Example 24: Verifying deflection of a thin cantilever beam

1 Description of the problem

To verify the mathematical model of *ELPLA* for computing plane stresses, results of a cantilever beam having a thin rectangular cross section introduced by *Timoshenko/ Goodier* (1970), Example 21, page 41, are compared with those obtained by *ELPLA*. The cantilever carries a point load of P = 150 [kN] applied at the end as shown in Figure 59.

2 Cantilever dimensions

The cantilever has the following dimensions:

Cantilever length	L = 6.0	[m]
Cross section depth	h = 1.6	[m]
Cross section width	b = 0.2	[m]

3 Cantilever material

Material of the cantilever has the following parameters:

Young's modulus	E_b	= 2.0×10^7	[kN/m^2]
Poisson's ratio	ν_b	= 0.15	[-]
Unit weight	γ_b	= 0	[kN/m^3]

The self-weight of the cantilever is ignored.

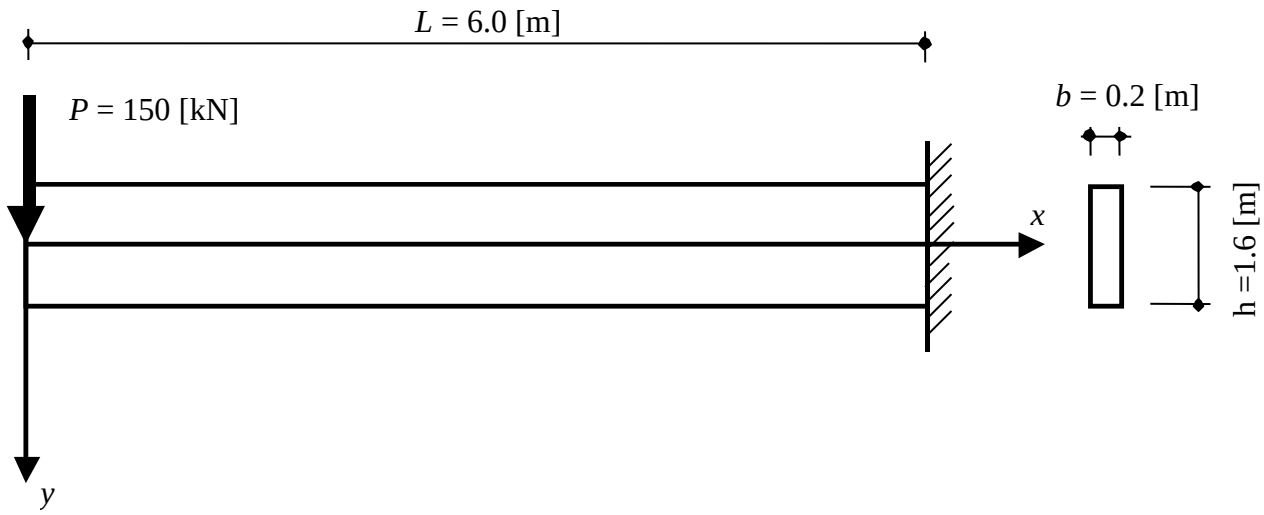

Figure 59 Cantilever beam loaded at the end

4 Analysis and Results

Because the cross section of the cantilever is thin, the cantilever may be considered as a plane stress problem. According to *Timoshenko*/ *Goodier* (1970), the equation of the deflection curve is expressed as:

$$(w)_{y=0} = \frac{Px^3}{6E_bI} - \frac{PL^2x}{2E_bI} + \frac{PL^3}{3E_bI} \qquad (19)$$

where:

w	Vertical deflection of the centerline of the cantilever [m]
x	Distance of deflection from the free end [m]
P	End load [kN]
E_b	*Young's* modulus of the cantilever material [kN/m^2]
L	Cantilever length [m]
I	Moment of inertia of the cantilever cross section [m^4]

Results of *ELPLA* are compared with the exact solution using Eq. 19 in Table 34. From this table, it can be noticed that results of deflection obtained by *ELPLA* are the same as those obtained from Eq. 19. A sufficient accuracy for results obtained by *ELPLA* may be considered at mesh size of 0.2 × 0.2 [m].

Table 34 Comparison of vertical deflection obtained by *ELPLA* and Eq. 19

Distance *x* [m]	Deflection obtained from Eq. 19	Deflection obtained by *ELPLA*		
		Mesh size		
		0.1 × 0.1 [m^2]	0.2 × 0.2 [m^2]	0.3 × 0.3 [m^2]
0	0.007910	0.008205	0.007895	0.007339
0.6	0.006728	0.006960	0.006709	0.006241
1.2	0.005569	0.005781	0.005572	0.005183
1.8	0.004457	0.004648	0.004480	0.004167
2.4	0.003417	0.003585	0.003455	0.003215
3	0.002472	0.002615	0.002521	0.002346
3.6	0.001645	0.001763	0.001699	0.001582
4.2	0.000961	0.001050	0.001013	0.000944
4.8	0.000443	0.000502	0.000484	0.000452
5.4	0.000115	0.000141	0.000136	0.000127
6	0.0000000	0.0000000	0.000000	0.0000000