Example 21: Examination of influence of load geometry

1 Description of the problem

A simple example was carried out to show the influence of load geometry on the values of settlements and internal forces for the different subsoil models. To carry out the comparison between the different soil models, three different soil models are used to analyze the raft. The three mathematical models Simple assumption, *Winkler's* and Continuum models are represented by five calculation methods as shown in Table 29.

Table 29 Calculation methods and soil models

Method No.	Calculation method	Soil model
1	Linear contact pressure method	Simple assumption model
2	Modulus of subgrade reaction method	*Winkler's* model
5	Isotropic elastic half-space	Continuum model
7	Modulus of compressibility method	Continuum model
8	Rigid slab	Continuum model

A square raft with the dimensions of 10 × 10 [m^2] is chosen and subdivided into 144 square elements. Each element has dimensions of 0.833 × 0.833 [m^2] yielding to 13 × 13 nodal points for the raft and the soil as shown in Figure 41a.

2 Soil properties

The raft rests on a homogeneous soil layer of thickness 10 [m] equal to the raft length, overlying a rigid base as shown in Figure 41b. The raft thickness is d = 0.4 [m].

The soil material is supposed to have the following parameters:

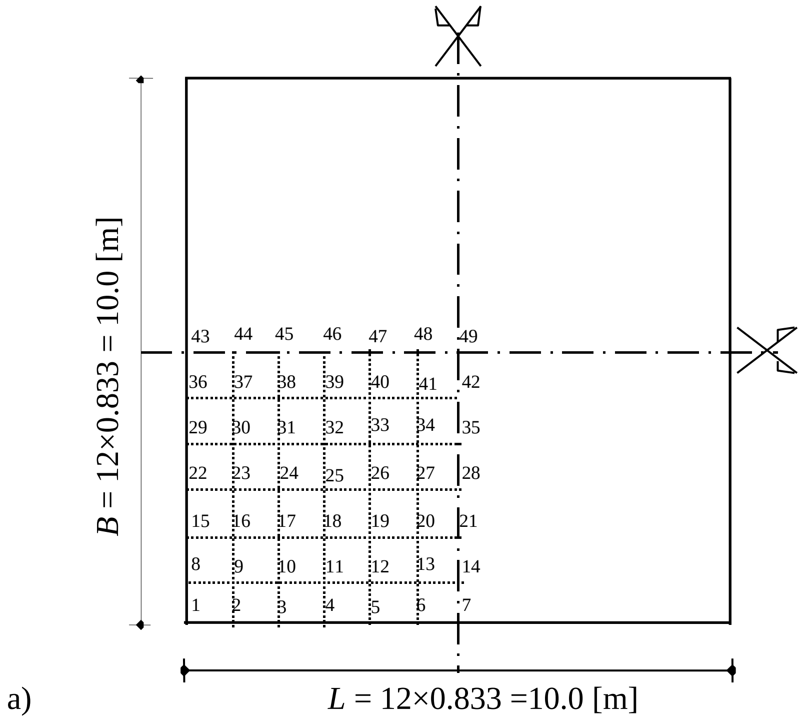
Modulus of compressibility	E_s	= 10 000	[kN/m^2]
Poisson's ratio	ν_s	= 0.2	[-]
Unit weight	γ_s	= 18	[kN/m^3]

3 Raft material

The raft material is supposed to have the following parameters:

Young's modulus	E_b	= 2 × 10^7	[kN/m^2]
Poisson's ratio	ν_b	= 0.25	[-]
Unit weight	γ_b	= 0.0	[kN/m^3]

Unit weight of the raft is chosen γ_b = 0.0 to neglect the self-weight of the raft.



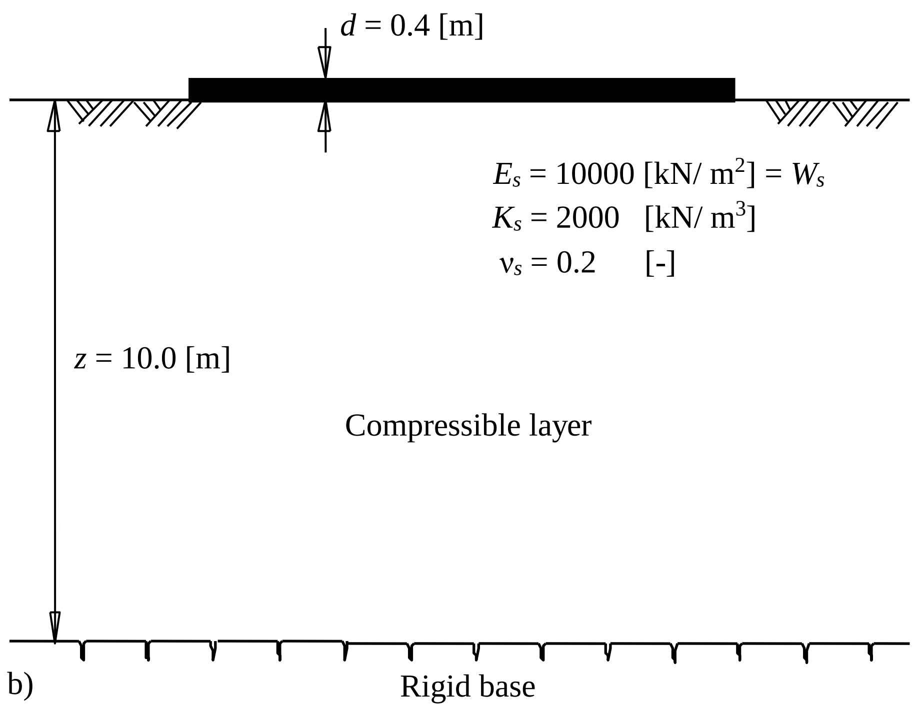


Figure 41 a) Slab numbering and dimensions
b) Soil cross section

4 Loads

To illustrate the raft behavior under various load arrangements, four different types of external load geometry are chosen such that each type gives 2000 [kN] total applied load and average contact pressure 20 [kN/m^2]. In addition, all loading cases are supposed to be symmetrical about the raft axes as shown in Figure 42.

The four load geometries are:

(a): A uniform load of intensity 20 [kN/m^2] on the entire raft
(b): Four inner loads, each 500 [kN]
(c): A concentrated central load 2000 [kN]
(d): Four corner loads, each 500 [kN]

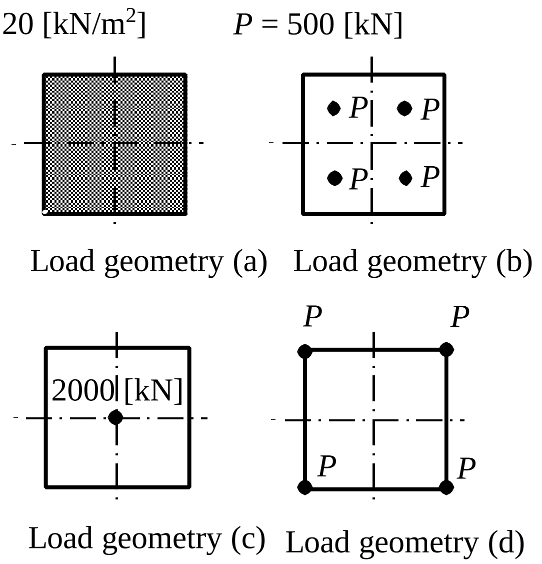


Figure 42 Arrangement of loads in the load cases (a) to (d)

5 Analysis of the raft

Taking advantage of the symmetry in shape and load geometry about both *x*- and *y*-axes, the analysis was carried out by considering only a quarter of the raft (Figure 41a). There is a total of only 49 nodal points; each node has three unknown displacements. Therefore, the total number of equations is reduced to 147.

5.1 Examination sections

For evaluation and comparison of the mathematical models, the numerical results for the four load geometries (a) to (d) were presented at three selected sections of the raft as follows:

Section I-I: at the edge of the raft (nodes 1 - 7)
Section II-II: at the quarter of the raft (nodes 22 - 28)
Section III-III : at the middle of the raft (nodes 43 - 49)

5.2 Mathematical models

The analysis was carried out by the modulus of Compressibility method 7 first and then the same raft with the same load geometries was analyzed using Simple assumption model 1, *Winkler's* model 2, Isotropic elastic half-space model 5 and Rigid slab 8.

5.3 Modulus of subgrade reaction

To make a comparison between *Winkler's* model and Continuum model the modulus of subgrade reaction was taken here so as to give nearly the same value for the average settlement, which was calculated by the Continuum model 7. This value for the modulus of subgrade reaction which is assumed to be constant at all foundation nodes is k_s = 2000 [kN/ m^3].

6 Results

6.1 Figures

The results of this example are plotted in Figure 43 to Figure 52 as follows:

- Figure 43 to Figure 45 show the settlements *s* (or deformation) at the middle of the raft (section III-III) for the four load geometries (a) to (d)

- Figure 46 to Figure 48 show the contact pressures *q* at edge of the raft (section I-I) for the four load geometries (a) to (d)

- Figure 49 to Figure 52 show the moments m_x in the three critical sections I, II and III of the raft for the four load geometries (a) to (d). From the assumption of the rigid slab, moments cannot be calculated. Therefore, moments in these figures are plotted only for methods 1, 2, 5 and 7

6.2 Tables

Furthermore, the results of this example are tabulated. Table 30 to Table 31 show the maximum values of the settlements s_{max} and the contact pressures q_{max} at the critical nodes by application of the different subsoil models for the four types of load geometries. The results of five calculation methods are given in these tables in order to observe the difference clearly.

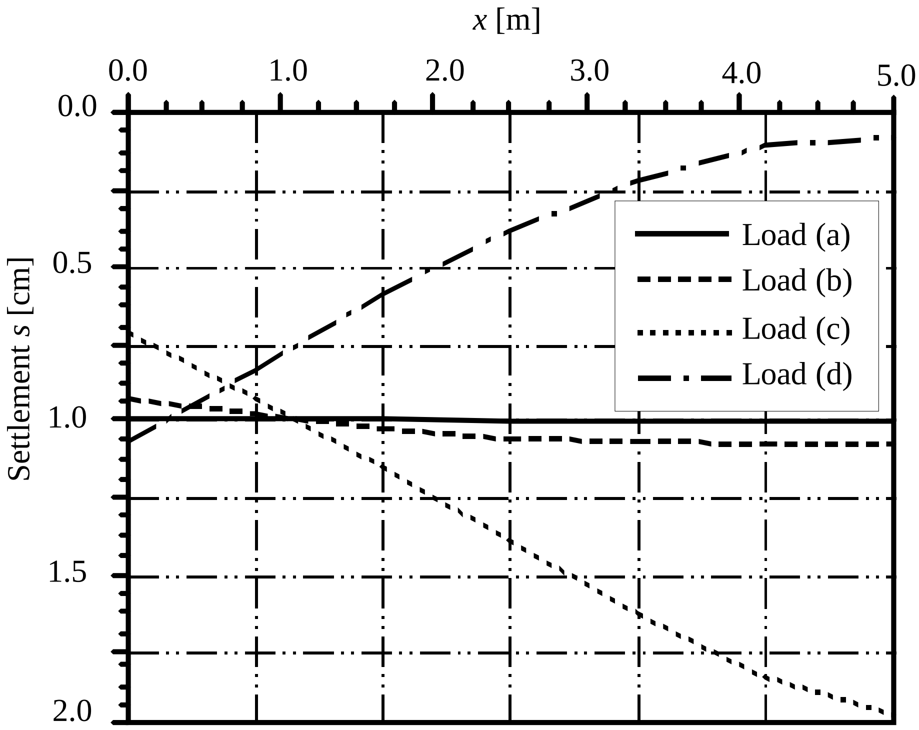


Figure 43 Settlement *s* [cm] at the middle section of the raft (method 2)

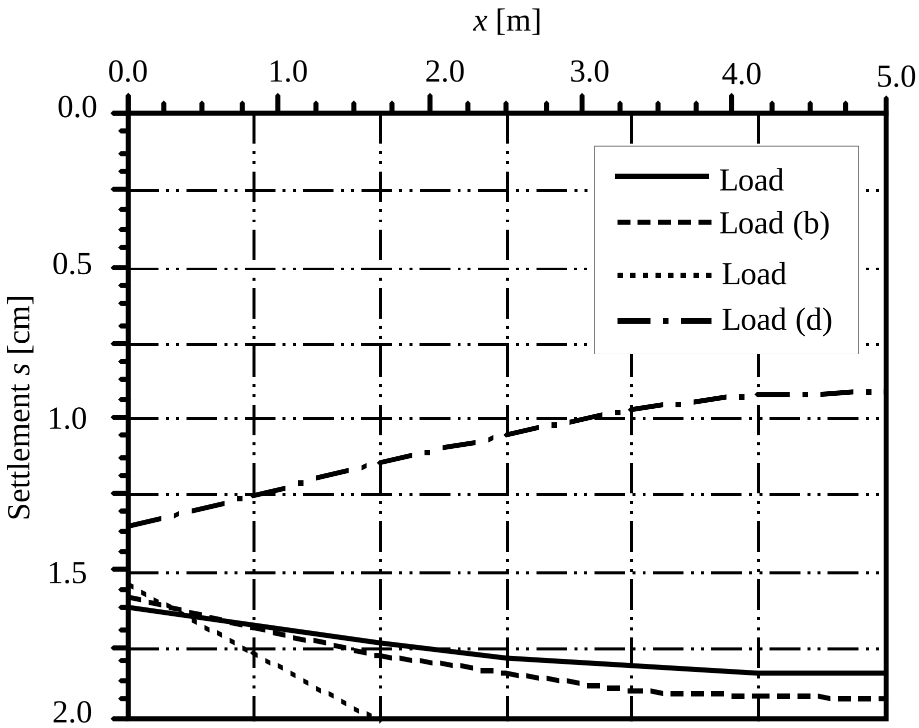


Figure 44 Settlement *s* [cm] at the middle section of the raft (method 5)

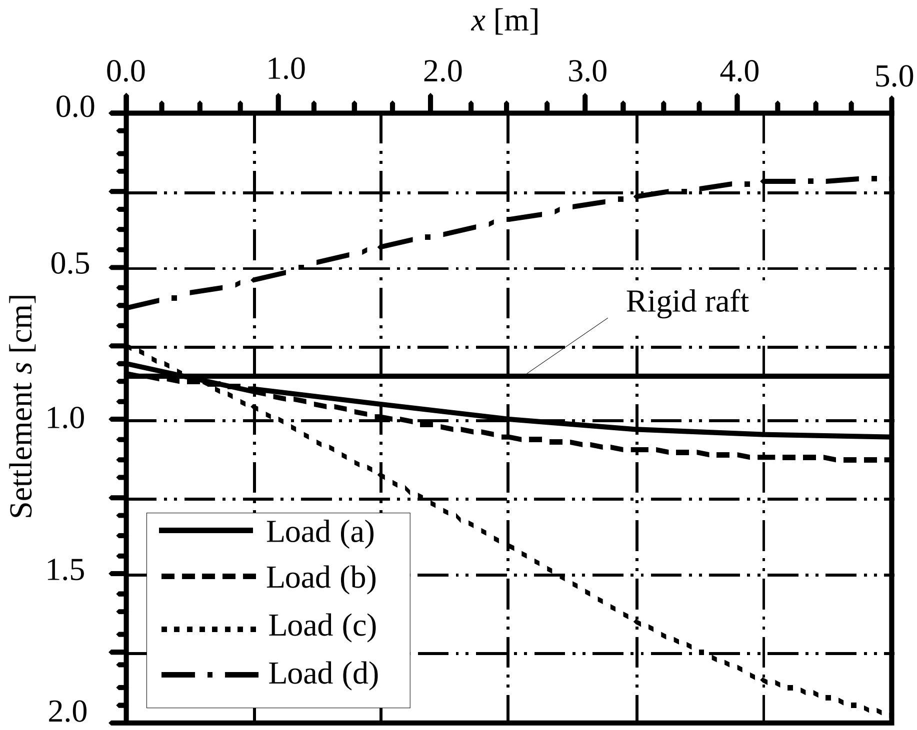


Figure 45 Settlement *s* [cm] at the middle section of the raft (methods 7 and 8)

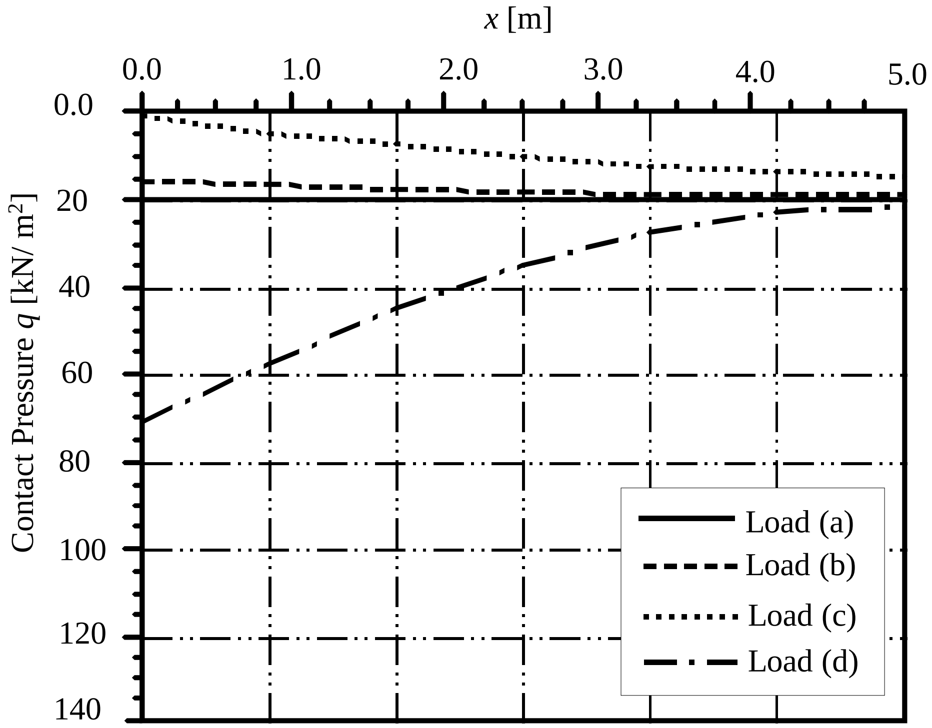


Figure 46 Contact pressure *q* [kN/m^2] at the raft edge (method 2)

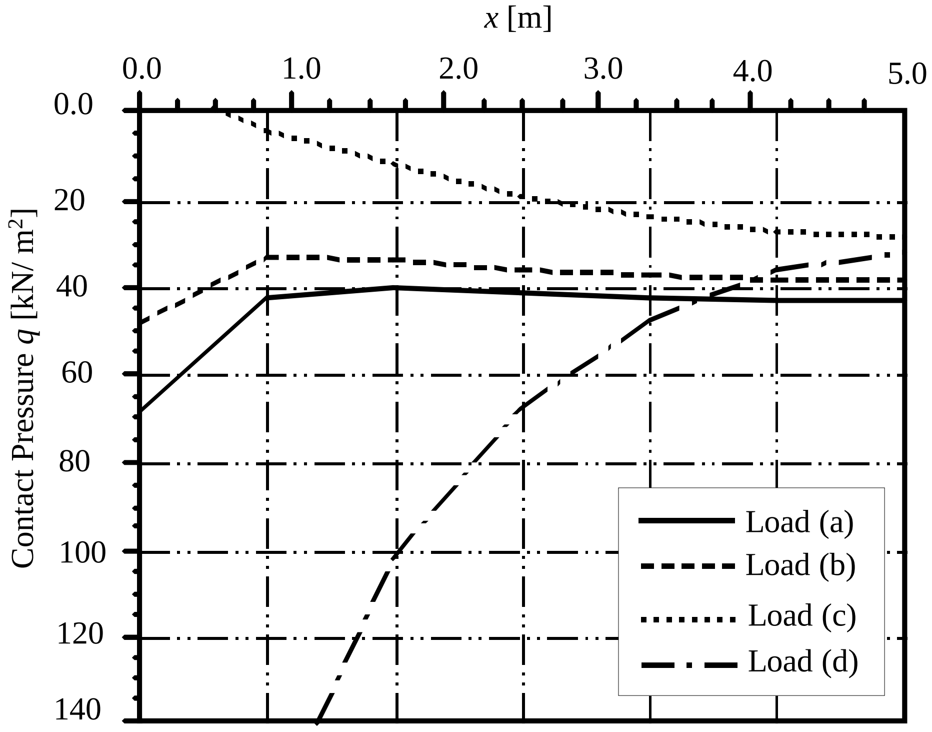


Figure 47 Contact pressure q [kN/m^2] at the raft edge (method 5)

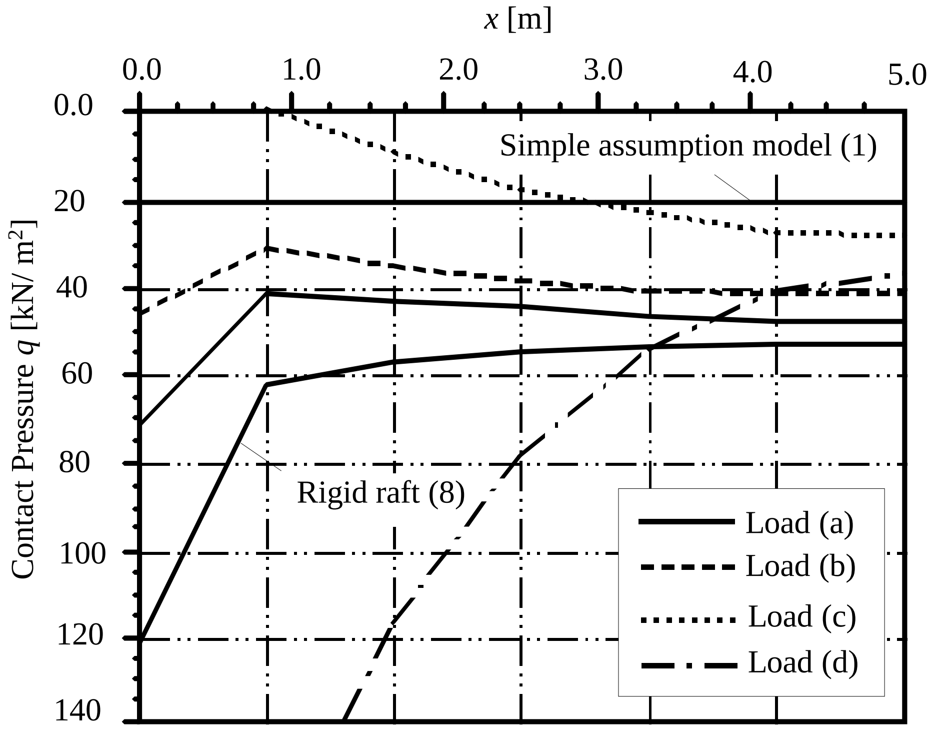


Figure 48 Contact pressure q [kN/m^2] at the raft edge (methods 1, 7 and 8)

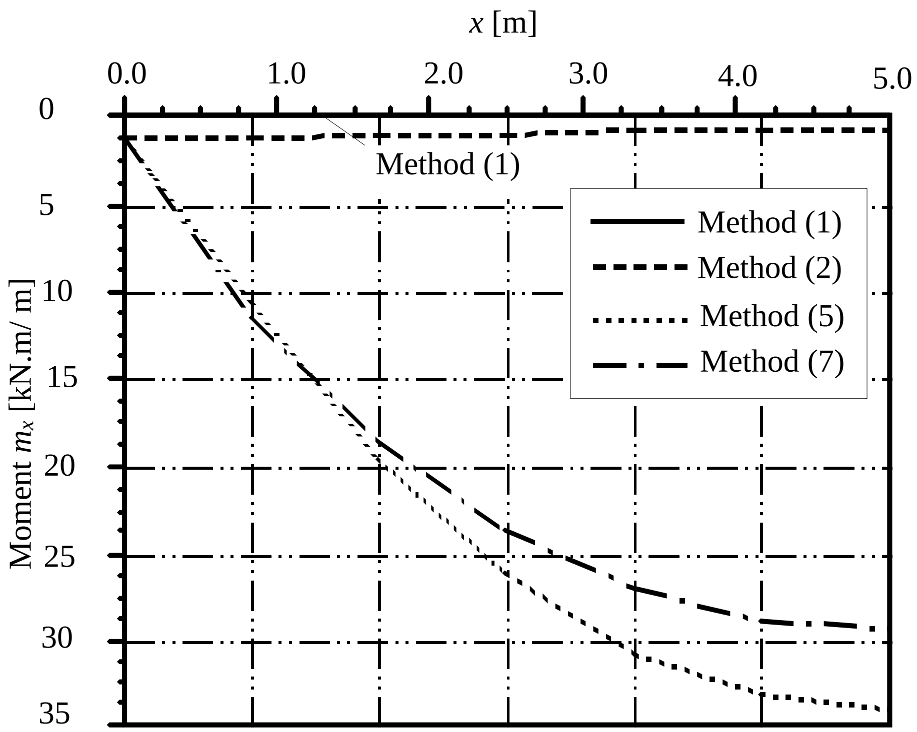


Figure 49 Moment m_x [kN.m/ m] at section III-III by application of different soil models, load geometry (a)

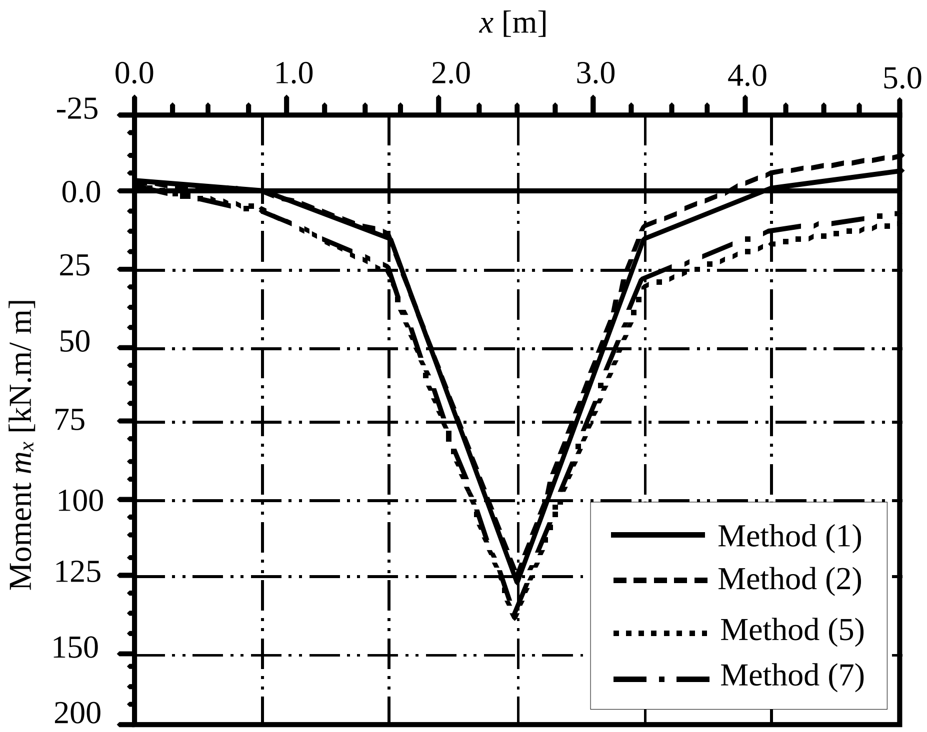


Figure 50 Moment m_x [kN.m/m] at section II-II by application of different soil models, load geometry (b)

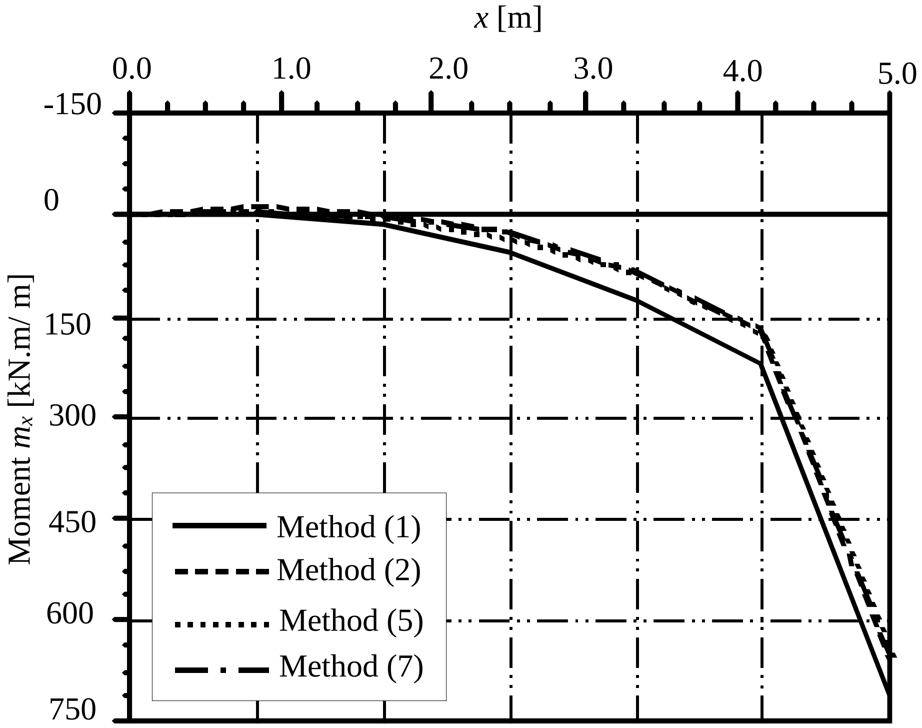


Figure 51 Moment m_x [kN.m/m] at section III-III by application of different soil models, load geometry (c)

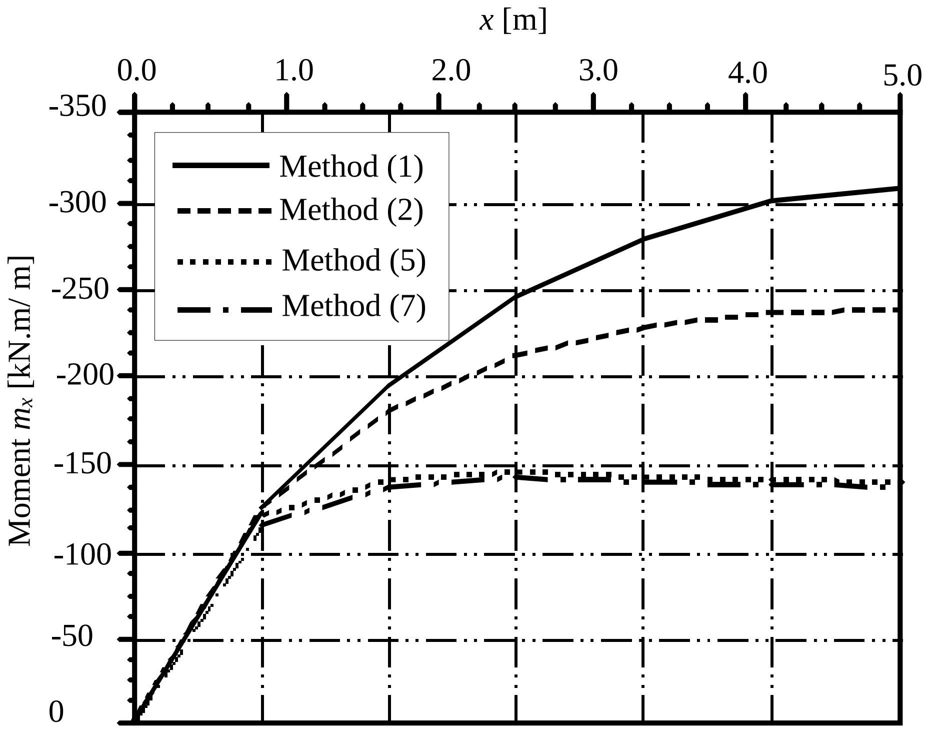


Figure 52 Moment m_x [kN.m/m] at section I-I by application of different soil models, load geometry (d)

Table 30 Maximum settlement *s* [cm] at critical nodes by applying calculation methods 2, 5, 7 and 8

Calculation method	Load geometry			
	(a)	(b)	(c)	(d)
Method 2	1.00 Center	1.08 center	1.96 center	3.57 corner
Method 5	1.86 Center	1.94 center	2.83 center	2.97 corner
Method 7	1.06 Center	1.12 center	1.97 center	2.20 corner
Method 8	0.85 all nodes	0.85 all nodes	0.85 all nodes	0.85 all nodes

Table 31 Maximum contact pressures *q* [kN/m^2] at critical nodes by applying calculation methods 1, 2, 5, 7 and 8

Calculation method	Load geometry			
	(a)	(b)	(c)	(d)
Method 1	20 all nodes	20 all nodes	20 all nodes	20 all nodes
Method 2	≈20 all nodes	22 center	39 center	71 corner
Method 5	68 corner	48 corner	51 center	360 corner
Method 7	71 corner	46 corner	58 center	442 corner
Method 8	121 corner	121 corner	121 corner	121 corner

7 Evaluation of the results

Settlement *s*

- Because the Simple assumption model (method 1) has no interaction between the soil and the raft, the soil settlement cannot be calculated

- For the elastic raft (methods 2, 5 and 7), the settlement distribution is concentrated near the external loads

- The Rigid raft (method 8) under the four types of external loads has a uniform settlement of s = 0.85 [cm] on the entire raft

- It is clear that the maximum differential settlement is due to load geometry (c) and the minimum is due to load geometry (a) for methods 2, 5 and 7, while for method 8 (Rigid raft) the settlement is uniform

- Isotropic elastic half-space (method 5) shows a higher settlement than that of method 7 due to the assumption of infinite thickness of the compressible soil layer by method 5

- The little difference between the results of both method 5 and that of method 7 is due to the compressible soil layer of this example is relatively thick ($z = L$)

Contact pressure *q*

- The Rigid raft (method 8) shows that the contact pressure is the same for the four types of external loads

- By load geometry (a), the Continuum model (methods 5, 7 and 8) shows that the distribution of the contact pressure is very different from that resulting of Simple assumption model (method 1) and *Winkler's* model (method 2)

- By load geometry (a), the distribution of the contact pressure by Simple assumption model (method 1) and *Winkler's* model (method 2) are nearly in agreement and equal to the applied load intensity 20 [kN/m^2] on the entire raft

- The Simple assumption model (method 1) for the four types of external loads has a uniform contact pressure of 20 [kN/m^2] on the entire raft

- For the methods 2, 5, 7 and 8, which have interaction between the soil and the raft, the values of contact pressure are different from a section to another

- For the elastic raft (methods 2, 5 and 7), the contact pressure is concentrated near the external loads

- For the elastic raft (methods 2, 5 and 7), the contact pressure near the load is higher for methods 5 and 7 than that for method 2

- The Continuum model (methods 5, 7 and 8) would predict contact pressure of infinite magnitude beneath the edges of the raft. Especially, if the raft is small or is loaded heavily at the middle

Moment *m*

- Applying methods 1 and 2 for analyzing load geometry (a) - uniform load on the raft -, gives also a uniform contact pressure. Therefore, there are no moments or shear forces on the raft. Thus, indicating the behavior of the raft by applying method 1 is similar to that of method 2 by this type of loading

- The high moment at the center of the raft by Continuum model (methods 5 and 7) is due to the high ordinates of the contact pressure distribution at the edge of the raft

- Figure 50 and Figure 51 show little difference between the results of moment by method 2 and that of methods 5 and 7 in case of load geometry (b) and (c), in spite of the contact pressure distribution is not the same for the three methods

- For load geometry (d), the maximum negative moment is small for higher values of contact pressure at the raft edges and high for smaller values of contact pressure at the raft edges. Therefore, the maximum negative moment for method 1 is higher than that of methods 2, 5 and 7

- It is clear that the maximum moment is due to load geometry (c) and the minimum is due to load geometry (a) for methods 1, 2, 5 and 7, while for method 8 (Rigid raft) the moment cannot be calculated

It can be concluded from the above comparisons that to be on the safe side, it is recommended to use the type of soil model for analysis of the raft according to the suitable case of study as shown in Table 32.

Table 32 Recommended soil model according to the suitable case

Case	Designing soil model
Uniform load on the entire raft	Continuum model (methods 5 or 7)
Edge loads	*Winkler's* model (method 2)
Small foundation	Simple assumption model (method 1) or *Winkler's* model (method 2)
Thin compressible soil layer over rigid base	*Winkler's* model (method 2)
Heavily loaded raft at the middle	Simple assumption model (method 1) or *Winkler's* model (method 2)
Influence of external foundation	Continuum model (methods 5,7 and 8)
Subsoil of different soil material	Continuum model (methods 7 or 8)
Influence of temperature change	Continuum model (methods 5 and 7) or *Winkler's* model (method 2)
Influence of the superstructure	Continuum model (methods 5 and 7) or *Winkler's* model (method 2)
Influence of tunneling or additional settlement	Continuum model (methods 5,7 and 8) or *Winkler's* model (2)
Very weak soil or a thick raft	Continuum model (method 8)
Infinite thickness of soil layer	Continuum model (method 5)