Example 4: Immediate settlement under a rectangular loaded area on layered subsoil

1 Description of the problem

To verify the mathematical model of ELPLA for computing the immediate (elastic) settlement under a rectangular loaded area on layered subsoil, the immediate settlement of saturated clay layers under a rectangular loaded area calculated by Graig (1978), Example 6.4, page 175, is compared with that obtained by ELPLA.

Janbu Bjerrum Kjaernsli (1956) presented a solution for the average settlement under an area carrying a uniform pressure \( q \) [kN/m\(^2\)] on the surface of a limited soil layer using dimensionless factors. Factors are determined for Poisson’s ratio equal to \( \nu_s = 0.5 \) [-]. The average vertical settlement \( s_a \) [m] is given by

\[
    s_a = \mu_0 \mu_1 qB \frac{E_s}{E_s}
\]

(5)

where:

- \( \mu_0 \), \( \mu_1 \) Coefficients for vertical displacement according to Janbu Bjerrum Kjaernsli (1956)
- \( E_s \) undrained modulus of the soil [kN/m\(^2\)]
- \( B \) lesser side of a rectangular area [m]
- \( q \) Load intensity [kN/m\(^2\)]

Eq. 5 can be used to estimate the immediate (elastic) settlement of loaded areas on saturated clays; such settlement occurs under undrained conditions. The principle of superposition can be used in cases of a number of soil layers each having a different undrained modulus \( E_s \).

A foundation 4 [m] \( \times \) 2 [m], carrying a uniform pressure of \( q = 150 \) [kN/m\(^2\)], is located at a depth of \( d_f = 1.0 \) [m] in a layer of clay 5.0 [m] thick for which the undrained modulus of the layer \( E_s \) is 40 [MN/m\(^2\)]. The layer is underlain by a second clay layer 8.0 [m] thick for which the undrained modulus of the layer \( E_s \) is 75 [MN/m\(^2\)]. A hard stratum lies below the second layer. A plan of the foundation with dimensions and FE-Net as well as a cross section through the soil under the foundation are presented in Figure 4. It is required to determine the average immediate settlement under the foundation.
Examples to verify and illustrate \textit{ELPLA}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{a) Cross section through the soil under the foundation \hspace{1cm} b) Plan of the foundation with dimensions and FE-Net}
\end{figure}

\section{Hand calculation of the immediate settlement}

According to \textit{Graig} (1978), the average immediate settlement under the foundation can be obtained by hand calculation as follows:

\textbf{Determination of the coefficient $\mu_0$}

\[
\frac{d_f}{B} = \frac{1}{2} = 0.5 [-] \text{ and } \frac{L}{B} = \frac{4}{2} = 2 [-]
\]
Examples to verify and illustrate ELPLA

From charts of *Janbu/ Bjerrum/ Kjaernsli* (1956)

\[ m_y0 = 0.9 \ [\text{-}] \]

a) Considering the upper clay layer, with \( E_s = 40 \ [\text{MN/m}^2] \) and thickness \( H = 4.0 \ [\text{m}] \)

\[
\frac{H}{H} = \frac{4}{2} = 2 \ [\text{-}] \text{ and } \frac{L}{R} = \frac{4}{2} = 2 \ [\text{-}]
\]

then \( m_y1 = 0.7 \ [\text{-}] \)

Hence from Eq. 5

\[ s_{a1} = 0.9 \times 0.7 \times \frac{150 \times 2}{40000} = 0.0047 \ [\text{m}] = 0.47 \ [\text{cm}] \]

b) Considering the two layers combined, with \( E_s = 75 \ [\text{MN/m}^2] \) and thickness \( H = 12.0 \ [\text{m}] \)

\[
\frac{H}{H} = \frac{12}{2} = 6 \ [\text{-}] \text{ and } \frac{L}{R} = \frac{4}{2} = 2 \ [\text{-}]
\]

then \( m_y1 = 0.9 \ [\text{-}] \)

Hence from Eq. 5

\[ s_{a2} = 0.9 \times 0.9 \times \frac{150 \times 2}{75000} = 0.0032 \ [\text{m}] = 0.32 \ [\text{cm}] \]

c) Considering the upper layer, with \( E_s = 75 \ [\text{MN/m}^2] \) and thickness \( H = 4.0 \ [\text{m}] \)

\[
\frac{H}{H} = \frac{4}{2} = 2 \ [\text{-}] \text{ and } \frac{L}{R} = \frac{4}{2} = 2 \ [\text{-}]
\]

then \( m_y1 = 0.7 \ [\text{-}] \)

Hence from Eq. 5

\[ s_{a3} = 0.9 \times 0.7 \times \frac{150 \times 2}{75000} = 0.0025 \ [\text{m}] = 0.25 \ [\text{cm}] \]

Hence, using the principle of superposition, the average immediate settlement \( s_a \) of the foundation is given by

\[ s_a = s_{a1} + s_{a2} - s_{a3} = 0.47 + 0.32 - 0.25 = 0.54 \ [\text{cm}] \]
Examples to verify and illustrate *ELPLA*

For rectangular flexible foundation the average settlement $s_a$ is equal to 0.85. Then, the central immediate settlement $s_c$ of the foundation is given by

$$s_c = \frac{1}{0.85} s_a = \frac{0.54}{0.85} = 0.64 \text{ [cm]}$$

Christian/ Carrier (1978) carried out a critical evaluation of the factors $\mu_0$ and $\mu_1$ of Janbu/ Bjerrum/ Kjaernsli (1956). The results are presented in a graphical form. The interpolated values of $\mu_0$ and $\mu_1$ from these graphs are given in Table 4. The average settlement $s_c$ according to this table is $s_c = 0.60 \text{ [cm]}$.

**Table 4** Factors $\mu_0$ and $\mu_1$ according to Christian/ Carrier (1978)

<table>
<thead>
<tr>
<th>Variation of $\mu_0$ with $d/B$</th>
<th>Variation of $\mu_1$ with $L/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d/B$</td>
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<td>--------</td>
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</tbody>
</table>

3 Immediate settlement by *ELPLA*

The available method "Flexible foundation 9" in *ELPLA* is used to determine the immediate settlement under the center of the foundation. A net of equal square elements is chosen. Each element has a side of 0.5 [m] as shown in Figure 4b. The immediate settlement obtained by *ELPLA* under the center of the raft is $s_c = 0.65 \text{ [cm]}$ and nearly equal to that obtained by hand calculation.