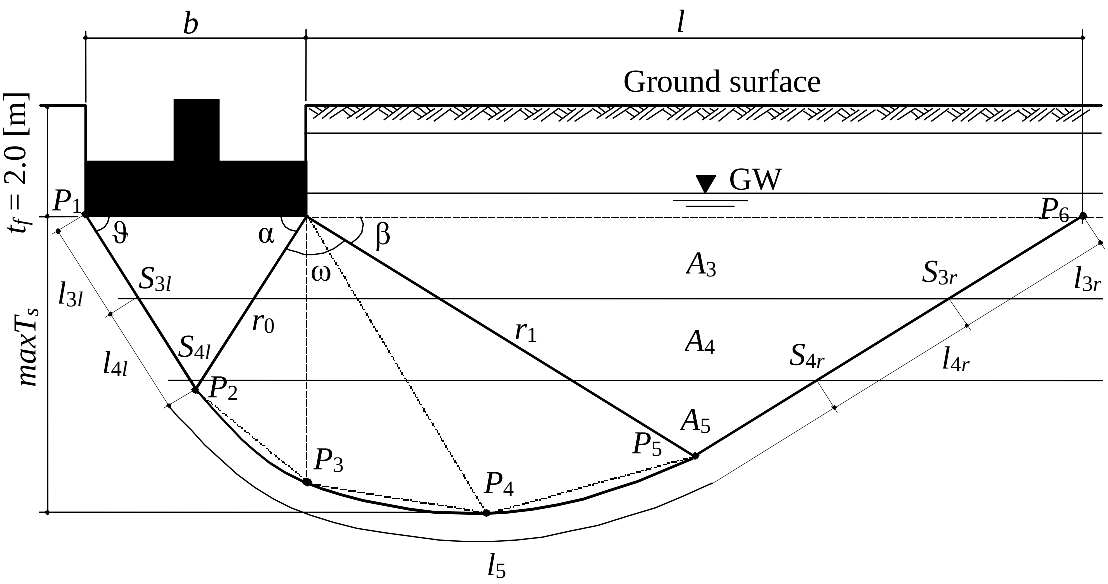

Figure 16 Ultimate bearing capacity for multi-layers system

To simplify the analytical calculation, the slip line is approximated by a polygon. Accordingly, by dividing the angle ω of the logarithmic spiral into three sub angles, the polygon P_1 to P_6 can be drawn. Then, the layer boundaries with the polygonal sequence are determined. The intersection points can be determined also graphically, when the bottom failure shape is considered and hence the intersection points are taken from the drawing. Considering a Cartesian coordinate system in which the origin coordinate is point P_1, the following intersection points are given

S_{3l} (0.87, 1.50), S_{3r} (18.56, 1.50), S_{4l} (1.73, 3.00), S_{4r} (15.96, 3.00)

Due to intersection of polygon points with soil layers, the following proportional lengths are determined

$$l_3 = l_{3l} + l_{3r} = 1.73 + 3 = 4.73 \text{ [m]}$$

$$l_4 = l_{4l} + l_{4r} = 4.73 \text{ [m]}$$

$$l_5 = 16.12 \text{ [m]}$$

$$\text{total length } l_{tot.} = 25.58 \text{ [m]}$$

From these proportional lengths, the main value of the angle of the internal friction for the first iteration can be determined as follows

$$\tan \varphi_{m1} = \frac{l_3 \tan \varphi_3 + l_4 \tan \varphi_4 + l_5 \tan \varphi_5}{l_3 + l_4 + l_5}$$

$$\tan \varphi_{m1} = \frac{4.73 \tan 30 + 4.73 \tan 25 + 16.12 \tan 22.5}{25.58}$$

or

$$\varphi_{m1} = 24.42 \text{ [°]}$$

The deviation Δ_i of the output value φ_{m1} from the input value φ_{m0} is

$$\Delta_i = \frac{\varphi_{m0} - \varphi_{m1}}{\varphi_{m0}} \times 100 = \frac{30 - 24.42}{30} \times 100 = 18.60 \text{ [\%]}$$

The deviation Δ_i is greater than 3 [%]. Therefore, a further iteration is necessary. The new angle of internal friction for the 2^{nd} iteration step is given by

$$\varphi_{m1} = \frac{\varphi_{m0} + \varphi_{m1}}{2} = \frac{30 + 24.42}{2} = 27.21 \text{ [°]}$$

2nd Iteration step

The failure shape for φ_{m1} = 27.21 [°] is determined. Then, the calculation is carried out analog to the first iteration step. The calculated proportional lengths are

$$l_3 = 4.64 \text{ [m]}$$

$$l_4 = 4.64 \text{ [m]}$$

$$l_5 = 13.49 \text{ [m]}$$

The main angle of the internal friction is given by

$$\tan \varphi_{m2} = \frac{4.64 \tan 30 + 4.64 \tan 25 + 13.49 \tan 22.5}{22.77}$$

$$\varphi_{m2} = 24.61 \text{ [°]}$$

The deviation Δ_i = 9.55 [%] is still greater than 3 [%]. Therefore, a further iteration step is to be carried out with

$$\varphi_{m2} = \frac{\varphi_{m1} + \varphi_{m2}}{2} = \frac{24.61 + 27.21}{2} = 25.91 \text{ [°]}$$

3rd Iteration step

The results of the 3rd iteration step give

$$\varphi_{m3} = 24.70 \text{ [°]}$$

The deviation Δ_i = 4.66 [%] is still greater than 3 [%]. Therefore, a further iteration step is to be carried out with

$$\varphi_{m3} = \frac{\varphi_{m2} + \varphi_{m3}}{2} = \frac{24.70 + 25.91}{2} = 25.31 \text{ [°]}$$

4th Iteration step

The results of the 4th iteration step give φ_{m4} = 24.74 [°]. The deviation Δ_i = 2.22 [%] is less than 3 [%]. Therefore, the iteration process will stop here. The mean value of the angle of internal friction is given by

$$\varphi_{m4} = \frac{\varphi_{m3} + \varphi_{m4}}{2} = \frac{24.70 + 25.31}{2} = 25 \text{ [°]}$$

Determination of the soil constant c_m

In this step the geometry of the failure shape for φ_m = 25.00 [°] can be determined. Then, the proportional lengths are

$$l_3 = 4.57 \text{ [m]}$$

$$l_4 = 4.57 \text{ [m]}$$

$$l_5 = 15.62 \text{ [m]}$$

The mean cohesion c_m is given from proportional lengths by

$$c_m = \frac{l_3\, c_3 + l_4\, c_4 + l_5\, c_5}{l_3 + l_4 + l_5}$$

$$c_m = \frac{4.57 \times 0 + 4.57 \times 5 + 15.62 \times 2}{24.76} = 2.19 \text{ [kN/m}^2\text{]}$$

Determination of the soil constant γ_m

a) Mean unit weigh of the soil γ_m under the foundation level

Due to intersection of polygon points with soil layers the following proportional areas A_3, A_4 and A_5 can be determined

$$A_3 = 23.13 \text{ [m}^2\text{]}$$

$$A_4 = 18.17 \text{ [m}^2\text{]}$$

$$A_5 = 15.62 \text{ [m}^2\text{]}$$

$$\text{total area } A_{tot.} = 56.92 \text{ [m}^2\text{]}$$

The mean unit weight of the soil under the foundation level γ_m is given from proportional areas by

$$\gamma_m = \frac{A_3\, \gamma_3 + A_4\, \gamma_4 + A_5\, \gamma_5}{A_3 + A_4 + A_5}$$

$$\gamma_m = \frac{23.13 \times 11 + 18.17 \times 12 + 15.62 \times 10}{56.92} = 11.05 \text{ [kN/m}^3\text{]}$$

b) Mean unit weigh of the soil γ'_m above the foundation level

The mean unit weight of the soil above the foundation level γ'_m is given from proportional areas above the foundation level by

$$\gamma'_m = \frac{0.5 \times 18 + 1.1 \times 18.5 + 0.4 \times 11}{2} = 16.88 \text{ [kN/m}^3\text{]}$$

Now, from the above calculated mean soil constants φ_m, c_m, γ_m and γ'_m, the bearing capacity factors can be determined for homogenous subsoil. Formulae used to determine the bearing capacity factors are described in DIN 4017 Part 1. From these formulae, the bearing capacity factors for φ_m = 25.00 [°] are

$$N_d = 10.7$$

$$N_c = 20.8$$

$$N_b = 4.5$$

while the shape factors for φ_m = 25.00 [°] and a = 4.0 [m], b = 5.0 [m] are

$$ny_d = 1.34$$

$$ny_c = 1.37$$

$$ny_b = 0.76$$

The ultimate bearing capacity of the soil q_{ult} can be determined according to DIN 4017 from

$$q_{ult} = c\, N_c\, ny_c + \gamma_1\, t_f\, N_d\, ny_d + \gamma_2\, B\, N_b\, ny_b$$

$$q_{ult} = 2.19 \times 20.8 \times 1.37 + 16.88 \times 2 \times 10.7 \times 1.34 + 11.05 \times 4 \times 4.5 \times 0.76$$

$$q_{ult} = 698 \text{ [kN/m}^2\text{]}$$

3 Ultimate bearing capacity by *ELPLA*

To determine the ultimate bearing capacity by *ELPLA*, one of the available calculation methods 2 to 8 used to carry out the nonlinear analysis of foundations may be used. Here the nonlinear analysis of foundation requires to know the ultimate bearing capacity of the soil. The ultimate bearing capacity obtained by *ELPLA* is q_{ult} = 701 [kN/m^2] and nearly equal to that obtained by hand calculation according to DIN 4017.