Example 13: Verifying main modulus of subgrade reaction k_{sm}

1 Description of the problem

It is known that the modulus of subgrade reaction k_s is not a soil constant but is a function of the contact pressure and settlement. It depends on foundation loads, foundation size and stratification of the subsoil. The main modulus of subgrade reaction k_{sm} for a rectangular foundation on layered subsoil can be obtained from dividing the average contact pressure q_o over the settlement s_o under the characteristic point on the foundation, which has been defined by *Graßhoff* (1955). Clearly, this procedure is valid only for rectangular foundations on a layered subsoil model. Determining the main modulus of subgrade reaction k_{sm} for irregular foundation on an irregular subsoil model using another analysis is also possible by *ELPLA*.

In this example, settlement calculations at the characteristic point on the raft, using *Steinbrener's* formula (1934) for determining the settlement under the corner of a rectangular loaded area with the principle of superposition, are used to verify *ELPLA* analysis for determining the main modulus of subgrade reaction k_{sm}.

Consider the square raft in Figure 18, with area of $A_f = 8 \times 12$ [m^2] and thickness of $d = 0.6$ [m].

2 Soil properties

The soil under the raft consists of three layers as shown in Figure 18 and Table 16. *Poisson's* ratio is $\nu_s = 0.0$ [-] for the three layers. The foundation level of the raft is $d_f = 2.0$ [m].

Table 16 Soil properties

Layer No.	Type of soil	Depth of layer z [m]	Modulus of compressibility E_s [kN/m^2]	Unit weight of the soil γ_s [kN/m^3]
1	Clay	9.0	8 000	18
2	Medium sand	14.0	100 000	-
3	Silt	20.0	12 000	-

3 Loads

The raft carries 12 column loads, each is $P = 1040$ [kN].

4 Raft material

The raft material (concrete) has the following properties:

Young's modulus	E_b	= 2.0 × 10^7	[kN/m²]
Poisson's ratio	ν_b	= 0.25	[-]
Unit weight	γ_b	= 0.0	[kN/m³]

Unit weight of the raft material is chosen γ_b = 0.0 [kN/m³] to neglect the self-weight of the raft.

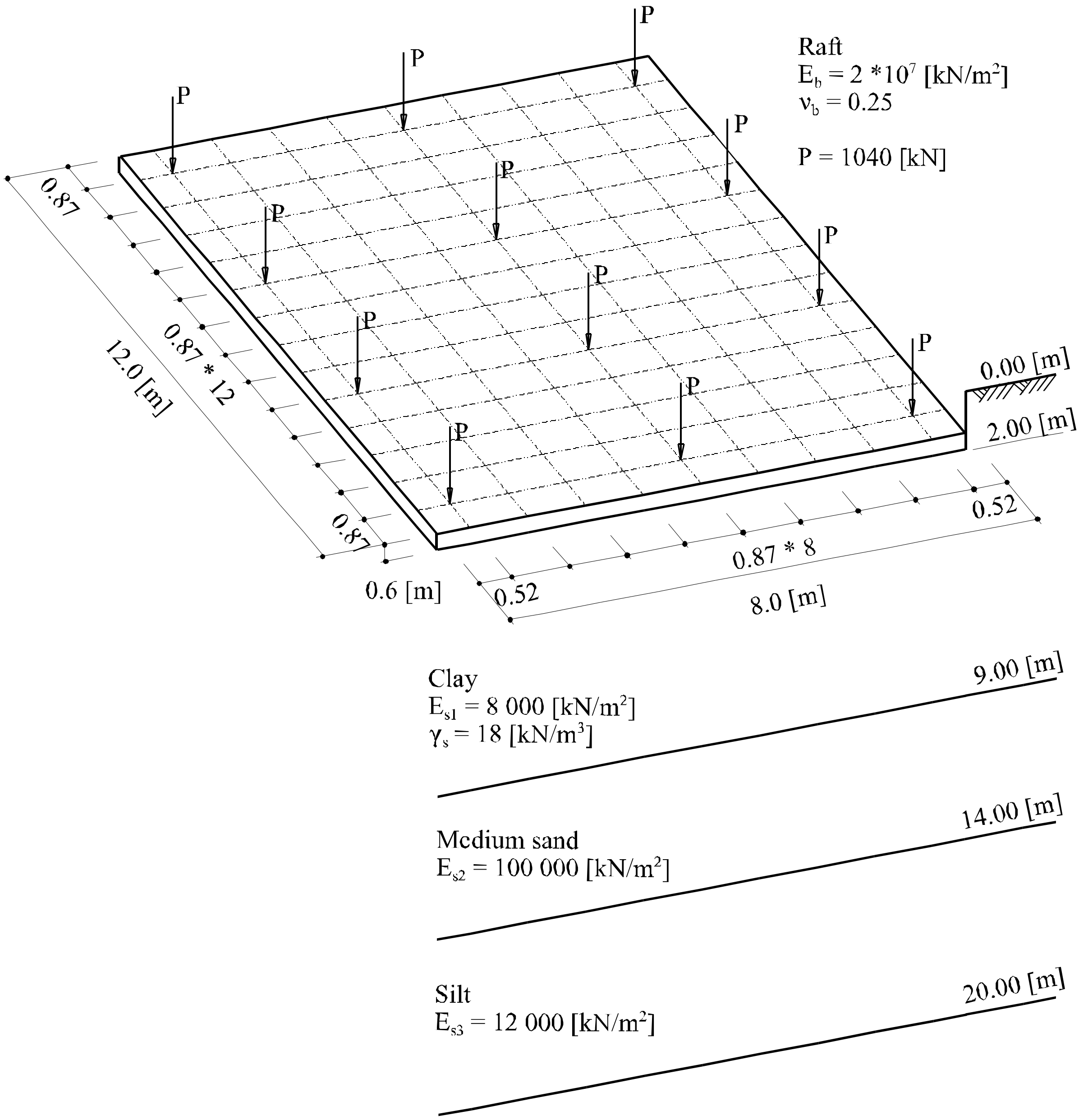

Figure 18 Raft dimensions, loads, FE-Net and subsoil

5 Settlement calculations

The average contact pressure q_o is given by

$$q_o = \Sigma P/A_f = 12\times1040 / (8 \times 12) = 130 \text{ [kN/ m}^2\text{]}.$$

The raft settlement is obtained at the characteristic point *o* by hand calculation. This point *o* takes the coordinates a_c = 0.87 *A* and b_c = 0.87 *B* as shown in Figure 19. The raft is divided into four rectangular areas I, II, III and IV as shown in Figure 19. The settlement of point *o* is then the sum of settlements of areas I, II, III and IV.

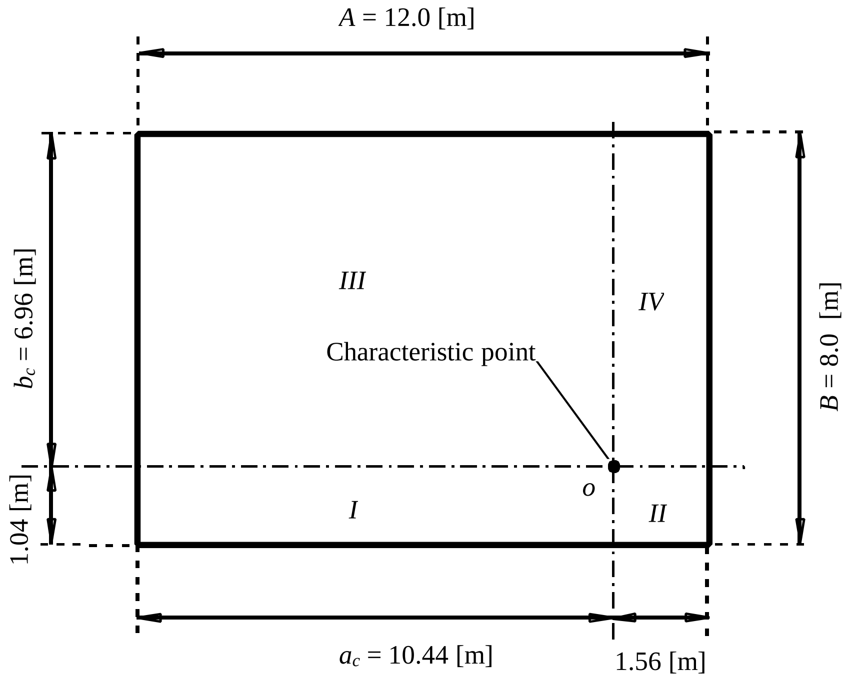

Figure 19 Characteristic point o of the settlement on the raft

According to *Steinbrenner* (1934) the settlement *s* of a point lying at a depth *z* under the corner of a rectangular loaded area *a* × *b* and intensity *q* is given by

$$s = \frac{q(1-\nu_s^2)}{2\pi E_s}\left(b.\ln\frac{(c-a)(m+a)}{(c+a)(m-a)} + a.\ln\frac{(c-b)(m+b)}{(c+b)(m-b)}\right) + \frac{q(1-\nu_s-2\nu_s^2)}{2\pi E_s}\left(z\tan^{-1}\frac{a.b}{z.c}\right) \quad (16)$$

The above equation can be rewritten as:

$$s = \frac{q(1-\nu_s^2)}{2\pi E_s}\,(B_n+A_n+D_n) = \frac{q(1-\nu_s^2)}{2\pi E_s}C_n = \frac{q}{E_s}f \quad (17)$$

Where $m = \sqrt{(a^2+b^2)}$ and $c = \sqrt{(a^2+b^2+z^2)}$

The settlement calculations of the 1st soil layer are carried out in Table 17.

Table 17 Settlement calculations of the 1st soil layer (z_1 = 7 [m])

Area	a [m]	b [m]	m [m]	c [m]	B_n	A_n	D_n	C_n
I	6.96	1.56	7.133	9.994	4.183	0.904	1.078	6.165
II	1.04	1.56	1.875	7.247	1.500	2.030	0.224	3.754
III	6.96	10.44	12.547	14.368	2.013	3.803	4.380	10.196
IV	1.04	10.44	10.492	12.613	0.351	3.788	0.857	4.996
ΣC_n								25.111

The settlement coefficient f_1 for the 1st layer is given by:

$$f_1 = \Sigma C_n / 2\pi = 25.111/ (2\pi) = 3.997$$

The settlement s_1 for the 1st soil layer is given by:

$$s_1 = q_o\, f_1 / E_{s1} = 130 \times 3.997/ 8000 = 0.06494 \text{ [m]}$$

In similar manner, the settlement coefficient f_2 for a soil layer until depth z =12 [m] is

$$f_2 = 5.2$$

The settlement s_2 for the 2nd soil layer is given by:

$$s_2 = q_o\, (f_2 - f_1)/ E_{s2} = 130\ (5.2 - 3.997)/ 100000 = 0.00156 \text{ [m]}$$

The settlement coefficient f_3 for a soil layer until depth z = 18 [m] is

$$f_3 = 6.038$$

The settlement s_3 for the 3rd soil layer is given by:

$$s_3 = q_o\, (f_2 - f_3)/ E_{s3} = 130\ (6.038 - 5.2)/ 12000 = 0.00908 \text{ [m]}$$

The total settlement s_o for all layers is given by:

$$s_o = s_1 + s_2 + s_3 = 0.06494 + 0.00156 + 0.00908 = 0.07558 \text{ [cm]}$$

The main modulus of subgrade reaction k_{sm} is given by:

$$k_{sm} = q_o / s_o = 130/ 0.07558 = 1720 \text{ [kN/m}^3\text{]}$$

6 Comparison of results

Table 18 compares the values of modulus of subgrade reaction obtained by using *Steinbrenner's* formula (1934) with that of *ELPLA*. It shows that the main modulus k_{sm} computed by using *Steinbrenner's* formula and that by *ELPLA* are nearly the same.

Table 18 Main modulus of subgrade reaction k_{sm} computed by using *Steinbrenner's* formula and *ELPLA*

Item	Hand calculation	*ELPLA*	Difference [%]
Main modulus k_{sm} [kN/m^3]	1720	1727	0.41