Example 25: Verifying forces in piles of a piled group

1 Description of the problem

To verify the mathematical model of *ELPLA* for determining pile forces of pile groups under a pile cap, results of a pile group obtained by *Bakhoum* (1992), Example 5.19, page 592, are compared with those obtained by *ELPLA*.

A pile cap on 24 vertical piles is considered as shown in Figure 60. It is required to determine the force in each pile of the group due to a vertical load of N = 8000 [kN] acting on the pile cap with eccentricities e_x = 1.4 [m] and e_y = 1.8 [m] in both x- and y-directions.

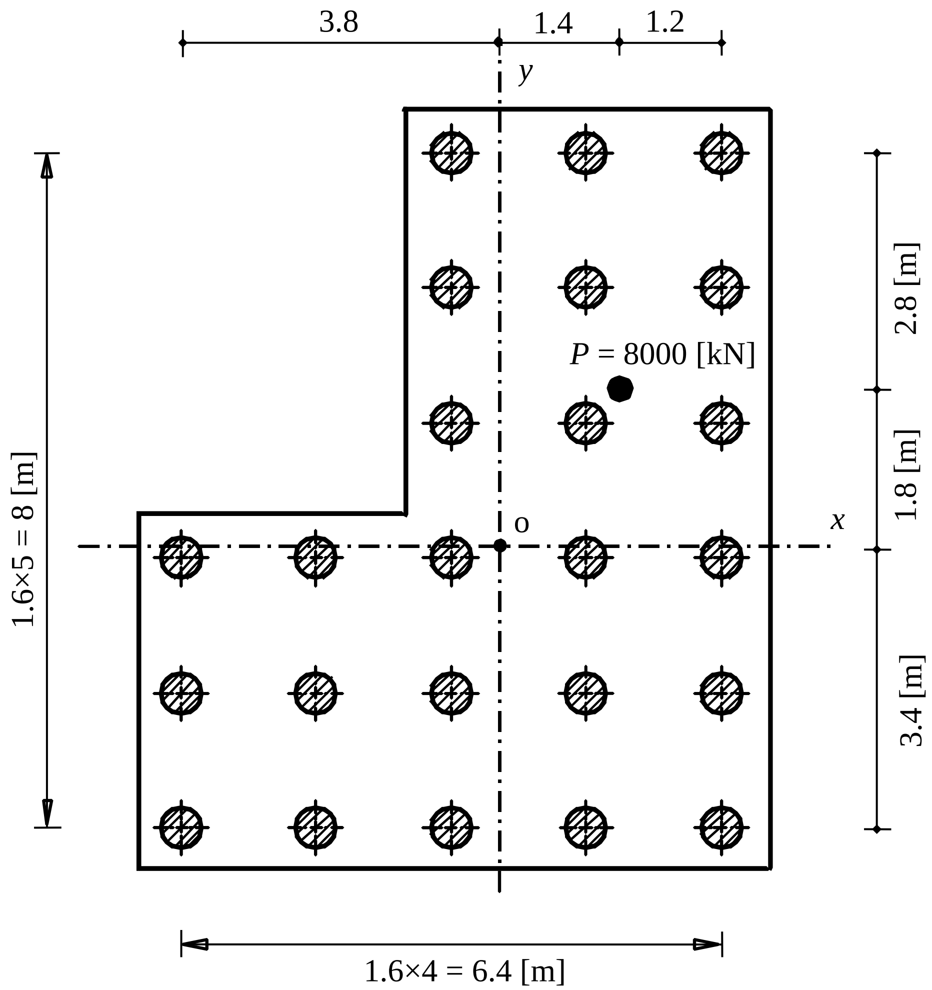

Figure 60 Pile cap dimensions and pile arrangements

The simple assumption model assumes a linear distribution of contact pressure on the base of the foundation. In general case of vertical piles under a pile cap forming linear contact forces, the force in any pile, analogous to *Navier's* solution, can be obtained from

$$P_i = \frac{N}{n} + \frac{M_y I_x \text{-} M_x I_{xy}}{I_x I_y \text{-} I_{xy}^2} x_i + \frac{M_x I_y \text{-} M_y I_{xy}}{I_x I_y \text{-} I_{xy}^2} y_i \qquad (20)$$

Where $I_x = \sum_1^n y_i^2$, $I_y = \sum_1^n x_i^2$ and $I_{xy} = \sum_1^n x_i y_i$

and:

P_i	Force in pile *i* [kN]
N	Sum of all vertical applied loads on the pile cap [kN]
x_i	Coordinate of pile *i* from the centroidal axis *x* [m]
y_i	Coordinate of pile *i* from the centroidal axis *y* [m]
M_x	Moment due to *N* about the *x*-axis, $M_x = N\, e_y$ [kN.m]
M_y	Moment due to *N* about the *y*-axis, $M_y = N\, e_x$ [kN.m]
e_x	Eccentricity measured from the centroidal axis *x* [m]
e_y	Eccentricity measured from the centroidal axis *y* [m]
n	Number of piles under the pile cap [-]

2 Hand calculation of pile forces

According to *Bakhoum* (1992), the force in each pile in the pile group can be obtained by hand calculation as follows:

Step 1: Compute moments

$M_x = 8000 \times 1.8 = 14400$ [kN.m]

$M_y = 8000 \times 1.4 = 11200$ [kN.m]

Step 2: Compute properties I_x, I_y and I_{xy}

Determining properties of I_x, I_y and I_{xy} are listed in 0.

Table 35 Properties I_x, I_y and I_{xy}

Pile number	x_i [m]	y_i [m]	x_i^2 [m^2]	y_i^2 [m^2]	$x_i\,y_i$ [m^2]
1	-3.8	-3.4	14.44	11.56	12.92
2	-2.2	-3.4	4.84	11.56	7.48
3	-0.6	-3.4	0.36	11.56	2.04
4	1.0	-3.4	1.00	11.56	-3.40
5	2.6	-3.4	6.76	11.56	-8.84
6	-3.8	-1.8	14.44	3.24	6.84
7	-2.2	-1.8	4.84	3.24	3.96
8	-0.6	-1.8	0.36	3.24	1.08
9	1.0	-1.8	1.00	3.24	-1.08
10	2.6	-1.8	6.76	3.24	-4.68
11	-3.8	-0.2	14.44	0.04	0.76
12	-2.2	-0.2	4.84	0.04	0.44
13	-0.6	-0.2	0.36	0.04	0.12
14	1.0	-0.2	1.00	0.04	-0.20
15	2.6	-0.2	6.76	0.04	-0.52
16	-0.6	1.4	0.36	1.96	-0.84
17	1.0	1.4	1.00	1.96	1.40
18	2.6	1.4	6.76	1.96	3.64
19	-0.6	3.0	0.36	9.00	-1.80
20	1.0	3.0	1.00	9.00	3.00
21	2.6	3.0	6.76	9.00	7.80
22	-0.6	4.6	0.36	21.16	-2.76
23	1.0	4.6	1.00	21.16	4.60
24	2.6	4.6	6.76	21.16	11.96
$\sum$			I_y = 106.56	I_x = 170.56	I_{xy} = 43.2

Step 3: Compute pile force

The force P_i in any pile i at location (x_i, y_i) from the geometry centroid is obtained from

$$P_i = \frac{N}{n} + \frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2} x_i + \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2} y_i$$

$$P_i = \frac{8000}{24} + \frac{(11200)(170.56) - (14400)(43.2)}{(170.56)(106.56) - (43.2)^2} x_i + \frac{(14400)(106.56) - (11200)(43.2)}{(170.56)(106.56) - (43.2)^2} y_i$$

$$P_i = 333.333 + 78.988 x_i + 64.421 y_i$$

3 Pile forces by *ELPLA*

The available method "Linear Contact pressure 1@ in *ELPLA* is used to determine the force in each pile in the pile group. A net of equal square elements is chosen. Each element has a side of 1.6 [m]. The pile forces obtained by *ELPLA* are compared with those obtained by *Bakhoum* (1992) in Table 36. It is obviously from this table that pile forces obtained by *ELPLA* are equal to those obtained by hand calculation.

Table 36 Comparison of pile forces obtained by *ELPLA* and Eq. 20

Pile number	*Bakhoum* (1992)						*ELPLA*
	x_i [m]	y_i [m]	N/n [kN]	78.988 x_i [kN]	64.421 y_i [kN]	P_i [kN]	P_i [kN]
1	-3.8	-3.4	333.33	-300.16	-219.03	185.86	-185.85
2	-2.2	-3.4	333.33	-173.77	-219.03	-59.47	-59.47
3	-0.6	-3.4	333.33	-47.39	-219.03	66.91	66.91
4	1.0	-3.4	333.33	78.99	-219.03	193.29	193.29
5	2.6	-3.4	333.33	205.37	-219.03	319.67	319.67
6	-3.8	-1.8	333.33	-300.16	-115.96	-82.79	-82.78
7	-2.2	-1.8	333.33	-173.77	-115.96	43.50	43.60
8	-0.6	-1.8	333.33	-47.39	-115.96	169.98	169.98
9	1.0	-1.8	333.33	78.99	-115.96	296.36	296.36
10	2.6	-1.8	333.33	205.37	-115.96	422.74	422.72
11	-3.8	-0.2	333.33	-300.16	-12.88	20.29	20.29
12	-2.2	-0.2	333.33	-173.77	-12.88	146.68	146.67
13	-0.6	-0.2	333.33	-47.39	-12.88	273.06	273.06
14	1.0	-0.2	333.33	78.99	-12.88	399.44	399.44
15	2.6	-0.2	333.33	205.37	-12.88	525.82	525.82
16	-0.6	1.4	333.33	-47.39	90.19	376.13	376.13
17	1.0	1.4	333.33	78.99	90.19	502.51	502.51
18	2.6	1.4	333.33	205.37	90.19	628.89	628.89
19	-0.6	3.0	333.33	-47.39	193.26	479.20	479.20
20	1.0	3.0	333.33	78.99	193.26	605.58	605.59
21	2.6	3.0	333.33	205.37	193.26	731.96	731.97
22	-0.6	4.6	333.33	-47.39	296.34	582.28	582.28
23	1.0	4.6	333.33	78.99	296.34	708.66	708.66
24	2.6	4.6	333.33	205.37	296.34	835.04	835.04