## Example 7: Consolidation settlement under a circular footing

### 1 Description of the problem

To verify the consolidation settlement calculated by *ELPLA*, the final consolidation settlement of a clay layer under a circular footing calculated by *Das* (1983), Example 6.3, page 371, is compared with that obtained by *ELPLA*.

A circular footing 2 [m] in diameter at a depth of 1.0 [m] below the ground surface is considered as shown in Figure 8. Water table is located at 1.5 [m] below the ground surface. The contact pressure under the footing is assumed to be uniformly distributed and equal to $q$ = 150 [kN/m$^2$]. A normally consolidated clay layer 5 [m] thick is located at a depth of 2.0 [m] below the ground surface. The soil profile is shown in Figure 8, while the soil properties are shown in Table 7. It is required to determine the final settlement under the center of the footing due to consolidation of the clay.

Table 7 Soil properties

| Layer No. | Type of Soil | Depth of the layer under the ground surface $z$ [m] | Unit weight of the soil $\gamma$ [kN/m$^3$] | Compression index $C_c$ [-] | Void ratio $e_o$ [-] |
|---|---|---|---|---|---|
| 1 | Sand | 1.5 | 17.00 | - | - |
| 2 | Sand | 2.0 | 9.19 | - | - |
| 3 | Clay | 7.0 | 8.69 | 0.16 | 0.85 |

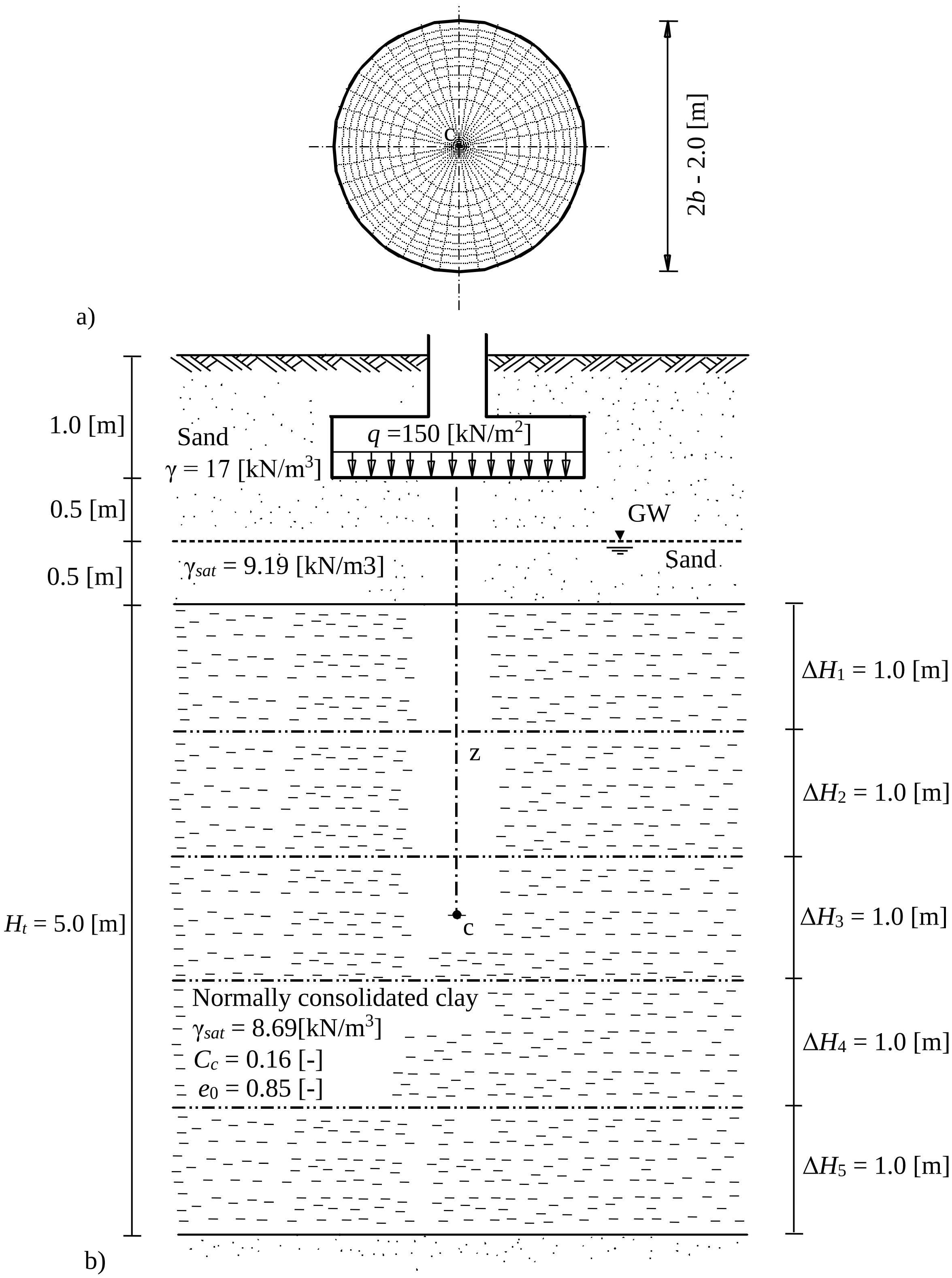


Figure 8 a) Plan of the footing with dimensions and FE-Net

b) Cross section through the soil under the footing

## 2 Hand calculation of consolidation

According to *Das* (1983), the consolidation of the clay layer can be obtained by hand calculation as follows:

The clay layer is thick relative to the dimensions of the footing. Therefore, the clay layer is divided into five layers each 1.0 [m] thick.

### Calculation of the effective stress $\sigma'_{o(i)}$

The effective stress $\sigma'_{o(1)}$ at the middle of the first layer is

$$\sigma'_{o(1)} = \gamma_1 z_1 + \gamma_2 z_2 + \gamma_3 \frac{\Delta H_1}{2}$$

$$\sigma'_{o(1)} = 17 \times 1.5 + 9.19 \times 0.5 + 8.69 \times \frac{1}{2} = 34.44 \text{ [kN/m}^2\text{]}$$

The effective stress $\sigma'_{o(2)}$ at the middle of the second layer is

$$\sigma'_{o(2)} = \sigma'_{o(1)} + \gamma_3 \left(\frac{\Delta H_1}{2} + \frac{\Delta H_2}{2}\right)$$

$$\sigma'_{o(2)} = 34.44 + 8.69 \left(\frac{1}{2} + \frac{1}{2}\right) = 43.13 \text{ [kN/m}^2\text{]}$$

Similarly

$$\sigma'_{o(3)} = 43.13 + 8.69 = 51.82 \text{ [kN/m}^2\text{]}$$

$$\sigma'_{o(4)} = 51.82 + 8.69 = 60.51 \text{ [kN/m}^2\text{]}$$

$$\sigma'_{o(5)} = 60.51 + 8.69 = 69.20 \text{ [kN/m}^2\text{]}$$

### Calculation of the increase of effective stress $\Delta\sigma'_i$

For a circular loaded area of radius *b* and load *q*, the increase of effective stress $\Delta\sigma'_i$ below the center at depth *z* is given by (*Das* (1983))

$$\Delta\sigma'_i = q\left(1 - \frac{1}{\left[\left(\frac{b}{z}\right)^2 + 1\right]^{3/2}}\right) \quad (9)$$

Hence

$$\Delta\sigma'_{1} = 150\left(1 - \frac{1}{\left[\left(\frac{1}{1.5}\right)^2 + 1\right]^{3/2}}\right) = 63.59\ [\mathrm{kN/m^2}]$$

$$\Delta\sigma'_{2} = 150\left(1 - \frac{1}{\left[\left(\frac{1}{2.5}\right)^2 + 1\right]^{3/2}}\right) = 29.93\ [\mathrm{kN/m^2}]$$

$$\Delta\sigma'_{3} = 150\left(1 - \frac{1}{\left[\left(\frac{1}{3.5}\right)^2 + 1\right]^{3/2}}\right) = 16.66\ [\mathrm{kN/m^2}]$$

$$\Delta\sigma'_{4} = 150\left(1 - \frac{1}{\left[\left(\frac{1}{4.5}\right)^2 + 1\right]^{3/2}}\right) = 10.46\ [\mathrm{kN/m^2}]$$

$$\Delta\sigma'_{5} = 150\left(1 - \frac{1}{\left[\left(\frac{1}{5.5}\right)^2 + 1\right]^{3/2}}\right) = 7.14\ [\mathrm{kN/m^2}]$$

**Calculation of consolidation settlement $s_c$**

The steps of the calculation of consolidation settlement $s_c$ are given in Table 8 and Figure 8.

Table 8 Steps of calculation of consolidation settlement $s_c$

| Layer No. | Layer thickness $\Delta H_i$ [m] | Effective stress $\sigma'_{o(i)}$ [kN/m$^2$] | Increase of effective stress $\Delta\sigma'_i$ [kN/m$^2$] | Decrease of void ratio $\Delta e_{(i)}$ [-] | Consolidation settlement $s_{c(i)}$ [m] |
|---|---|---|---|---|---|
| 1 | 1.0 | 34.44 | 63.59 | 0.07270 | 0.0393 |
| 2 | 1.0 | 43.13 | 29.93 | 0.03660 | 0.0198 |
| 3 | 1.0 | 51.82 | 16.66 | 0.01940 | 0.0105 |
| 4 | 1.0 | 60.51 | 10.46 | 0.01110 | 0.0060 |
| 5 | 1.0 | 69.20 | 7.14 | 0.00682 | 0.0037 |
| Total consolidation settlement Σ | | | | | 0.0793 |

In Table 8 the decrease of void ratio $\Delta e_{(i)}$ and the consolidation settlement $s_{c(i)}$ are given by

$$\Delta e_{(i)} = C_c \log\left(\frac{\sigma'_{0(i)} + \Delta\sigma'_{(i)}}{\sigma'_{0(i)}}\right) \qquad (10)$$

$$s_{c(i)} = \frac{\Delta e_{(i)}}{1 + e_0} \Delta H_i \qquad (11)$$

The total consolidation settlement obtained by hand calculation is

$$s_c = 0.0793 \text{ [m]} = 7.93 \text{ [cm]}$$

## 3 Consolidation by *ELPLA*

Taking advantage of the symmetry in shape and load geometry about both $x$- and $y$-axes, the analysis was carried out by considering only a quarter of the footing. The footing rests on two different soil layers. The first layer is sand of 2.0 [m] thickness, while the second layer is clay 5.0 [m] thick as shown in Figure 8. As it is required to determine the settlement due to the consolidation of the clay only, the settlement due to the sand can be eliminated by assuming very great value for modulus of compressibility of the sand $E_{s1}$. Consequently, the settlement due to the sand tends to zero. The settlement due to the sand becomes nearly equal to zero when for example $E_{s1} = 1 \times 10^{20}$ [kN/m$^2$]. *ELPLA* can consider the clay layer as a whole and calculate the consolidation settlement directly in terms of Compression index $C_c$ and Void ratio $e_o$. The contact pressure of the footing in this example is known. Also, the footing rigidity is not required. Therefore, the available method "Flexible foundation 9" in *ELPLA* may be used here to determine the consolidation of the clay. The effective stress $\sigma'_o$ and the increase of effective stress $\Delta\sigma'$ at mid-depth of the clay layer calculated in Table 8 can be also obtained by *ELPLA* through the option "Determination of limit depth", where the limit depth calculation is required to know the stress on soil against the depth under the foundation. The effective stress $\sigma'_o$ and increase of effective stress $\Delta\sigma'$ against depth obtained by *ELPLA* are plotted and compared with those obtained by hand calculation in Figure 8. The final consolidation settlement of the clay under the center of the footing obtained by the program *ELPLA* is $s_c$ = 8.09 [cm] and nearly equal to that obtained by hand calculation.

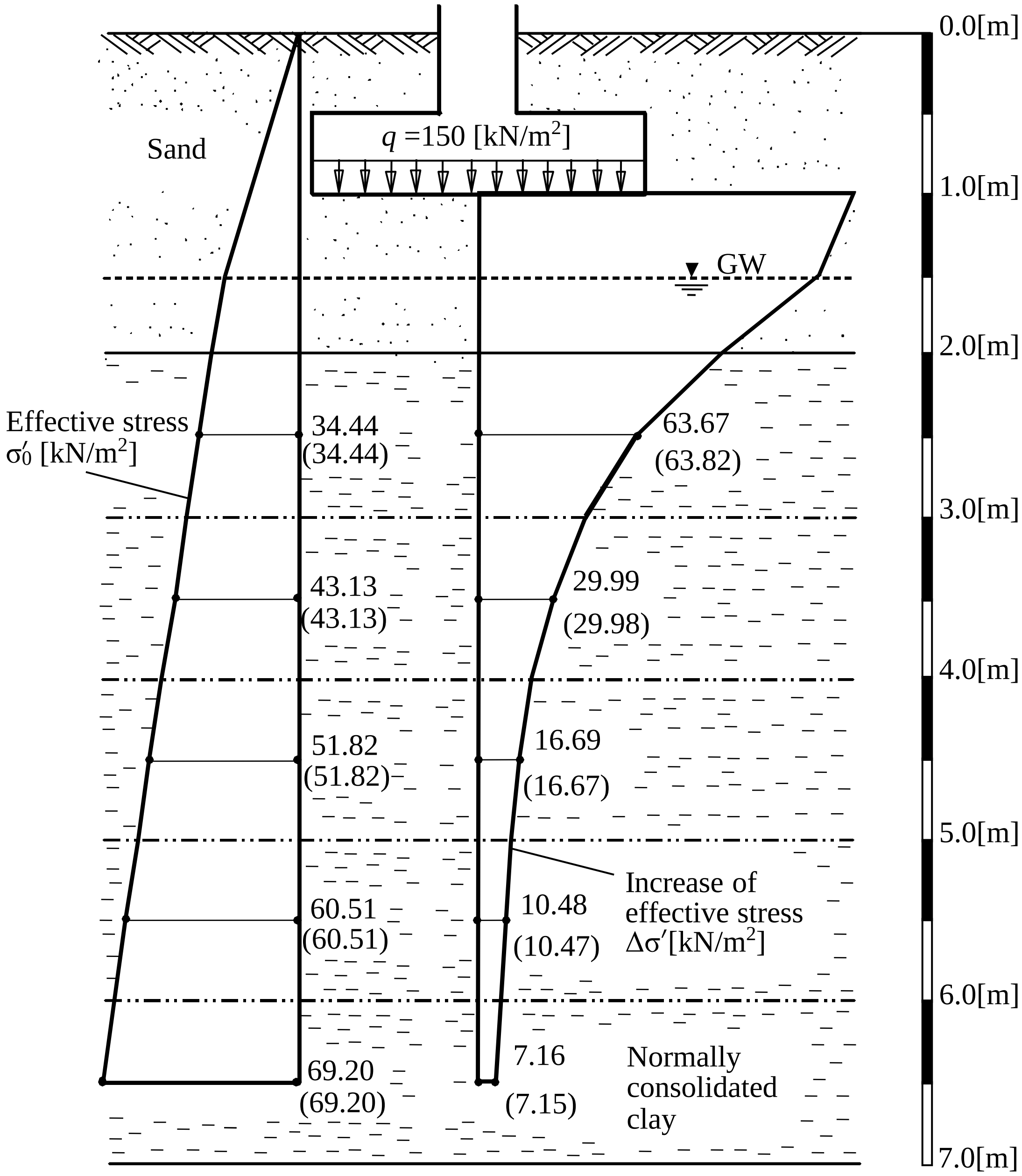


Figure 9 Effective stress $\sigma'_o$ [kN/m$^2$] and increase of effective stress $\Delta\sigma'$ [kN/m$^2$]

(Results of $\Delta\sigma'$ without brackets obtained from *ELPLA*
while with brackets obtained by hand calculation)