Example 12: Verifying simple assumption model for irregular raft

1 Description of the problem

To verify the simple assumption model of *ELPLA*, the contact pressure distribution of an irregular foundation obtained by *Bowles* (1977), Example 9-6, page 265, is compared with that obtained by *ELPLA*.

A square foundation that has 10 [m] side is chosen. The foundation is subjected to a column load of 540 [kN] at the center. It is required to determine the distribution of the contact pressure when the corner is notched as shown in Figure 17. The notch has the following properties:

Area	A =	4.5	[m^2]
Center of gravity from *o* in x-direction	x' =	3.5	[m]
Center of gravity from *o* in y-direction	y' =	4.25	[m]

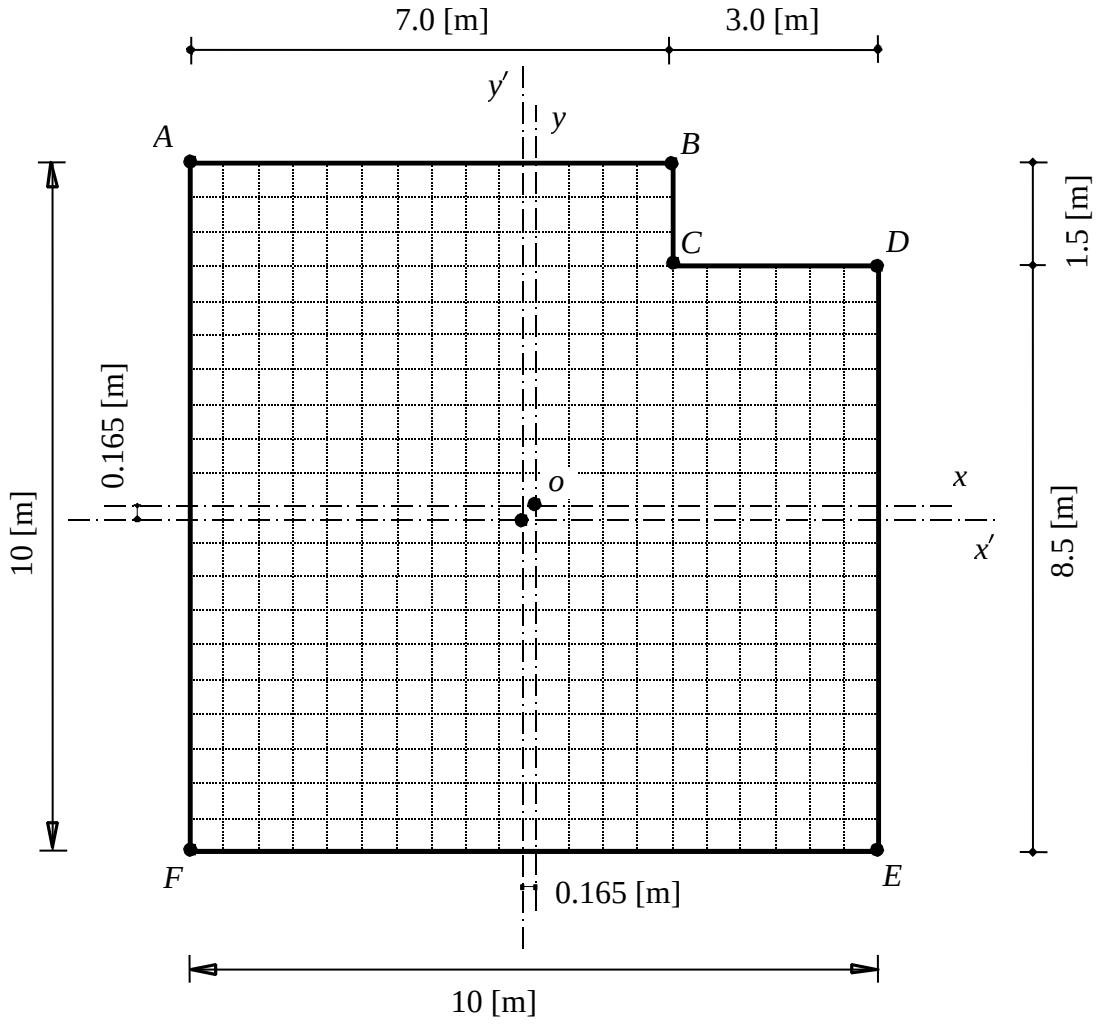

Figure 17 Foundation dimensions and FE-Net

The simple assumption model assumes a linear distribution of contact pressure on the base of the foundation. In the general case of a foundation with an arbitrary unsymmetrical shape and loading, based on *Navier´s* solution, the contact pressure q_i [kN/m²] at any point (x_i, y_i) [m] from the geometry centroid on the bottom of the foundation is given by:

$$q_i = \frac{N}{A_f} + \frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2} x_i + \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2} y_i \qquad (15)$$

where:

N	Sum of all vertical applied loads on the foundation [kN]
A_f	Foundation area [m^2]
M_x	Moment due to N about the x-axis [kN.m]
M_y	Moment due to N about the y-axis [kN.m]
I_x	Moment of inertia of the foundation about the x-axis [m^4]
I_y	Moment of inertia of the foundation about the y-axis [m^4]
I_{xy}	Product of inertia [m^4]

2 Hand calculation of contact pressure

According to *Bowles* (1977), the contact pressure distribution under the foundation can be obtained by hand calculation as follows:

Step 1: Find new *x*-, *y*-axis

$$\bar{x} = \frac{-15.75}{95.5} = -0.165 \text{ [m]}$$

$$\bar{x} = \frac{-19.13}{95.5} = -0.20 \text{ [m]}$$

which gives the location of new axes x' and y' as shown in Figure 17

Step 2: Compute new properties $I_{x'}$, $I_{y'}$ and $I_{xy'}$

Determining properties of foundation parts are listed in Table 14.

Table 14 Properties of foundation parts

Part	Area A [m^2]	x [m]	Y [m]	Ax^2 [m]	Ay^2 [m]	I_{ox} [m^4]	I_{oy} [m^4]
Uncut	100	-0.165	-0.20	2.72	4.00	833.3	833.3
Notch	-4.5	3.66	4.45	-60.3	-89.1	-0.84	-3.38
Total	95.5						

$$I_x = I_{ox} - I_{ox\ notch} + A_y^2$$

$$I_x = 833.3 - 0.84 + 4.0 - 89.0 = 747.5\ [m^4]$$

$$I_y = I_{oy} - I_{oy\ notch} + A_x^2$$

$$I_y = 833.3 - 3.38 + 2.73 - 60.5 = 772.15\ [m^4]$$

$$I_{xy} = I_{oxy} + A_{\bar{x}\bar{y}}$$

Step 3: Compute moments

$$M_y = 540 \times 0.165 = 89.1\ [kN.m]$$

$$M_x = 540 \times 0.2 = 108\ [kN.m]$$

Step 4: Compute contact pressure at selected locations

The contact pressure q_i at any point (x_i, y_i) from the geometry centroid on the bottom of the foundation is obtained from

$$q_i = \frac{N}{A_f} + \frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2} x_i + \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2} y_i$$

$$q_i = \frac{540}{95.5} + \frac{(89.1)(747.5) - (108)(-70)}{(747.5)(772.15) - (-70)^2} x_i + \frac{(108)(772.15) - (89.1)(-70)}{(747.5)(772.15) - (-70)^2} y_i$$

$$q_i = 5.65 + 0.13 x_i + 0.157 y_i$$

3 Contact pressure by *ELPLA*

The available method "Linear Contact pressure 1" in *ELPLA* is used to determine the contact pressure distribution under the foundation. A net of equal square elements is chosen. Each element has a side of 0.5 [m] as shown in Figure 17. The contact pressures at the foundation corners obtained by *ELPLA* are compared with those obtained by *Bowles* (1977) in Table 15. It is obviously from this table that contact pressures obtained by *ELPLA* are equal to those obtained by hand calculation.

Table 15 Contact pressures at foundation corners

Point	*Bowles* (1977)						*ELPLA*
	x_i [m]	y_i [m]	N/A_f [kN/m^2]	0.13 x_i [kN/m^2]	0.157 y_i [kN/m^2]	q [kN/m^2]	q [kN/m^2]
A	-4.84	5.20	5.65	-0.63	0.82	5.84	5.84
B	2.16	5.20	5.65	0.28	0.82	6.75	6.75
C	2.16	3.70	5.65	0.28	0.58	6.51	6.52
D	5.16	3.70	5.65	0.67	0.58	6.90	6.90
E	5.16	-4.80	5.65	0.67	-0.75	5.57	5.57
F	-4.84	-4.80	5.65	-0.63	-0.75	4.27	4.28